Torsional Vibrations in the Drive Train of Motor Vehicles Principle Considerations

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Today more and more attention is being paid to torsional drive line vibrations. For one thing, modern automotive design is producing stronger excitation. Secondly, drive trains that have been optimized for weight and efficiency are highly sensitive to excitation. This situation results in a number of comfort problems. Gear rattle and boom, particularly in low speed ranges that are desireable for fuel efficiency, pose an annoying problem. Besides this, tip-in/back-out reactions can take all the fun out of driving. These low-frequency torsional vibrations are also known by the descriptive term "Bonanza" or bucking bronco effect.

Traditionally, torsional vibrations have been studied primarily from the standpoint of reducing noise and improving comfort. However, one must also bear in mind that this kind of drive train vibration, and resonance in particular, can generate loads that far exceed the maximum engine torque. Rapid tip-ins generate short-term loads that may equal up to twice the engine torque. In such cases, the clutch can even slip temporarily. Given current emphasis on drive train weight optimization and cost reduction, avoiding these torque peaks will certainly become more important in the future [1].

Excitation of Torsional Vibration in Motor Vehicle Drive Trains

Many sources of excitation must be considered with respect to torsional vibrations. Figure 1 lists major causes, without claiming to be all-inclusive. The main source is engine irregularity resulting from the ignition cycle. This basic excitation in the motor vehicle drive train is the subject of almost all the following presentations. Irregular ignition or even misfiring can also lead to serious vibration problems, but these factors will not be treated in this series of presentations because they can usually be eliminated by optimizing ignition itself.

Driver-induced torque changes can generate the dreaded "Bonanza" effect.

ENGINE	IGNITION IRREGULAR IGNITION TORQUE CHANGE
CLUTCH	CHATTER
TORSION DAMPER	PERIODIC CHANGE IN DAMPING CHARACTERISTIC
TRANSMISSION	GEAR MESHING GEAR PITCH ERROR TORQUE SPIKES DUE TO SHIFTS
CRANKSHAFTS	BENDING ANGLE
TIRES	ROAD SURFACE CONDITION

Figure 1:
Sources of torsional vibration excitation in a motor vehicle drive train

A chattering clutch can also become an additional source of vibration. The physical principles affecting this phenomenon will be the subject of a separate presentation.

Axial displacement between the engine and the transmission input shaft can lead to additional radial loads that cause changes in the damping characteristic synchronous to the engine speed. This generates an associated excitation.

Gear meshing and gear pitch error in the transmission itself can also excite torsional vibrations [2]. In automatic transmissions, gear shifts can have an effect similar to a change in engine torque.

Drive shafts with bending angles can also be a source of excitation [3]. Irregular road surfaces are capable of exciting the drive train, as can tire slippage resulting from changes in friction coefficients between the tire and the road surface.

Engine Irregularity

The main source of excitation for torsional vibrations is discrete engine ignition [4]. It is also the primary cause of low-speed gear rattle and boom. The following discussion deals with the basic principles involved in this phenomenon.

Torque at the crankshaft is a periodic function of time or – to use the terminology favored by the engine specialists – a function of the crankshaft angle. This torque curve is plotted as a function of time and is determined by the gas forces in the cylinder, by the geometry of the crank drive, and by the acceleration moments of the crank drive and its changing mass moments of inertia during one rotation [5, 6].

Significant mass forces only occur at higher speeds, at which point they can even become dominant. Because gear rattle and boom occur primarily at low engine speeds, inertial forces are negligible for these observations. This results in the very simple torque relation shown in Figure 2 below.

According to this equation, the time-dependent torque acting on the crankshaft is the product of the piston displacement, a geometry factor dependent on the angle of the crankshaft and the pressure curve of the cylinder. The graph shown at the bottom of Figure 2 illustrates this principle for one cylinder. In a four stroke engine, this curve is repeated twice per revolution.

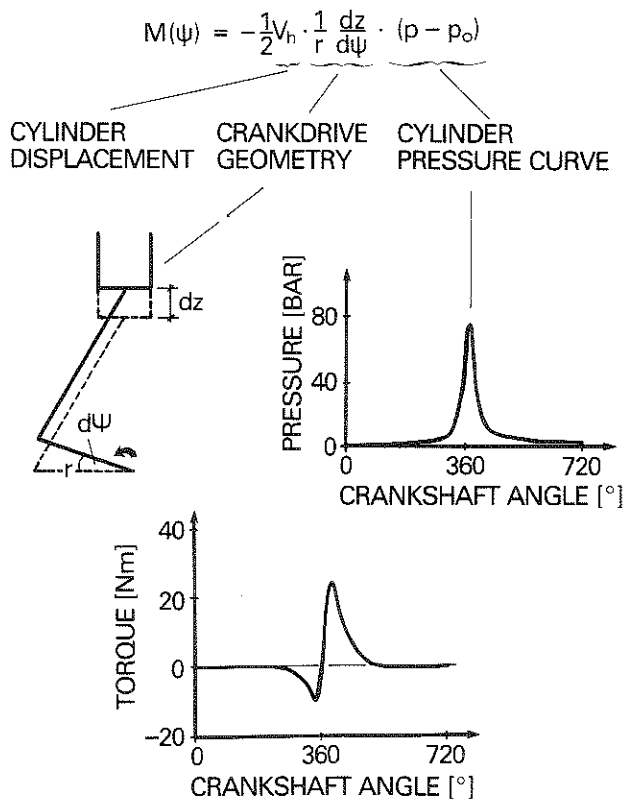

Figure 2:

Engine excitation due to gas forces

For four, five, six or eight cylinder engines, the characteristic curve for each individual cylinder as shown in Figure 2 must be superimposed according to the appropriate phase angle in order to obtain the total engine characteristic. As an example, Figure 3 shows the torque curves for a four, a six and an eight cylinder engine, each with identical piston displacements.

