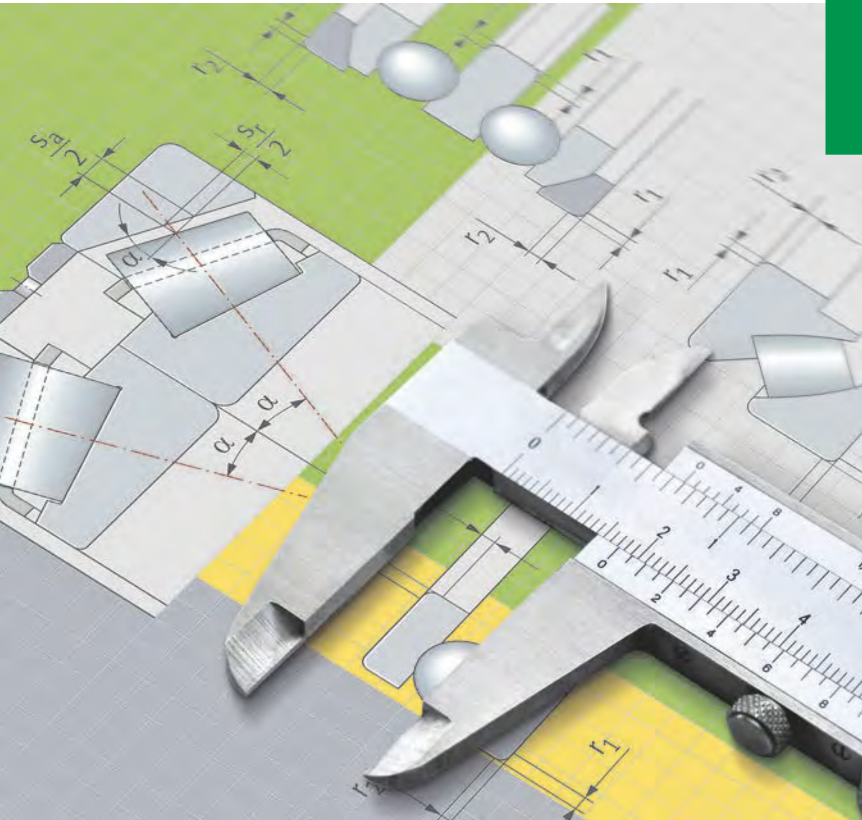
Technical Pocket Guide



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PDF only

Full list of contents

Contents

Foreword

Schaeffler has always and on principle been committed to its responsibility for sustainable training and global transfer of knowledge. Even if the transition from an industry-based to a knowledge-based society was not so clearly defined in the mid-1980s, it was already clear to the attentive observer that education was becoming an extremely valuable commodity in which it was worth investing. From this notion came the idea for the INA Technical Pocket Guide ITT, which was produced under the technical direction of Prof. Dr.-Ing. Ernst-Günter Paland, presented for the first time at the Hannover Messe (English: Hanover Fair) in 1985 and contained 96 pages.

Over the years, this reference work with its equations and laws has been increased to 370 pages and published in several editions. More than 750 000 copies have been distributed since its initial publication.
In addition to its function as a reference work, it has also proven itself as a work book for use in training and further education by designers, engineers, technicians, and students of technical and scientific disciplines.

As confirmed by the high demand for this publication, it was completely revised by Schaeffler in 2013 and has been published since then under the name Schaeffler Technical Pocket Guide STT. For the purposes of technical coordination, it was possible to obtain the services of Prof. Dr.-Ing. Harald Meerkamm of the Friedrich-Alexander University of Erlangen-Nuremberg and formerly Head of the Chair for Engineering Design. Without his exceptional support, experience, diligence and patience, it would not have been possible to produce the Schaeffler Technical Pocket Guide in this form.

Since 2021, Prof. Dr.-Ing. Sandro Wartzack, Head of the Chair for Engineering Design at the Friedrich-Alexander University of Erlangen-Nuremberg, has been continuing to oversee the technical coordination of the Schaeffler Technical Pocket Guide and made valuable contributions towards its enhancement. As a product of our collaboration, we have added numerous new topics to this version and updated established topics. The current revised and expanded version imparts extensive, up-to-date knowledge over around 760 pages.

We would like to express our heartfelt gratitude to Prof. Dr.-Ing Sandro Wartzack and his highly motivated team of scientific personnel for their extraordinary commitment and the high level of trust they have demonstrated throughout our collaboration. We would also like to thank all of the readers who pointed out corrections and amendments.

We are delighted that the Schaeffler Technical Pocket Guide is so highly valued and that more than 500 000 copies have been requested from us to date. Interested parties can request copies of the new edition via ➔ *http://www.schaeffler.de/std/1D50* in our Media Library or as a download in PDF format. The Schaeffler Technical Pocket Guide is also available as an app at ➔ *apps.schaeffler.com*.
We are convinced that this new edition will meet with the same positive response as the previous versions.

Herzogenaurach, March 2025
The Publisher

Preface

For over 25 years, the INA Technical Pocket Guide ITT from Schaeffler has been a standard work among apprentices in metal-processing and electrical engineering professions, as well as designers, technicians, engineers and also students involved in technical and scientific disciplines. With its brief, compact and concise layout, it successfully bridges the gap between comprehensive textbook and purely tabular work.

As a result of the huge demand for this guide – over three quarters of a million copies have been distributed since its initial publication more than two and a half decades ago – Schaeffler decided to produce a fully revised version and reissue this as the Schaeffler Technical Pocket Guide STT. The key priority in this process was to retain the former character of the guide as a reference work which provides rapid access to detailed information and to ensure scientific relevance, intelligibility, clarity and presentability, plus the addition of topical subjects. As a result, all of the contents have been updated, the data on standards, tolerances and fits have been brought up to date and the current subject of mechatronics has been added.

The chapter Design elements – which has been expanded to include rotatory and translatory bearings and examples of applications which use these products – has been completely reconfigured and significantly extended in accordance with the scientifically founded "function-oriented approach". The technical principles of rolling bearings, where were previously covered in two chapters of the guide, have been consolidated in this chapter. The "function-oriented approach" follows the concept of structuring the field of machine elements according to their respective function.

As it was not the intention to produce extensive descriptions of the subject areas in the manner found in textbooks, the statements have been kept deliberately concise, in the sense of a reference work, with the focus on fast, practical usability. This enables readers to quickly locate the required information in a condensed form and gain familiarity with the specialist knowledge. The layout of the chapters, the use of colour as a breakdown and control parameter, a reader-friendly typography, and well-structured tables and formulae all help to provide easy access to information.

As drawings are often involved in understanding technical and scientific relationships, the pictures in this work have been designed to make their content clearer and more readily accessible. This opens up new possibilities in the transfer of information involving diagrammatic elements, adhering to the principles of modern didactic textbook design and supporting the uptake of information.