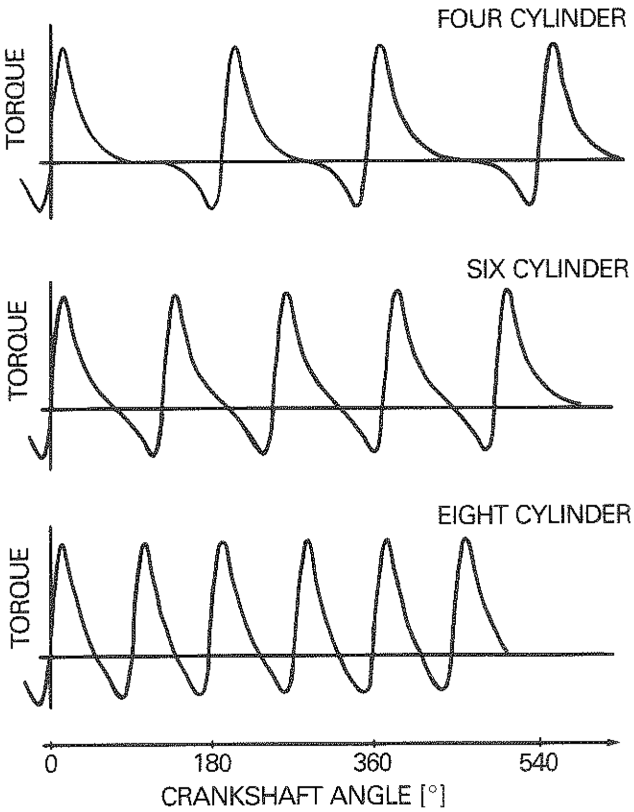

Figure 3:
Crankshaft torque due to gas forces

Engines with higher numbers of cylinders have several advantages. Total cylinder displacement is distributed over several cylinders. This reduces piston displacement and the resulting excitation per cylinder accordingly. Furthermore, excitation occurs at a higher frequency. As we will demonstrate later, this simplifies torsion damper tuning.

Figure 2 showed that the torque curve is proportionate to piston displacement per cylinder. It follows then that Figure 4 shows the measured values for the positive and negative torque peaks plotted with respect to the piston displacement of one cylinder operating under full load. The measurements were conducted in a speed range starting from about 1,500 rpm. Figure 4 includes data for both gasoline and diesel engines with four, six and eight cylinders. As one would expect from the equation shown in Figure 2, the measured values for full load actually do cluster around straight lines passing through the point of origin.

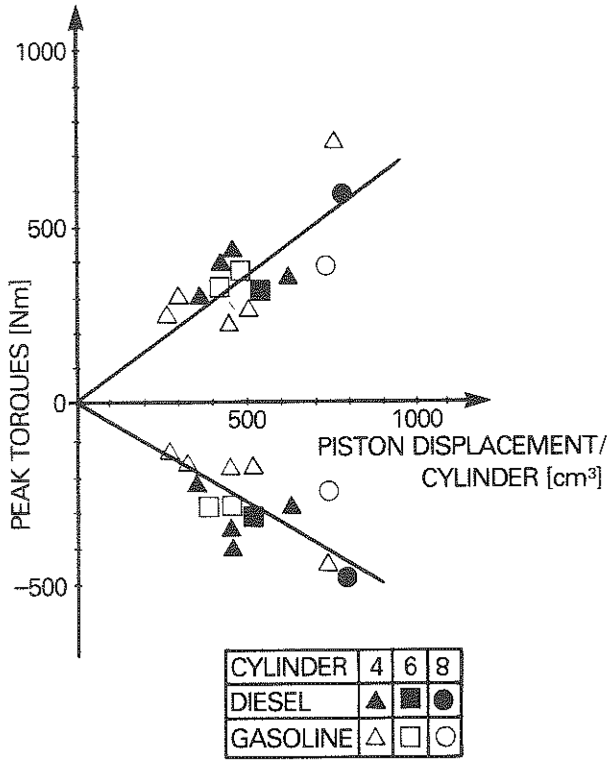

Figure 4:
Peak crankshaft torques
– full load –

Peak torque magnitude by itself does not reveal anything about engine irregularity. This value is determined based on the mass moment of inertia J_m of the crankshaft together with components such as the flywheel and the clutch, which are rigidly attached to it. The angular acceleration $\dot{\omega}_m$ is determined using the equation:

$$M = J_m \cdot \dot{\omega}_m$$

This value exhibits an almost constant amplitude at the low speeds we are interested in. The curve for the angular velocity or the speed as a function of time can be plotted using integration:

$$\omega_m = \int \dot{\omega}_m \, dt$$

Given the information summarized in Figure 4, we can predict engine excitation under full load with a fair degree of accuracy. Moreover, it is possible to make comparisons between different engines and their degrees of irregularity.

However, in idle mode, the torques acting on the crankshaft do not present such a uniform picture (Figure 5).

Only the torque spikes for diesel engines are proportionate to the piston displacement of a cylinder. In the case of gasoline engines, they are much lower and they do not reveal any clearcut relationship.

Comparison of Figures 4 and 5 reveals that excitation torques under full load are similar for gasoline and diesel engines, while significant differences appear in idle mode.

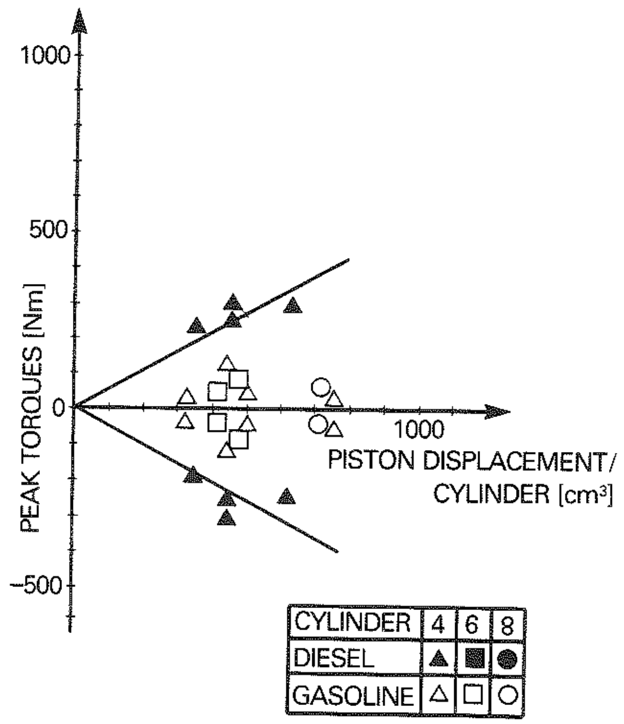

Figure 5:

Peak crankshaft torques – idle mode –

The Motor Vehicle Drive Train Viewed as a Torsional Vibration System

A motor vehicle drive train represents a torsional vibration system [7 – 10]. It can be roughly described as a chain of rotating inertias and torsion springs. Let us disregard the fact that the gear sets generate reaction forces on the transmission case, which is attached to the body via a more or less elastic suspension. Now we can even assume that we are dealing with a linear chain as shown in Figure 6. For purposes of clarity, we will try to get by with as few different rotating inertias and torsion springs as possible. Figure 6 illustrates the transition from a chain consisting of five rotating inertias (engine, transmission, differential, wheel and vehicle) to a chain having only three rotating inertias (engine, transmission and vehicle) [11].

This kind of simple model provides adequate information for many vibration problems. Of course, it is important to emphasize that it cannot be used to illustrate all drive train problems.

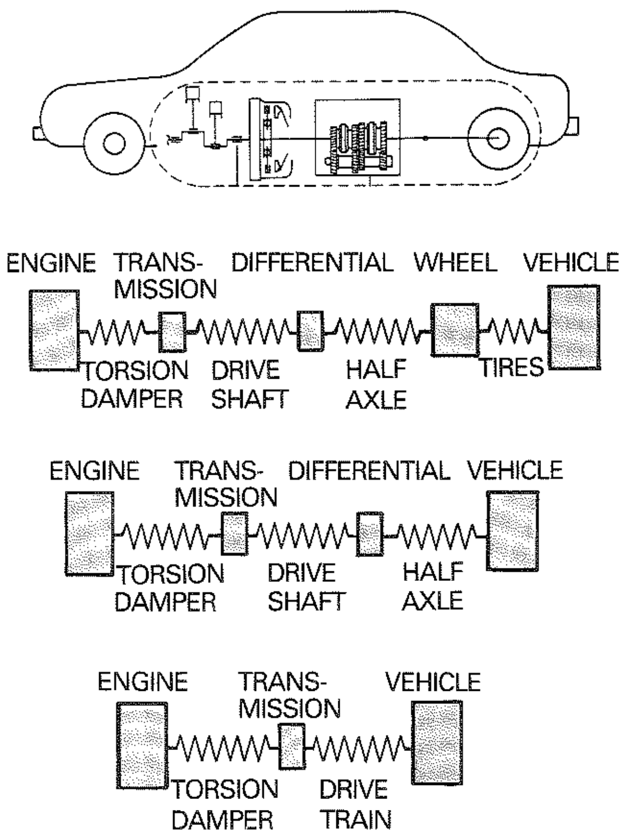

Figure 6:

Analytical model for vehicle drive train vibrations

Anyone who has ever performed vibration calculations will welcome a simple model. It is extremely difficult to come by the necessary data for rotating inertias and spring rates and to represent the results in an easy, understandable form.

This kind of procedure always poses the danger of over simplification and associated significant errors or even false assertions. Consequently, the formulation of a model must always be accompanied by actually measuring torsional vibrations.

Figure 7 shows the possible vibration modes for a simple three-mass vibration system [12].

During the "surging" vibration mode, the engine vibrates essentially in opposition to the vehicle. The transmission is close to a vibration node and consequently only vibrates at a low amplitude. This vibration mode occurs in conjunction with vehicle surging, for instance during tip-in. Depending on the vehicle model and the selected gear, natural frequencies associated with surging range from 2 – 5 Hz, occasionally even higher.

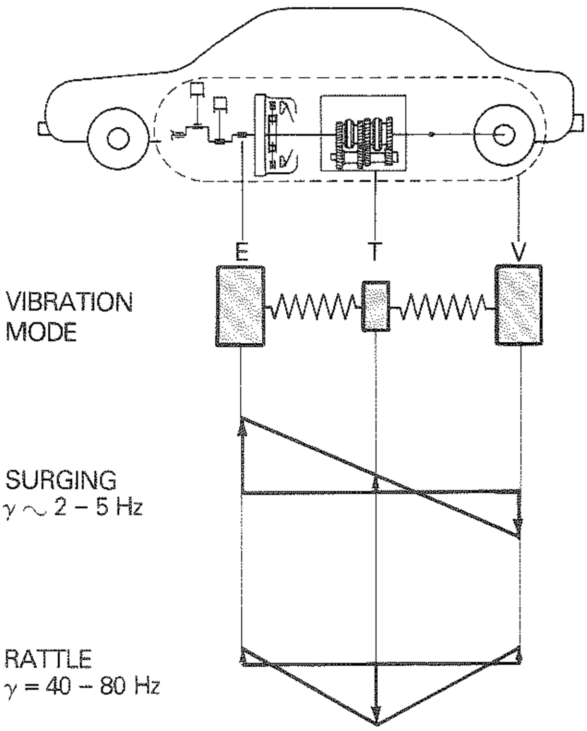

Figure 7:

Vibration modes for a three-mass vibration system

In the case of "rattle," the transmission vibrates at high amplitudes. Natural frequencies of 40 – 80 Hz are typical. This vibration mode occurs in the case of gear rattle.

Resonance Speed

Whenever engine irregularity excites the drive train vibration system, resonance can occur if the excitation frequency equals the natural frequency.

Figure 8 shows the resonance curve for a single inertia vibration system, with its typical resonance peak. Excitation frequencies are proportional to speed. Consequently, the graphing format shown in Figure 9, where excitation frequency is shown as a function of speed, is appropriate for our purposes. Because four cylinder engines feature two ignition cycles per revolution, they primarily excite vibrations of the 2nd order. This 2nd order vibration is represented in the heavy straight line passing through the origin.

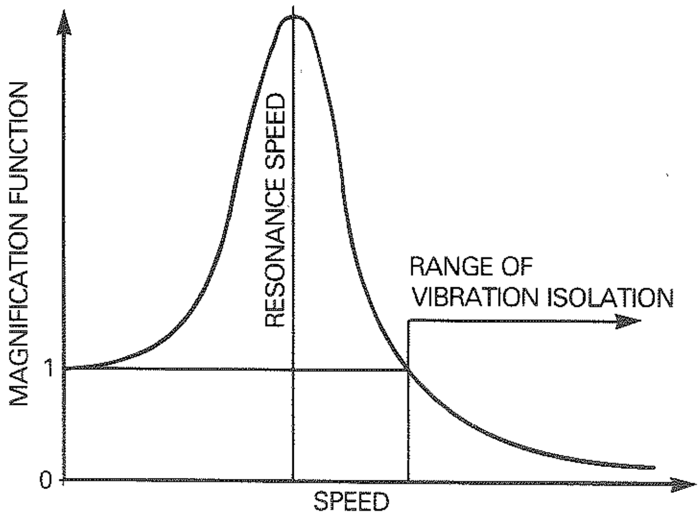

Figure 8 Resonance curve for simple oscillator

Because engine excitation is not sinusoidal, additional high frequency components must occur in the excitation, but their intensity generally decreases with increased frequency. High frequency components also exhibit a fixed ratio to the engine speed. The 4th and 6th order vibrations are plotted on the graph as well. In the event of irregular ignition, the 1st or 0.5th order can also occur. All these orders are represented by straight lines with different slopes.