I would like to thank the publisher's employees who have worked on this edition for their comments, suggestions, tips, amendments and corrections and for the trusting and stimulating working relationship we have shared, the excellent support they have provided and the conscientious, operative implementation of this project. I would also like to sincerely thank my STT Team at the Chair for Engineering Design, who have given me excellent support throughout all stages of this project.

I hope that this proven work book and reference work, in its new edition as the Schaeffler Technical Pocket Guide STT, will assist all readers in tackling their everyday tasks.

Erlangen, March 2014
Prof. Dr.-Ing. Harald Meerkamm
Friedrich-Alexander University of Erlangen-Nuremberg

Preface to the 4th edition
Digitalisation in general and digital engineering in particular is assuming an increasingly important role in everyday engineering activities. In this new edition, we therefore give particular priority to the topics associated with this area. We have expanded the Mechatronics chapter to include the subchapters Sensors, Actuators, Modelling and simulation, and Condition monitoring of machines with rolling bearings by means of vibration analysis. The chapter on Product development is also a new addition, covering not only the methodical approach but also the use of CAx, knowledge-based engineering and knowledge discovery in databases/data mining. Additions were also necessary to the Technical statistics chapter, most notably on the subject of design of experiments. These new additions were accompanied by adjustments necessitated by changes to central standards and the introduction of new standards. As in every edition, we have also made any necessary corrections, as identified, not least, by the readers of this publication.

The Schaeffler Technical Pocket Guide continues to impress with its conciseness, which has been successfully maintained despite the wealth of material contained within its pages, and is excellently qualified as a support tool in everyday industrial engineering activities. I am also delighted to have received positive feedback from students, who prefer this Technical Pocket Guide to textbooks on account of its compact size and clearly arranged contents – features which have enabled it to bridge an important gap between tabular works and textbooks.

I am especially grateful for the ongoing excellent and trusting collaboration between Schaeffler and the STT team at the Chair for Engineering Design at the Friedrich-Alexander University of Erlangen-Nuremberg and would like to extend my particular thanks at this point to Dr.-Ing. Marcel Bartz, Dr.-Ing. Stefan Götz and Prof. Dr.-Ing. Benjamin Schleich.

I hope you find the Schaeffler Technical Pocket Guide an interesting read and a useful support tool in your daily technical work or studies.

Erlangen, March 2025
Prof. Dr.-Ing. Sandro Wartzack
Friedrich-Alexander University of Erlangen-Nuremberg

Schaeffler – We pioneer motion

There is something that the legendary VW Beetle, the DTM Electric demo vehicle, the Airbus A380 (the world's largest passenger aircraft), wind turbines across the globe, cobots and robots, the "London Eye" Ferris wheel tourist attraction, artificial intelligence applications in production, and cars with electrified drives all have in common: they each rely on technology developed by Schaeffler.

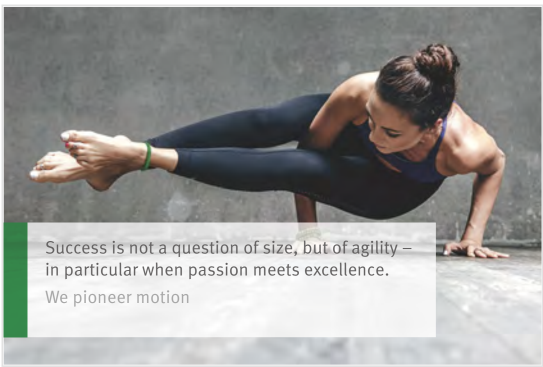
As a leading global automotive and industrial supplier, the Schaeffler Group has been driving technological progress for over 75 years, delivering groundbreaking and marketable innovations to produce more efficient, intelligent and sustainable motion and mobility solutions. The technology company offers high-precision components, systems and services in the fields of electromobility, CO_2-efficient drives, chassis applications, digitalisation and renewable energies as well as lifetime solutions and rolling and plain bearing solutions for a multitude of industrial applications. Schaeffler has been ranked among the most innovative companies in Germany by the German Patent and Trademark Office for many years.

Figure 1
Headquarters in Herzogenaurach

Shaping the transformation

The year 2020 marked a further development in the company's corporate strategy. Building on the strategic "Roadmap 2025", the company embraced its new corporate claim "We pioneer motion", which reflects Schaeffler's vision of becoming the preferred automotive and industrial supplier. As part of this quest, the company with its three divisions – Automotive Technologies, Automotive Aftermarket and Industrial – has five focus fields in its sight: CO_2-efficient drives (e-mobility and hydrogen technology), chassis applications, industrial machinery and equipment, renewable energies, and aftermarket and service solutions.

The four corporate values – "sustainability", "innovation", "excellence" and "passion" – form the pillars on which these focus fields are based. For Schaeffler, sustainable corporate success means assuming ecological and social responsibility – in its production processes, for the use of its products by customers, and in supplier engagement. As part of this commitment, the company has set itself the target, across all regions, of achieving CO_2-neutral production by 2030 and climate-neutral operations by 2040. The Schaeffler Group was named one of the 50 Sustainability & Climate Leaders by the United Nations in 2021.



The Divisions of the Schaeffler Group

Automotive Technologies

The Automotive Technologies Division develops and manufactures groundbreaking components and system solutions for vehicles with powertrains comprising internal combustion engines as well as for hybrid and electric vehicles, in which Schaeffler's traditional strengths – a high level of vertical integration and excellence in production – play a fundamental role. The company's portfolio ranges from drives for partially and fully electrified mobility solutions through to hydrogen fuel cell technologies. Complete systems such as the 4in1 e-axle are among the developments that will represent the future of full fleet electrification.

Electric mobility, chassis technologies and hydrogen power: Schaeffler continues to rely on innovative mobility solutions and consistently applied sustainability in its bid to maintain the position of preferred technology partner to the global automotive industry in the future.

Figure 2
Schaeffler develops components and systems for electric mobility

Figure 3
As a highly integrated and compact complete system, the 4in1 e-axle requires significantly less installation space than non-integrated solutions

Automotive Aftermarket

The Automotive Aftermarket Division delivers components and comprehensive repair solutions for the automotive aftermarket worldwide. New technologies and systems in modern vehicles are presenting workshops with ever greater challenges, as repair work becomes more extensive and increasingly requires an understanding of the entire system. Thanks to comprehensive system knowledge and an extensive range of services, Schaeffler can support workshops in carrying out complex repairs. Whatever the requirement – whether innovative repair solutions for clutch and release systems or components for engines, gearboxes and chassis applications – Schaeffler can offer a full range of automotive aftermarket parts and associated services under the LUK, INA, FAG and REPXPERT brands.