Figure 9 also shows the natural frequencies for a passenger car for "rattle" and "surging" in gears 1 – 5. Of course, since the natural frequencies are not dependent on speed, they result in horizontal lines.

Resonance can – but does not have to – occur at each intersection point. The numerous possible resonance points provide a picture of complex drive train vibration behavior.

Rattle vibration is excited primarily by the 2nd and the 4th order. Associated resonance speeds typically occur in the range between about 700 and 2,000 rpm. The lower the selected gear, the higher these speeds. Surging vibration can actually only be excited by engine irregularity of the 0.5th order and then only in higher gears. Excitation in this vibration mode usually occurs as broad-band excitation resulting from a tip-in/back-out jerk.

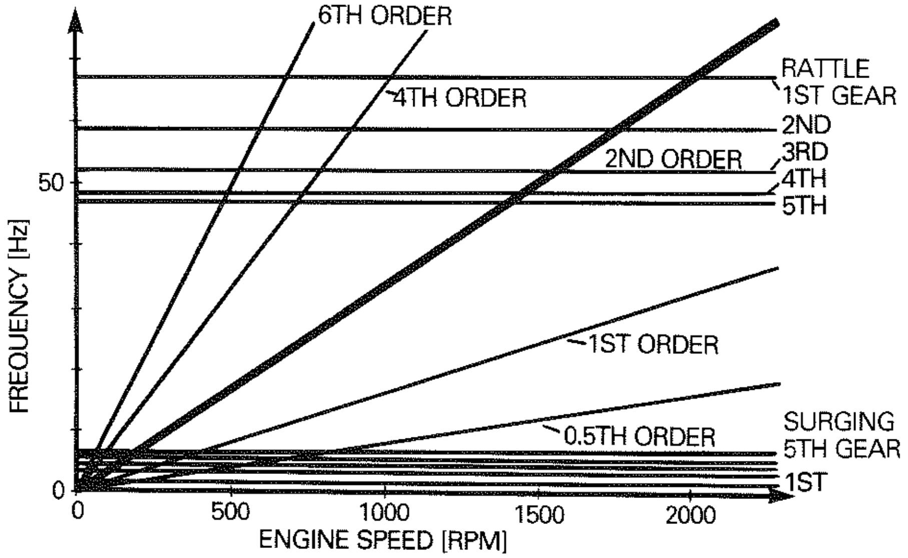

Figure 9 Excitation frequencies as a function of engine speed for a four cylinder engine

Isolation of Torsional Vibration

In theory, there are several options for modifying the vibration performance of a motor vehicle drive train in order to reduce undesirable torsional vibrations. Unfortunately, however, compelling engineering considerations prevent us from changing most of the components making up the drive train. Therefore, with the exception of the flywheel mass, it is hardly possible to change the mass moments of inertia involved in the system. Nor is it readily feasible to alter the spring rates (stiffness) of the tires, half axles and drive shafts. Consequently, as shown in Figure 10, the only actual option left to target for positive change is the connection between the engine and the transmission.

An appropriately damped, torsionally elastic coupling – in other words, a torsion damper – can be used to shift the resonance speeds, damp resonance amplitudes, and in some cases, even achieve genuine vibration isolation.

Generally speaking, gear rattle in drive, coast and idle modes requires different torsional elasticity and damping characteristics. Consequently, most torsion dampers include precisely tuned multi-stage characteristic curves, with each stage optimized individually for their respective load ranges (Figure 10).

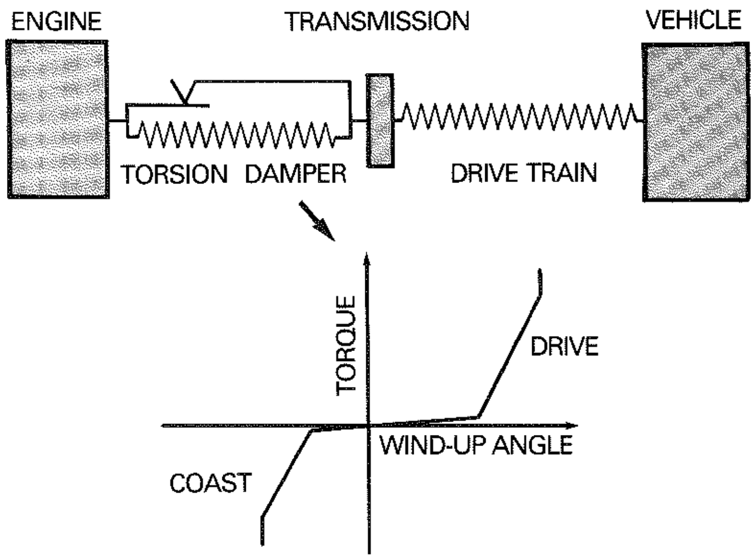

Figure 10 Torsion damper characteristic curve in analytical model

Based on a calculated example, Figure 11 illustrates how vibration performance can be influenced by changing torsional spring rate and damping. The individual sections of the matrix graph each show the vibration amplitude of the engine and the transmission speed plotted as a function of the mean engine speed. Engine speed is plotted as a solid line, and transmission speed as a broken line. Each section was based on the calculation for a torsion damper with different frictional hysteresis and torsional spring rate. The torsional spring rate increases along the horizontal axis, and hysteresis increases on the vertical axis. The graph in the upper right-hand section reveals that at a torsional spring rate of 20 Nm/° without frictional hysteresis, we encounter several high amplitude resonance points. At about 1,600 rpm there is a 2nd order resonance caused by the main engine excitation. The 4th order resonance occurs at about 800 rpm, which is attributable to the first engine harmonic. An additional 2nd order resonance occurs between these two values, but is associated with a different vibration form. Vibration isolation does not occur until we reach the speed range above natural frequency in the 2,000 rpm range.

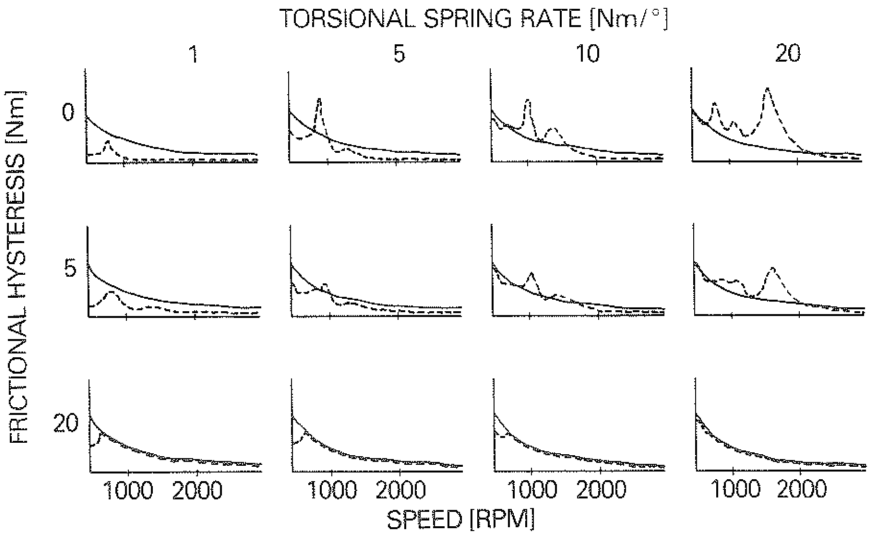

Figure 11 Effect of torsional spring rate and frictional hysteresis on the resonance curve of a four-cylinder passenger car engine

If we reduce the torsional spring rate, we can shift the resonances to lower speeds and introduce the vibration isolation range sooner. At very low torsional spring rates of about 1 Nm/°, the vibration amplitude also decreases significantly in the resonance range. With this design, the very flexible coupling hardly passes any vibrations at all through to the rest of the drive train. This would constitute the ideal torsion damper. Unfortunately, however, it is impossible to accommodate this kind of flat spring rate in the space available for installing the clutch.

Modern long-travel clutch disc designs such as those being developed at LuK can achieve about 10 Nm/° in passenger car engines with moderate torque ratings. Consequently, we must reduce resonance amplitude by introducing carefully defined damping. And this is achieved at the cost of vibration isolation in the range above natural frequency (Figure 11, bottom). With high hysteresis values, the torsion damper exhibits increasingly rigid performance.

The matrix in Figure 11 clearly shows that it is impossible to achieve vibration isolation in drive mode over the entire speed range using an elastic coupling such as a conventional torsion damper. As a result, torsion damper tuning always represents a compromise. Low hysteresis results in high resonance amplitudes at low speeds and good vibration isolation at high speeds, whereas high hysteresis leads to rigid performance. If a satisfactory compromise proves impossible, other vibration damping procedures must be adopted, such as a dual mass flywheel, a torque control isolation system (TCI) or a hydrodynamic coupling.

Torsion Measurement System

It has already been pointed out that modelling drive train vibration analysis poses the risk of incorrect projections. Sometimes systems are oversimplified when defining the analytical model, or mass moments of inertia, spring rates and damping factors do not accurately reflect the system. For instance, transmission damping factors cannot be determined directly at all, but have to be established by comparing vibration measurement with vibration calculations. The resulting damping factor must be varied in the calculations until the calculated results agree with measured results with respect to amplitude and phase. The damping value derived in this way is then introduced into the final calculation.

Therefore it is very important to measure torsional vibrations. Calculation and measurement go hand in hand and are mutually interdependent.

For this reason, LuK has for many years considered it essential to have access to appropriate modelling procedures and measurement capability as well. It is particularly important that measuring equipment be easy to use and produce quick results because tuning vehicles are generally only available for brief periods.

Because no appropriate measuring equipment was available off the shelf, LuK has worked together with AFT, a LuK affiliate, to develop a mobile torsional vibration measuring computer. This system uses simple sensors and very user-friendly software and provides immediate output of almost all important information needed for analysis of torsional vibration systems.

Figure 12 shows a block diagram of this measuring computer. The standard measurement is recorded using special magnetoresistive sensors that recognize rotational direction as well as vibration. These sensors provide digital measurement of the time elapsed between successive gear teeth on the flywheel and in the transmission. It is also possible to implement other measuring points, such as on the differential gear.

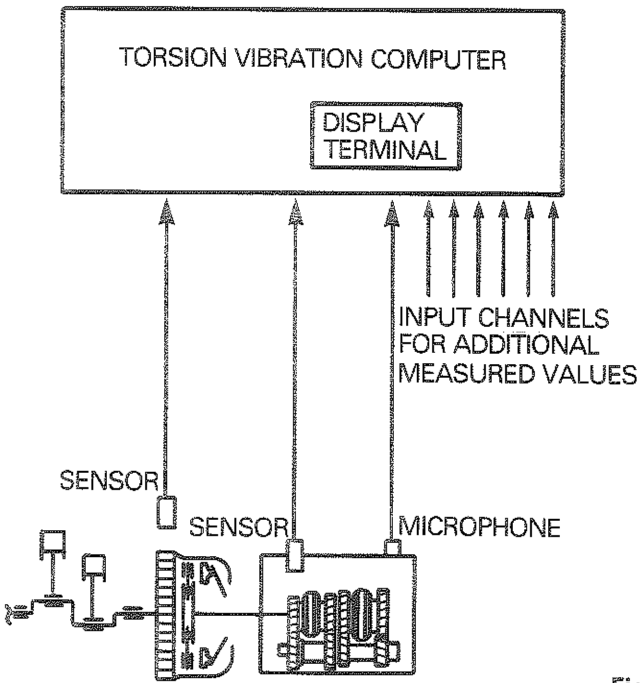

Figure 12 Measuring computer

A sensor measures structure-borne noise at an appropriate point on the transmission. A custom-designed electronic evaluation circuit provides high time resolution of structure-borne noise intensity.

Additional input channels can be used to gather data on temperatures, acceleration, throttle position, clutch travel, etc. High resolution, high scanning rate analog-digital converters are used for this purpose.

Measured values can be checked directly on the computer display in the vehicle. A 20 MB hard disk drive stores data for further evaluation.

After speed data has been collected in the vehicle, they are processed at a stationary work station. This routine produces the angular acceleration values that are critical for calculation purposes and calculates the engine and transmission vibration angles that are superimposed on the average speed.

LuK has developed an evaluation algorithm for determining the relative torsional angle between the engine and the transmission. This program provides precise data on torsion damper movement and function. Consequently, in most cases it is possible to get by without measuring drive train torque.

All measured and calculated curves can be displayed synchronous to noise intensity and the output from other AD channels.

The following examples illustrate the system's capability.