Figure 4
Schaeffler Automotive Aftermarket – qualified partner to workshop customers

Figure 5
Replacing the belt in an auxiliary drive

Industrial The Industrial Division supplies rolling and plain bearings, linear and direct drive technology as well as maintenance products and monitoring systems to customers from a wide range of industrial sectors, both in the form of direct business and via a global network of certified sales partners. Schaeffler offers an extensive selection of bearing solutions for application areas including robotics, wind turbines, production and construction machinery, medical technology, and trains and planes. Engineers work closely with customers to develop optimum solutions tailored to the most diverse of requirements, while considering the entire system and the specific ambient influences and operating conditions associated with the application, to ensure reliable bearing performance even in extreme operating environments, such as cold, hot and stormy conditions, under exposure to permanent water or salt water, in desert climates, or even in space. Schaeffler offers industrial maintenance solutions from its Lifetime Solutions portfolio, such as condition monitoring systems and intelligent lubricators, which help to prolong the life of industrial systems, reduce operating costs and minimise the use of resources.

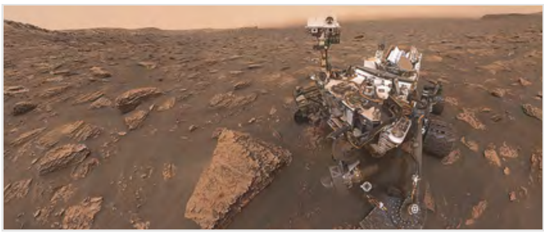


Figure 6
Deployment site on Mars: the research vehicle "Curiosity" containing Schaeffler technology

Figure 7
PSC-series precision planetary gearboxes for industrial robots

The path to entrepreneurship

It all started with two brothers – Dr. Wilhelm Schaeffler (1908–1981) and Dr.-Ing. E.h. Georg Schaeffler (1917–1996) – more than 75 years ago. In 1946, the two business graduates founded the company Industrie GmbH in Herzogenaurach, marking the start of the success story of a leading global supplier to the automotive and industrial sectors, which now employs more than 83 000 people at around 200 locations in more than 50 countries.

The invention of the cage-guided needle roller bearing by Georg Schaeffler, for which a patent was filed in 1950, marked the foundation of this success, with the company experiencing rapid growth in the 1950s. The sensational success of the INA needle roller bearing and the development of new products led to expansions in production capacity, the creation of new departments and the establishment of subsidiaries both at home and, soon after, abroad. The location in Herzogenaurach remains home to the company's headquarters to this day.

Figure 8
In 1950, Georg Schaeffler filed a patent application for his idea: a needle roller cage, in which the rolling elements are guided parallel to the axis. Over the course of 1950, this idea progressed from prototype to a rolling bearing ready for volume production

LuK became a member company of the Schaeffler Group in 1999, which was followed by the acquisition of FAG Kugelfischer Georg Schäfer AG in Schweinfurt in 2001, making the Schaeffler Group the world's second largest manufacturer of rolling bearings. The INA, LuK and FAG product brands continue to play an important role in sales today.

Since the death of Dr.-Ing E.h. Georg Schaeffler in 1996, his wife Maria-Elisabeth Schaeffler-Thumann and son Georg F. W. Schaeffler have maintained the family's ownership of the company as shareholders, successfully steering its growth alongside the management team.

In 2008, the Schaeffler family submitted a voluntary takeover bid to the shareholders of Continental AG and, with a stake of 46 percent, is now the majority shareholder. Schaeffler also became a stock corporation during this period. Schaeffler AG has been listed on the Frankfurt Stock Exchange since October 2015. All common shares and, with them, all voting rights belong to the family, ensuring that Schaeffler remains a family-owned company.

Figure 9
Schaeffler AG goes public in 2015. Georg F. W. Schaeffler, Jürgen R. Thumann, Maria-Elisabeth Schaeffler-Thumann and CEO Klaus Rosenfeld (f.l.t.r.) usher in a new era on 9th October in Frankfurt

A little known fact

It is a little known fact today that, alongside the metal division, carpets remained the second mainstay of the business until the late 1980s. By the early 1960s, the company had, according to its own figures, become the largest carpet manufacturer in the Federal Republic of Germany. In late 1989, Georg Schaeffler and his son Georg F. W. Schaeffler found a purchaser, divested themselves of the carpet business and began to focus solely on their expertise as automotive and industrial suppliers.

For a detailed account of the Schaeffler Group's history, including numerous photos, videos and contemporary witness accounts, visit: *www.schaeffler.com/history*

Training and studying – Professional future at Schaeffler

Training and studying have a long tradition at Schaeffler and are regarded as a top priority. Every year, approximately 900 young people around the globe – from Herzogenaurach to Taicang in China, and from Port Elizabeth in South Africa to Fort Mill in the USA – embark on a training program to land their dream job at Schaeffler.

The training portfolio traditionally focusses on industrial/technical subjects, such as mechanical engineering, electronics and mechatronics, leading to a range of possible career opportunities, including positions for development engineers, IT experts and specialists in production or administration. With Schaeffler's ongoing transformation and emerging industry trends such as robotics and hydrogen technology, the range of training and study programs is constantly expanding.

Schaeffler currently offers around 30 training and study programs across 48 locations in 16 countries.

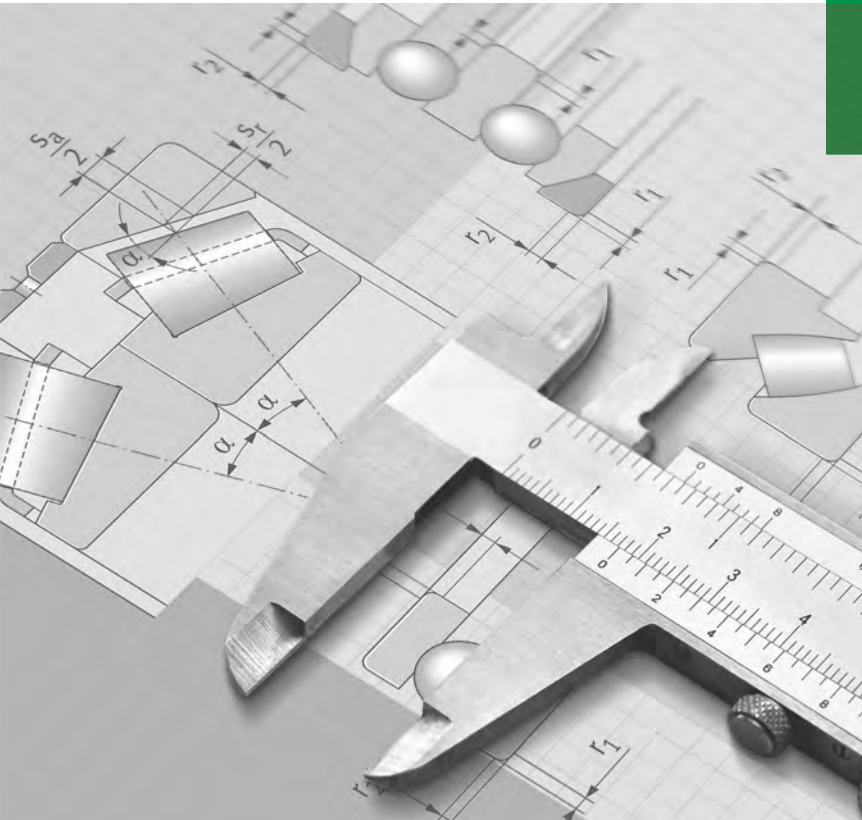
Each year in Germany – the largest training region – around 1 000 apprentices complete their training with a certificate from the Chamber of Commerce and Industry (IHK) after two to three and a half years. Additionally, around 300 students complete a dual bachelor or master's degree course in three to four and a half years. In addition to the academic qualification, the degree course includes intensive practical experience at Schaeffler or an accompanying training course certified by the Chamber of Commerce and Industry ("Two in One" study program).