Sample Applications for the Torsion Measuring Computer

Figure 13 shows a measurement made in idle mode.

At the beginning of the measurement interval, the torsion damper vibrates in the idle stage with good vibration isolation and no gear rattle. During measurement, the test driver pulls slightly on the gear shift lever in order to increase transmission drag torque. As a result, the torsion damper operating point migrates into the steep main stage. This eliminates the good vibration isolation and the transmission starts to rattle. As shown in the measured noise intensity curves, precisely one noise impulse is generated on the positive flank of the transmission speed curve for each engine excitation oscillation. In this case, the noise is generated while the input shaft accelerates after a slow-down phase. During this slow-down phase, the free-wheeling gears accelerate ahead of the input shaft.

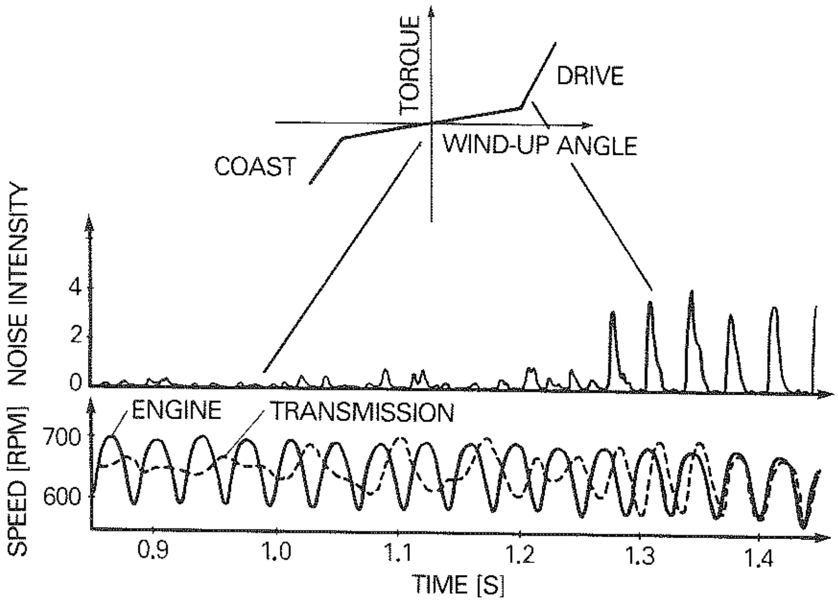

Figure 13 Changes in vibration behavior during idle mode as a result of shift lever loading

Figure 14 shows a torsional vibration measurement in drive mode under full load for a four-cylinder gasoline engine.

The graph is divided into four sections, representing the speed range 850 to 2,500 rpm. First we can clearly see the 4th order resonance at 1,000 rpm, at which point the transmission input shaft vibrates at twice the basic combustion frequency. At about 2,000 rpm, the 2nd order is extremely evident. In the intermediate resonance range, peak vibrations are somewhat lower at the transmission input shaft. As has been confirmed in many tests, gear rattle increases as angular acceleration at the transmission input shaft increases. For this reason, both resonance points can also be recognized subjectively in the form of pronounced rattle.

Figure 15 shows a tip-in/back-out procedure in which torsion damper torsional rotation was plotted throughout the measuring cycle.

First the driver steps down abruptly on the gas pedal. He accelerates from about 1,200 rpm to 3,200 rpm, then lets up abruptly on the gas, and the coast phase begins (bottom of Figure 15).

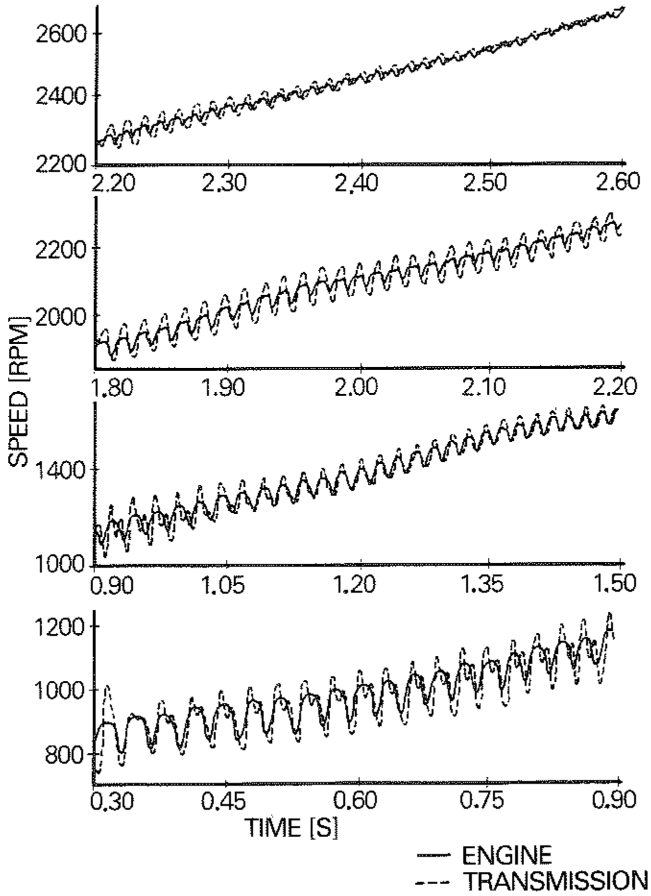

Figure 14:

Torsional vibrations in drive mode at speeds from 850 to 2600 rpm

At the beginning of the measurement cycle, the measuring computer sets the torsion angle equal to zero. Starting from this point, all changes in torsion damper position occurring during the measuring period are calculated. In the example shown in Figure 15, the torsion damper winds about 7° in the drive direction when the driver steps on the gas. During acceleration, it turns an additional 3° as speed and, with it, engine torque, increase. Once the acceleration process is completed, the torsion damper returns to its initial position after a few surge vibrations.

Changes in the torsion damper wind-up angle can be determined with an accuracy of about 0.1°. This procedure uses only the information generated by the two speed sensors and is therefore as simple as possible. Complex torque measurements are usually superfluous because the torsion damper wind-up angle is directly related to a torque.