The Schaeffler Group makes major investments in training and advanced training and collaborates with numerous universities on practice-oriented degree programs.

As a global mobility supplier, the company offers a wide range of professional development and career planning opportunities – including international exchanges between different Schaeffler Group companies.

For further information, visit: *www.schaeffler.com/careers*

Technical Pocket Guide



Contents

Contents

Contents

Page

Technical statistics

Contents

Contents

Page

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Unit systems

International system of units SI

Base units in SI

The “Law on Units of Measurement” of 2 July 1969 came into force in the Federal Republic of Germany on 5 July 1970. This law defined the legal units of measurement for commercial practice that were to be introduced no later than 31 December 1977.
Furthermore, this law defined the base values and base units of the International System of Units (Système International d’Unités, abbreviated to SI).

Defining constants

In 2018, a fundamental change to the International System of Units (SI) was adopted at the 26th General Conference on Weights and Measures[1)], which came into force in 2019 and includes seven “defining constants” that create the foundation on which the base units and other SI units are defined.

Name	Symbol	Value
Hyperfine transition frequency of the caesium atom	$\Delta\nu_{Cs}$	$9\,192\,631\,770\ s^{-1}$
Speed of light	c	$299\,792\,458\ m\ s^{-1}$
Planck’s constant	h	$6{,}626\,070\,15 \cdot 10^{-34}\ kg\ m^2/s$
Elementary charge	e	$1{,}602\,176\,634 \cdot 10^{-19}\ A\ s$
Boltzmann’s constant	k	$1{,}380\,649 \cdot 10^{-23}\ kg\ m^2/(s^2\ K)$
Avogadro’s constant	N_A	$6{,}022\,140\,76 \cdot 10^{23}\ mol^{-1}$
Luminous efficacy of radiation	K_{cd}	$683\ cd\ sr\ s^3/(kg\ m^2)$

Source: Die gesetzlichen Einheiten in Deutschland, 2. Auflage 2020,
Physikalisch-Technische Bundesanstalt, Nationales Metrologieinstitut.

[1)] CGPM: Conférence Génerale des Poids et Mesures (General Conference on Weights and Measures).

The following table shows the seven base values and base units of the SI.

Base value	Base unit		Definition
	Designation	Symbol	
Length	Metre	m	The metre, unit symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit m/s, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$[1].
Mass	Kilogram	kg	The kilogram, unit symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6{,}626\,070\,15 \cdot 10^{-34}$ when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$[1].
Time	Second	s	The second, unit symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$[1] to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.
Electric current	Ampere	A	The ampere, unit symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1{,}602\,176\,634 \cdot 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$[1].
Thermo-dynamic temperature	Kelvin	K	The kelvin, unit symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be $1{,}380\,649 \cdot 10^{-23}$ when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} K^{-1}, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$[1].

Continuation of table, see Page 20.

Source: Die gesetzlichen Einheiten in Deutschland, 2. Auflage 2020, Physikalisch-Technische Bundesanstalt, Nationales Metrologieinstitut

[1] The caesium frequency $\Delta\nu_{Cs}$ is the unperturbed ground-state hyperfine transition frequency of the caesium atom 133, see also table Defining constants, Page 18.

Continuation of table, Base units in SI, from Page 19.

Base value	Base unit		Definition
	Designation	Symbol	
Amount of substance	Mole	Mole	The mole, unit symbol mol, is the SI unit of amount of substance. One mole contains exactly 6,022 140 76 · 10^{23} elementary entities. This number is the fixed numerical value of the Avogadro constant NA when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
Luminous intensity	Candela	cd	The candela, unit symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the photometric radiation equivalent K_{cd} of monochromatic radiation of frequency 540 · 10^{12} Hz to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} m^{-2} s^3, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$[1)].

Source: Die gesetzlichen Einheiten in Deutschland, 2. Auflage 2020, Physikalisch-Technische Bundesanstalt, Nationales Metrologieinstitut

[1)] The caesium frequency $\Delta\nu_{Cs}$ is the unperturbed ground-state hyperfine transition frequency of the caesium atom 133, see also table Defining constants, Page 18.

Derived units

Further SI units can be derived from the base units. If this derivation leads only to the numerical factor 1, the derived units are **coherent** relative to the base units. Coherent units form a system of units.

Derivation of the unit "Newton"

In accordance with Newton's basic law, force is a value derived from the base values of mass, time and length.

This coherent value has the unit name "Newton" and the unit symbol "N":

- force = mass · acceleration
- 1 N = 1 kg · 1 m/s^2

SI units and derived units The following table shows an excerpt of common SI units and units derived therefrom (other derived values and units are given in the specific sections).

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Length	***l***	**Metre**	**–**	**m**	Micron 1 μ = 1 μm = 10^{-6} m Ångström 1 Å = 10^{-10} m X unit 1 xu = 10^{-13} m
		–	Nautical mile	1 NM = 1852 m	
Area	A	Square metre	–	m^2	
		–	Are	1 a = 100 m^2	
		–	Hectare	1 ha = 10^4 m^2	
Volume	V	Cubic metre	–	m^3	
		–	Litre	1 l = 10^{-3} m^3	
Elongation	ϵ	■	–	m/m	
Plane angle	α β γ	Radian	–	1 rad[4] = 1 m/m	Right angle 1 ∟ = (π/2) rad Gradian 1 g = 1 gon Centigon 1′ = 1 cgon
		–	Degree	1° = π/180 rad	
		–	Minute	1′ = π/10800 rad	
		–	Second	1″ = π/648000 rad	
		–	Gon	1 gon = π/200 rad	
Solid angle	Ω	Steradian	–	1 sr = 1 m^2/m^2	
Mass	***m***	**Kilogram**	**–**	**kg**	Gamma 1 γ = 1 μg Quintal 1 dz = 100 kg
		–	Gram	1 g = 10^{-3} kg	
		–	Tonne	1 t = 10^3 kg	
Mass of precious stones		–	Metric carat	1 Kt = 0,2 · 10^{-3} kg	
Mass/length	m'	■	–	kg/m	
Mass of textile fibres	–	–	Tex	1 tex = 10^{-6} kg/m	
Mass/area	m''	■	–	kg/m^2	

Continuation of table, see Page 22.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

1) Legal units as of 2 July 1970.
2) Units that legally may no longer be used as of 1 January 1978.
3) Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.
4) The unit rad can be replaced by “1” in calculation.