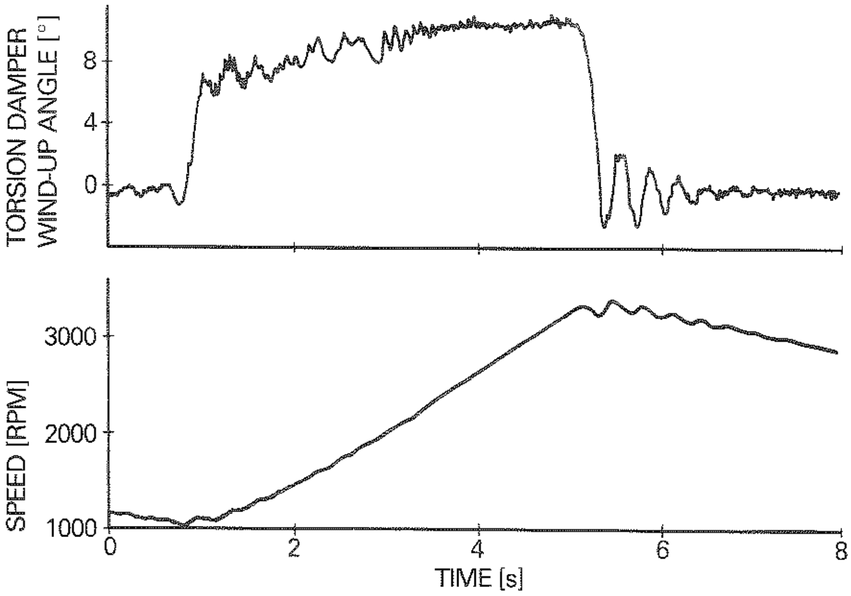

Figure 15 Tip-in/back-out

The Interaction between Measurement and Calculation during Tuning

Optimum torsion damper tuning requires mutual interaction of vibration measurement and calculation, as noted above. If we worked with only one of these procedures, the danger of incorrect interpretation would be too great. Measured data supports the vibration model selected. Without calculation, changes in the system parameters can never be systematically studied. Without this calculation procedure, we would probably have never been able to develop the dual mass flywheel.

The interaction between measurement and calculation during torsion damper tuning will be explained using the following examples:

A clutch disc with a well-functioning idle stage damper was installed in a passenger car. Figure 16 shows the schematic functional characteristic. However, as is frequently the case with idle-stage dampers, a short, annoying shut-off rattle occurs when the engine is shut off. LuK was supposed to improve shut-off performance without causing any deterioration in the idle stage.

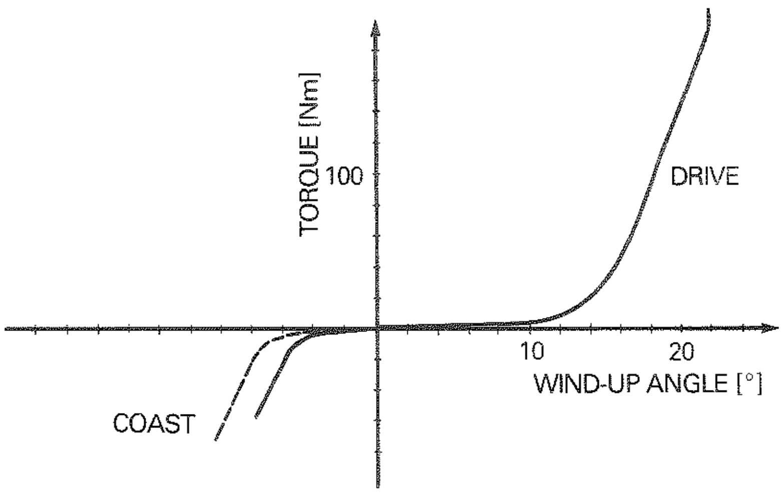

Figure 16 Idle stage torsion damper characteristics

Initial measurement of torsional vibration behavior (Figure 17) resulted in a series of important findings. First of all, speed measurement (Graph Section C) exhibits excellent isolation at idle speed. In other words, the idle stage damper is functioning well. When the ignition is shut off, the engine decelerates at a constant rate. However, it does not stop abruptly at 0 speed, but instead continues to vibrate in the opposite direction. The measuring computer detects and displays this phenomenon. Based on the inertia of the engine, the flywheel and the clutch, we can assume a constant time-delay moment of -30 Nm, which is generated by internal engine friction.

The good vibration isolation in idle deteriorates below a speed of about 600 rpm and even turns into resonance. The transmission virtually vibrates simultaneously with the final engine vibrations. It is obvious to associate this resonance with the short transmission rattle. Noise measurement recorded in the top graph section substantiates this assumption.

It is interesting to note that strong noise peaks occur specifically when the transmission is subjected to particularly strong negative acceleration. The plotted graph (Section D) shows strong acceleration peaks in the transmission. Obviously the torsion damper is operating in the steep coast stage.

The torsion damper wind-up angle appears in Graph Section B. Vibrations of about 16° occur during the short resonance interval, passing through almost the entire predamper range.

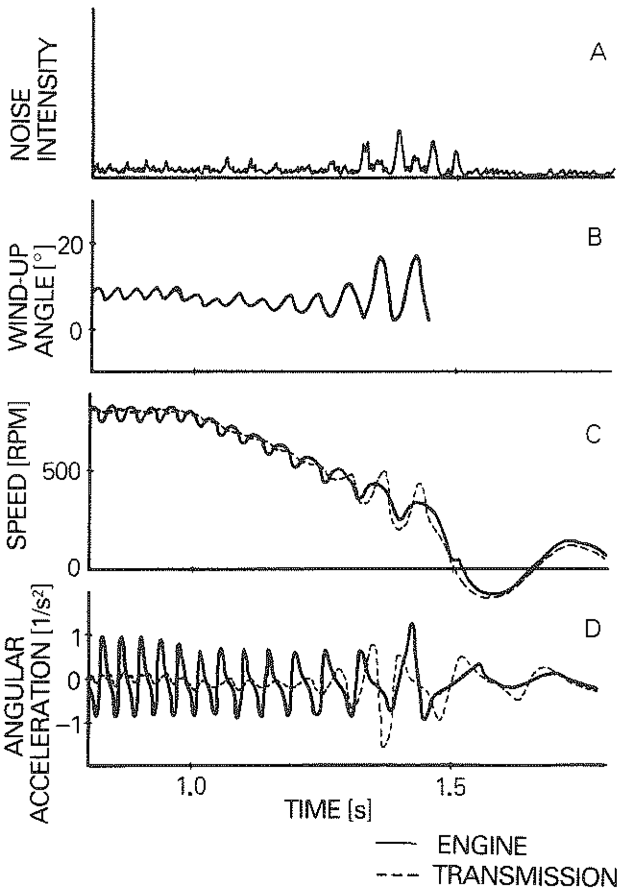

Figure 17:
Measuring the shut-off procedure

We attempted a vibration simulation based on the analysis of the shut-off cycle. We consider that we have a good correlation if the amplitude and phase position of the calculation approximate the measured values, as can be seen below in Figure 18.