Continuation of table, SI units and derived units, from Page 21.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Density	ρ	■	–	kg/m^3	The numerical value of the specific gravity in kp/m^3 is not always equal to the numerical value of the density, but is dependent on location, see also Equation 4, Page 30
Specific volume	v	■	–	m^3/kg	
Time	t[4]	**Second**	**–**	**s**	–
		–	Minute	1 min = 60 s	
		–	Hour	1 h = 3 600 s	
		–	Day	1 d = 86 400 s	
		–	Tropical year	1 a ≈ 365,24 d	
Rotational speed	n	■	–	1/s	rpm is still permissible, however it should be replaced by min^{-1}
		–	Revolution/ minute	1 rpm = 1 min^{-1}	
Frequency	f	Hertz	–	1 Hz = 1/s	–
Angular frequency	ω	–	1/s		
Velocity	v	■	–	m/s	
		–	Kilometre/ hour	1 km/h = (1/3,6) m/s	
		–	Knot	1 kn = 1 NM/h	
Acceleration	a	■		m/s^2	
Angular velocity	ω	■	–	rad/s	
Angular acceleration	$\dot{\omega}$	■	–	rad/s^2	
Volume flow	$\dot{V}$	■	–	m^3/s	
Mass flow	$\dot{m}$	■	–	kg/s	

Continuation of table, see Page 23.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

1) Legal units as of 2 July 1970.

2) Units that legally may no longer be used as of 1 January 1978.

3) Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.

4) The formula symbol *t* is also used for temperature.

Continuation of table, SI units and derived units, from Page 22.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Force	F	Newton	–	$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$	Kilopond $1\ \text{kp} = 9{,}806\,65\ \text{N}$
Impulse	p	■	–	$\text{kg} \cdot \text{m/s}$	
Angular momentum	L	■	–	$\text{kg} \cdot \text{m}^2/\text{s}$	Technical atmosphere $1\ \text{at} = 1\ \text{kp/cm}^2$
Pressure	p	Pascal	–	$1\ \text{Pa} = 1\ \text{N/m}^2$	Physical atmosphere $1\ \text{atm} = 1{,}013\,25\ \text{bar}$
Stress	σ τ	■	Newton/ square millimetre	$1\ \text{N/mm}^2 = 1\ \text{MPa}$	Water column $1\ \text{mm WC} = 1\ \text{kp/m}^2$
		■	Bar	$1\ \text{bar} = 10^5\ \text{Pa}$	Mercury column $1\ \text{mm Hg} = 1{,}333\,2\ \text{hPa}$
Work Energy	W E	Joule	–	$1\ \text{J} = 1\ \text{N} \cdot \text{m}$	Kilopond metre $1\ \text{kpm} = 9{,}81\ \text{J}$
Heat quantity	Q	Watt second	–	$1\ \text{W} \cdot \text{s} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$	Horsepower hour $1\ \text{hp} \cdot \text{h} = 0{,}735\,5\ \text{kW} \cdot \text{h}$
		–	Kilowatt hour	$1\ \text{kW} \cdot \text{h} = 3{,}6\ \text{MJ}$	Kilocalorie $1\ \text{kcal} = 4{,}186\,8\ \text{kJ}$
Moment of force	M	Newton metre	–	$\text{N} \cdot \text{m}$	Kilopond metre $1\ \text{kpm} = 9{,}81\ \text{N} \cdot \text{m}$
Power, energy flow Heat flow	P $\dot{Q}$	Watt	–	$1\ \text{W} = 1\ \text{J/s}$ $= 1\ \text{N} \cdot \text{m/s}$	Horsepower $1\ \text{hp} = 0{,}735\,5\ \text{kW}$ $1\ \text{kW} = 1{,}36\ \text{PS}$ $1\ \text{kcal/s} = 4{,}186\,8\ \text{kW}$
Dynamic viscosity	η	Pascal second	–	$1\ \text{Pa} \cdot \text{s} = 1\ \text{N} \cdot \text{s/m}^2$	Poise $1\ \text{P} = 0{,}1\ \text{Pa} \cdot \text{s}$ Centipoise $1\ \text{cP} = 1\ \text{mPa} \cdot \text{s}$
Kinematic viscosity	ν	■	–	m^2/s	Stokes $1\ \text{St} = 10^{-4}\ \text{m}^2/\text{s}$ Centistokes $1\ \text{cSt} = 1\ \text{mm}^2/\text{s}$

Continuation of table, see Page 24.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

1) Legal units as of 2 July 1970.

2) Units that legally may no longer be used as of 1 January 1978.

3) Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.

Continuation of table, SI units and derived units, from Page 23.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Electric current	***I***	**Ampere**	**–**	**A**	–
Voltage (potential difference)	U	Volt	–	1 V = 1 W/A	
Electrical resistance	R	Ohm	–	1 Ω = 1 V/A	
Electrical conductance	G	Siemens	–	1 S = 1/Ω	
Apparent power	S	–	Volt-ampere	1 W = 1 V · A	
Reactive power	Q	–	Var	1 var = 1 W	
Quantity of electricity, electrical charge	Q	Coulomb	–	1 C = 1 A · s	
	–	–	Ampere-hour	1 A · h = 3 600 C	
Electrical capacitance	C	Farad	–	1 F = 1 C/V	
Electrical flux	ψ	–	–	C	
Electrical flux density	D	–	–	C/m^2	
Electrical field strength	E	–	–	V/m	
Magnetic flux	Φ	Weber	–	1 Wb = 1 V · s	Maxwell 1 M = 10^{-8} Wb
Magnetic flux density	B	Tesla	–	1 T = 1 Wb/m^2	Gauss 1 G = 10^{-4} T
Magnetic field strength	H	–	–	A/m	Oerstedt 1 Oe = $10^3/(4\pi)$ A/m = 79,58 A/m
Inductance	L	Henry	–	1 H = 1 Wb/A	–

Continuation of table, see Page 25.

1) Legal units as of 2 July 1970.

2) Units that legally may no longer be used as of 1 January 1978.

3) Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.