At this point we can conduct iterative computer calculations. And we already suspect what the problem is: the primary noise is associated with operation in the coast stage.

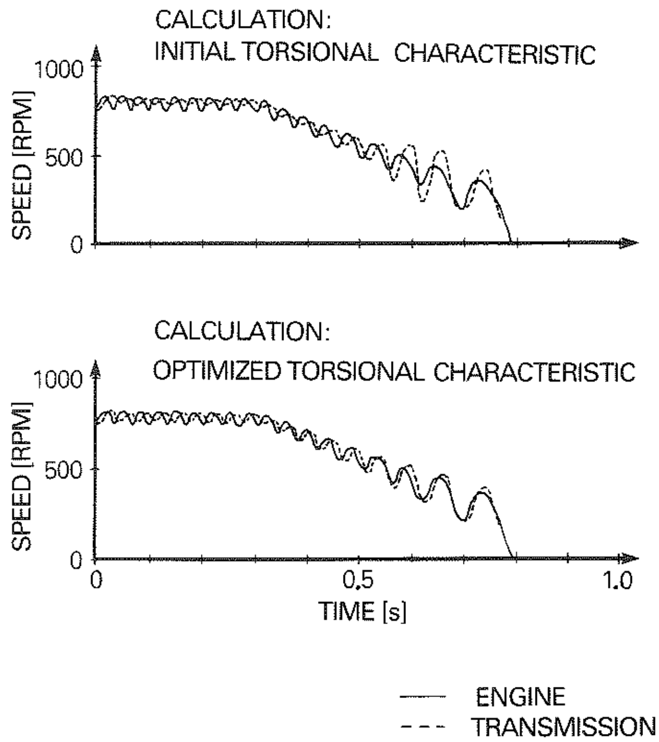

Figure 18:
Engine shut-off calculation

The bottom section of Figure 18 shows computer results with a reduced resonance amplitude. The coast stage – shown as a broken line in Figure 16 – is extended and an additional friction control plate added. The measured curve for this new characteristic actually shows a more favorable performance. In comparison, the top graph in Figure 19 again shows the speed curve from Figure 17. Subjective noise evaluation has improved by 3 rating points. The remaining rattle noise is no longer perceived as objectionable.

Success doesn't always come this quickly. The calculation process cannot predict what degree of vibration isolation is required in order to eliminate transmission noises. Rattle intensity depends to a great extent on the vehicle's tendency to transmit structure-borne noise, which cannot be predicted yet.

If results are unsatisfactory after the first round of tuning, vibration isolation has to be improved even further.

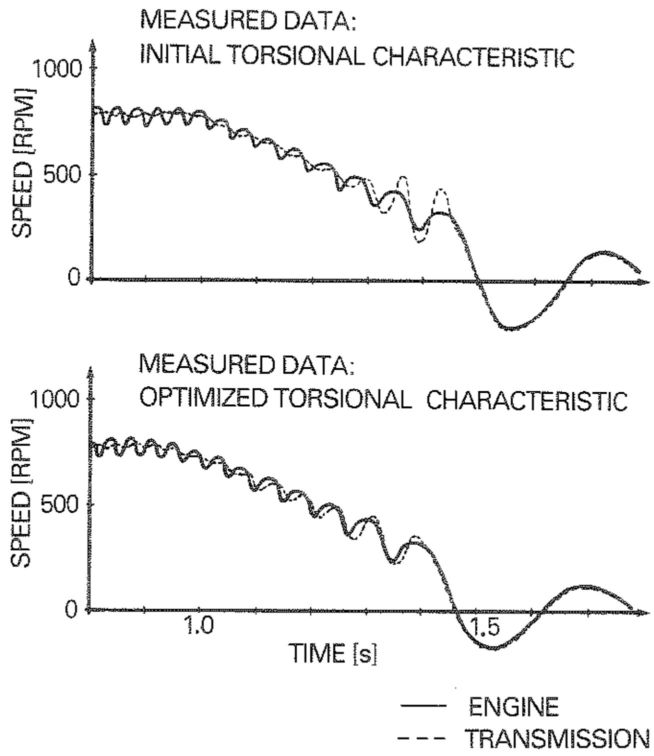

Figure 19:
Engine shut-off measured data

But other areas should not be allowed to deteriorate. For instance, a characteristic that is well suited for the idle stage can cause tip-in/back-out problems. We have to reach a compromise between both vibration modes. In such cases, tuning can become a time-consuming activity, even with electronic measuring systems and computer assistance.

For many years, the measurement and calculation procedures that support our successful torsion damper tuning had to be performed in-house in our testing labs. Nevertheless, our customers in the automotive industry frequently expressed a real need to be able to perform this work in their facilities or in the field. Frequently the limited availability of prototype vehicles, coupled with secrecy requirements, necessitates outside testing capability.

LuK has responded to this need by developing our mobile tuning lab. It contains all the basic tools needed to support a well-documented tuning program:

- A torsion test stand with computer analysis capability for plotting torsion damper and dual mass flywheel characteristic data
- A break-in test stand for testing torsion damper friction control assemblies
- A measuring computer for in-vehicle acquisition of torsional vibration data
- A computer for vibration simulation
- A small workbench, complete with basic tools.

With the mobile tuning lab, it is frequently possible to solve torsional vibration problems on-site at the vehicle manufacturer's own test facilities. This means that LuK can also help in cases where a secret prototype cannot be driven off a customer's test track.

Summary

The most important source of excitation for torsional vibrations in the vehicle drive train has been discussed. The main sources of excitation, gas forces in the engine, have been analyzed for low speeds, ignoring the effects of inertial forces. The alternating torques applied at the crankshaft under full load are proportional to the cylinder displacement. In idle mode, this only applies for diesel engines.

The drive train represents a vibration system with several resonance frequencies. Subjected to engine excitation, these frequencies can lead to numerous resonant speeds. With conventional clutch disc torsion dampers, it is possible to shift resonant speeds somewhat, and resulting resonances can be damped. Vibration isolation is only possible beyond speeds of about 2,000 rpm.

Measurement of the torsional vibrations causing gear rattle or boom is essential for the analysis of the problem. Measuring procedures have been described. However, to optimize torsion damper design, designers need simulation procedures in order to conduct systematic variations of system parameters. An example has been provided showing how measurement and calculation complement each other during the tuning process.

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