Continuation of table, SI units and derived units, from Page 24.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Temperature	T, Θ	**Kelvin**	**–**	**K**	–
Celsius temperature	t[4], ϑ, θ	–	Degree Celsius	1 °C = 1 K[5]	
Thermal diffusivity	a	–	–	m^2/s	
Specific heat capacity	c	–	–	J/(kg · K)	1 kcal/(kg · grd) = 4,187 kJ/(kg · K)
Entropy	S	–	–	J/kg	–
Specific entropy	s	–	–	J/(kg · K)	
Enthalpy	H	Joule	–	J	
Thermal conductivity	λ	–	–	W/(m · K)	1 kcal/(m · h · grd) = 1,163 W/(m · K)
Heat transfer coefficient	α	–	–	$W/(m^2 \cdot K)$	–
Heat transition coefficient	k	–	–	$W/(m^2 \cdot K)$	

Continuation of table, see Page 26.

[1] Legal units as of 2 July 1970.

[2] Units that legally may no longer be used as of 1 January 1978.

[3] Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.

[4] The formula symbol *t* is also used for time.

[5] The Celsius temperature *t* is the name for the particular difference between any thermodynamic temperature *T* and the temperature T_0 = 273,15 K. It is therefore $t = T - T_0 = T - 273{,}15$ K.
The degree Celsius is the name for the unit Kelvin when specifying Celsius temperatures.
In compound units, temperature differences must be specified in K, for example kJ/(m · s · K).
Tolerances for Celsius temperatures are presented, for example,
as t = (50 ± 2) °C or t = 50 °C ± 2 °C or t = 50 °C ± 2 K.

Continuation of table, SI units and derived units, from Page 25.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Amount of substance	***n***	**Mole**	**–**	**mol**	–
Atomic unit of mass	u	–	–	$1{,}6606 \cdot 10^{-27}$ kg	
Energy	W	Electron volt	–	1 eV = $1{,}6022 \cdot 10^{-19}$ J	
Activity of a radioactive substance	A	Becquerel	–	1 Bq = 1/s	Curie 1 Ci = $3{,}7 \cdot 10^{10}$/s
Absorbed dose	D	Gray	–	1 Gy = 1 J/kg	Rem 1 rem = 10^{-2} J/kg
Energy dose rate	$\dot{D}$	–	–	W/kg	–
Ion dose	J	–	–	C/kg	Röntgen 1 R = $258 \cdot 10^{-6}$ C/kg
Ion dose rate	j	–	–	A/kg	–
Dose equivalent	H	Sievert	–	1 Sv = 1 J/kg	
Luminous intensity	***I***	**Candela**	**–**	**cd**	
Luminance	L	–	–	cd/m^2	Stilb 1 sb = 10^4 cd/m^2 Apostilb 1 asb = $(1/\pi)$ cd/m^2
Luminous flux	Φ	Lumen	–	1 lm = 1 cd · sr	–
Luminous energy	Q	–	–	1 lm · s	
Illuminance	E	Lux	–	1 lx = 1 lm/m^2	
Refractive power of lenses	D	–	Dioptre	1 dpt = 1/m	

1) Legal units as of 2 July 1970.
2) Units that legally may no longer be used as of 1 January 1978.
3) Formula symbol standardised in accordance with DIN 1304 and DIN EN ISO 80000-1.

Internationally defined prefixes for units of measurement

Decimal fractions or multiples of SI units are designated by prefixes before the names of the units or by prefix symbols before the unit symbols. This factor, by which the unit is multiplied, is generally a power of ten with a positive or negative exponent.

The prefix symbol is placed immediately in front of the unit symbol, without a space, to form a coherent unit, such as the millimetre (mm). Compound prefixes, such as millikilogram (mkg), must not be used.

Prefixes must not be used for the following units: the time units of the minute, hour, day, year; the temperature unit of the degree Celsius; the angular units of the degree, second, minute.

The following table shows a selection of the most important prefixes:

Power of ten	Name		Prefix	Prefix symbol
	Long scale (Western Europe etc.)	Short scale (USA etc.)		
10^{-30}	Quintillionth	Nonillionth	Quecto	q
10^{-27}	Quadrilliardth	Octillionth	Ronto	r
10^{-24}	Quadrillionth	Septillionth	Yocto	y
10^{-21}	Trilliardth	Sextillionth	Zepto	z
10^{-18}	Trillionth	Quintillionth	Atto	a
10^{-15}	Billiardth	Quadrillionth	Femto	f
10^{-12}	Billionth	Billionth	Pico	p
10^{-9}	Milliardth	Trillionth	Nano	n
10^{-6}	Millionth		Micro	μ
10^{-3}	Thousandth		Milli	m
10^{-2}	Hundredth		Centi	c
10^{-1}	Tenth		Deci	d

Continuation of table, see Page 28.

Continuation of table, Internationally defined prefixes for units of measurement, from Page 27.

Power of ten	Name		Prefix	Prefix symbol
	Long scale (Western Europe etc.)	Short scale (USA etc.)		
10^1	Ten		Deca	da
10^2	Hundred		Hecto	h
10^3	Thousand		Kilo	k
10^6	Million		Mega	M
10^9	Milliard	Billion	Giga	G
10^{12}	Billion	Trillion	Tera	T
10^{15}	Billiard	Quadrillion	Peta	P
10^{18}	Trillion	Quintillion	Exa	E
10^{21}	Trilliard	Sextillion	Zetta	Z
10^{24}	Quadrillion	Septillion	Yotta	Y
10^{27}	Quadrilliard	Octillion	Ronna	R
10^{30}	Quintillion	Nonillion	Quetta	Q

Definitions of terms according to DIN 1305

According to DIN 1305:1988, the following definitions and terms are specified:

1 Scope

This standard applies to the field of classical physics and its application in engineering and economic usage.

2 Mass

The mass m describes the characteristic of a body, which manifests itself in inertia effects in relation to a change in its motion state and also in its attraction for other bodies.

The mass of a body is decisive for its inertial behaviour. In Newton's law of forces, the inert mass m must therefore be used, while the associated forces are inertia forces. However, the mass is also simultaneously the cause of gravity (weight force). In this case, the heavy mass m must be used. These are, in terms of phenomena, differing characteristics of mass but are equivalent in all relationships.

3 Measured value of weight

Equation 1

In the case of a weighing operation in a fluid (liquid or gas) of the density $\rho_{fl,}$ the measured value of weight W is defined by the following relationship:

$$W = m \frac{1 - \frac{\rho_{fl}}{\rho}}{1 - \frac{\rho_{fl}}{\rho_G}}$$

where ρ is the density of the weighed bulk and ρ_G is the density of the units of weight.

4
Conventional measured weight value

The conventional measured weight value W_{std} is calculated on the basis of the equation from section 3 Measured value of weight with the standard conditions $\rho_{fl} = 1{,}2\ kg/m^3$ and $\rho_G = 8\,000\ kg/m^3$. In this case, the density of the weighed bulk at +20 °C must be inserted for ρ.

5
Force

The force F is the product of the mass m of a body and the acceleration a which it experiences or would experience as a result of the force F:

Equation 2

$$F = m \cdot a$$

6
Weight force

The weight force F_G of a body of mass m is the product of the mass m and the gravitational acceleration g:

Equation 3

$$F_G = m \cdot g$$

7
Weight

The word "weight" is predominantly used in three different senses:

- instead of "measured value of weight"
- as a shortened form of "weight force"

If misunderstandings could arise, the word "weight" should be replaced by the relevant term "measured value of weight" or "weight force".

8
Load

The word "load" is used in different senses in engineering (e.g. for power, force or for an object).

If misunderstandings could arise, the word "load" should be avoided.

Explanations

We live and have weight resting on the base of an ocean of air. In weighing, a correction is hardly ever made for the buoyancy of the air, although this would actually be necessary. We almost always accept the uncorrected measured value, which is also the basis for settlement of accounts in commerce where goods are sold by weight. It is necessary, however, to distinguish between the mass and the result of a weighing operation in air, namely the measured value of weight.
Where weighed items are of low density, as in the case of mineral oils, the relative difference between the mass and the measured value of weight is equivalent to approximately 1 per mil. It is smaller where weighed items are of high density. Air has a measured weight value of zero.
Bodies of the same mass but of differing density will have different measured weight values. The measured weight value of a body will also change if there is a change in the density of the surrounding air.
The measured weight value is also dependent on the weather.

Unit systems no longer to be used

The Physical System of Units

Like the SI System of Units, the Physical System of Units used the base values of length, mass and time, but used the following base unit for these values:

Base value	Base unit	
	Designation	Symbol
Length	Centimetre	cm
Mass	Gram	g
Time	Second	s

The Technical System of Units

The Technical System of Units used the following base values and base units:

Base value	Base unit	
	Designation	Symbol
Length	Metre	m
Force	Kilopond	kp
Time	Second	s

In the Technical System of Units, force was defined as the base value with the unit name "kilopond" (kp). All forces were compared with the gravitational attraction of the Earth (weight). In contrast to mass, however, the acceleration of free fall (and therefore the weight) is dependent on the location.

The following definition was therefore given:
1 kilopond is the force with which the mass of 1 kilogram exerts pressure on its support surface at the location of the standard acceleration of free fall (g_n = 9,806 65 m/s^2).

Equation 4

$$1\ \text{kp} = 1\ \text{kg} \cdot 9{,}806\,65\ \text{m}/\text{s}^2 = 9{,}806\,65\ \frac{\text{kg m}}{\text{s}^2}$$

The relationship between the International System of Units SI and the Technical System of Units is given by virtue of the fact that, in the SI system, the derived, coherent quantity with the unit name "Newton" was defined for the force:

Equation 5

$$1\ \text{N} = 1\ \frac{\text{kg m}}{\text{s}^2}$$

Therefore, 1 kp = 9,806 65 N.

For technical conversions from one system to the other, it is normally sufficient to use 1 kp = 9,81 N.

Conversions into the International System of Units SI

Anglo-American systems

The Anglo-American units are based on older, English systems of units and are still in common use in the USA.
They include the units of the fps system ("foot-pound-second").

Units in the fps system

The following table shows the conversion of the most significant units in the fps system and SI:

	fps system		SI (MKS)	
Length	1 ft	= (1/3) yd = 12 in	1 ft	= 0,304 8 m
Area	1 ft^2	= 144 in^2	1 ft^2	= 0,092 903 m^2
Volume	1 ft^3 1 gal (US)	= 1728 in^3 = 6,228 2 gal (UK) = 0,832 68 gal (UK)	1 ft^3	= 0,028 316 9 m^3
Velocity	1 ft/s 1 knot	= 0,681 818 mile/h = 1,687 7 ft/s	1 ft/s	= 0,304 8 m/s
Acceleration	1 ft/s^2		1 ft/s^2	= 0,304 8 m/s^2
Mass	1 lb 1 slug	= cwt/112 = 32,174 lb	1 lb 1 slug	= 0,453 592 kg = 14,593 9 kg
Force	1 lbf 1 pdl	 = 0,031 081 lbf	1 lbf 1 pdl	= 4,448 22 N = 0,138 255 N
Work	1 ft · lb 1 btu	= 0,323 832 cal (IT) = 252 cal_{IT} = 778,21 ft · lb	1 ft · lb 1 btu	= 1,355 82 J = 1,055 06 kJ
Pressure	1 lb/ft^2 1 lb/in^2 1 atm	= 6,944 4 · 10^{-3} lb/in^2 = 0,068 046 atm = 29,92 in Hg = 33,90 ft water	1 lb/ft^2 1 lb/in^2 1 atm	= 47,88 N/m^2 = 6 894,76 N/m^2 = 1,013 25 bar
Density	1 lb/ft^3 1 lb/gal	= 5,787 04 · 10^{-4} lb/in^3 = 6,228 2 lb/ft^3	1 lb/ft^3 1 lb/gal	= 16,018 5 kg/m^3 = 99,763 3 kg/m^3
Temperature	32 °F 212 °F	= 0 °C = 100 °C	1 °F	= 0,555 6 °C
Power	1 ft · lb/s	= 1,814 8 · 10^{-3} hp = 1,281 82 · 10^{-3} btu/s	1 ft · lb/s	= 1,353 34 W
Specific heat capacity	1 btu/(lb · deg F)		1 btu/(lb · deg F)	= 4,186 8 kJ/(kg · K)
Coefficient of thermal conductivity	1 btu/(ft · h · deg F)		1 btu/(ft · h · deg F)	= 1,730 6 W/(m · K)
Heat transfer (heat transition) coefficient	1 btu/(ft^2 · h · deg F)		1 btu/(ft^2 · h · deg F)	= 5,677 8 W/(m^2 · K)
Kinematic viscosity	1 ft^2/s		1 ft^2/s	= 0,092 903 m^2/s
Dynamic viscosity	1 lb/(ft · s)		1 lb/(ft · s)	= 1,488 16 kg/(m · s)

Source: DIN 1301-3:2018.

Length, area and volume dimensions

The following table shows the conversion from German to English dimensions for length, area and volume:

German – English			English – German		
Length dimensions					
1 mm	= 0,039 370 14 inches		1 pt = 1/864 foot	= 1/72 inch	= 0,352 78 mm
1 cm	= 0,393 701 47 inches		1 inch		= 25,399 956 mm
1 m	= 3,280 851 feet		1 foot	= 12 inches	= 304,799 472 mm
1 m	= 1,093 616 yards				= 0,304 799 m
1 m	= 0,546 808 fathoms		1 yard	= 3 feet	
1 km	= 0,621 372 statute miles			= 36 inches	= 0,914 398 m
1 km	= 0,539 614 nautical miles		1 fathom	= 2 yards	
1 km	= 0,539 037 Admiralty miles			= 6 feet	
1 German statute mile		= 7,5 km		= 72 inches	= 1,828 797 m
1 German nautical mile		= 1,852 km	1 stat. mile	= 880 fathoms	
1 geographical mile		= 7,420 438 54 km (15 miles = 1 degree of arc)		= 1760 yards	
				= 5 280 feet	
1 degree of arc		= 111,306 6 km		= 1 English mile	= 1,609 341 km
1 degree of meridian		= 111,120 6 km	1 common English mile	= 5 000 feet	= 1,523 995 km
			1 nautical mile	= 6 080 feet	= 1,853 178 km
			1 Admiralty mile	= 6 086,5 feet	= 1,855 16 km
				= 1/4 geographical mile	
				= 1/60 of degree of arc	
Area dimensions					
1 mm^2	= 0,001 550 0 1 square inch ($inch^2$)		1 sq. inch		= 6,451 578 cm^2
1 cm^2	= 0,155 006 35 square inch ($inch^2$)		1 sq. foot	= 144 sq. inches	= 929,027 2 cm^2
1 m^2	= 10,763 983 28 square feet ($foot^2$)				= 0,092 903 m^2
1 m^2	= 1,195 995 96 square yards		1 sq. yard	= 9 sq. feet	= 8 361,244 80 cm^2
1 a	= 100 m^2	= 0,024 711 acres	1 acre	= 160 sq. poles	
1 ha	= 100 a	= 2,471 063 acres		= 4 840 sq. yards	
1 km^2	= 100 ha	= 0,386 100 square miles		= 40,468 4 a	= 4 046,842 5 m^2
1 geographical square mile		= 55,062 91 km^2	1 sq. mile	= 640 acres	= 2,59 km^2
			1 sq. pole		= 25,298 676 m^2
			1 circular inch	= $\pi/4$ sq. inches	= 5,067 057 cm^2

Continuation of table, see Page 33.

Continuation of table, Length, area and volume dimensions, from Page 32.

German – English		English – German		
Volume dimensions				
1 cm^3	= 0,061 024 cubic inches ($inch^3$)	1 cub. inch		= 16,386 979 cm^3
1 dm^3	= 0,035 315 cubic feet ($foot^3$)	1 cub. foot	= 1728 cub. inches	= 28,316 700 dm^3
	= 61,024 061 cubic inches	1 cub. yard	= 27 cub. feet	= 0,764 551 m^3
1 m^3	= 1,307 957 cubic yards	1 reg. ton	= 100 cub. feet	= 2,831 670 m^3
	= 35,314 850 cubic feet	1 imperial gallon	= 277,26 cub. inches	= 4,543 454 l
1 m^3	= 0,353 148 register tons			
1 l	= 0,220 097 imperial gallons	1 bushel	= 8 gallons	= 36,347 632 l
1 l	= 0,027 512 bushels	1 imperial quarter	= 8 bushels	= 64 gallons
1 l	= 0,003 439 imperial quarters		= 290,781 056 l	= 2,907 811 hl
1 hl	= 100 l			
	= 0,343 901 imperial quarters			

Units of temperature The following table shows the conversion into various units of temperature:

T_K K Kelvin	t_C °C Degree Celsius	t_F °F Degree Fahrenheit	T_R °R Degree Rankin
$T_K = 273,15 + t_C$	$t_C = T_K - 273,15$	$t_F = \frac{9}{5} \cdot T_K - 459,67$	$T_R = \frac{9}{5} \cdot T_K$
$T_K = 255,38 + \frac{5}{9} \cdot t_F$	$t_C = \frac{5}{9}\left(t_F - 32\right)$	$t_F = 32 + \frac{9}{5} \cdot t_C$	$T_R = \frac{9}{5}\left(t_C + 273,15\right)$
$T_K = \frac{5}{9} \cdot T_R$	$t_C = \frac{5}{9}\left(T_R - 273,15\right)$	$t_F = T_R - 459,67$	$T_R = 459,67 + t_F$
Conversion of some temperatures			
0,00	−273,15	−459,67	0,00
+255,37	−17,78	0,00	+459,67
+273,15	0,00	+32,00	+491,67
+273,16[1)]	+0,01	+32,02	+491,69
+300,00	+26,85	+80,33	+540,00
+310,94	+37,78	+100	+559,67
+373,15	+100,00	+212	+671,67
+400,00	+126,85	+260,33	+720,00
+500,00	+226,85	+440,85	+900,00

1) The triple point of water is +0,01 °C.
This is the temperature point of pure water at which solid ice, liquid water and water vapour occur simultaneously in equilibrium (at 1013,25 hPa).
Temperature differential: 1 Kelvin = 1 degree Celsius = 1,8 degree Fahrenheit = 1,8 degree Rankin.

Roman numeral system

Definition

In the Roman numeral system, a distinction is made between cardinal numbers and ordinal numbers:

Cardinal numbers				Ordinal numbers		
I = 1	X = 10	C = 100	M = 1000	V = 5	L = 50	D = 500

Conversion table

The following table shows some conversion examples:

I	1	VII	7	XL	40	XCIX	99	DC	600
II	2	VIII	8	L	50	C	100	DCC	700
III	3	IX	9	LX	60	CC	200	DCCC	800
IV	4	X	10	LXX	70	CCC	300	CM	900
V	5	XX	20	LXXX	80	CD	400	CMXCIX	999
VI	6	XXX	30	XC	90	D	500	M	1000

Rules

In the formation of Roman numerals, the following rules apply:

- Notation starts from the left.
- Identical cardinal numbers are added consecutively. A maximum of 3 identical cardinal numbers may appear consecutively, while ordinal numbers are only written once:
  Permissible: III = 3; XX = 20
  Not permissible: XXXX = 40; VV = 10
- Smaller numbers are **added** to the **right** of larger numbers: VI = 6.
  Smaller numbers are **subtracted** to the **left** of larger numbers: IV = 4.
- Ordinal numbers may be added but **not subtracted**:
  Permissible: LV = 55
  Not permissible: VL = 45
- Cardinal numbers may only be subtracted from the next largest cardinal number or ordinal number:
  Permissible: IV = 4; XL = 40; CD = 400
  Not permissible: IC = 99; XM = 990
- Examples:
  1673 = MDCLXXIII; 1891 = MDCCCXCI; 1981 = MCMLXXXI

Alphabets

Greek alphabet

The Greek alphabet and some variant shapes of specific letters are shown:

Α α Alpha (a)	Β β Beta (b)	Γ γ Gamma (c)	Δ δ Delta (d)	Ε ϵ Epsilon (e)	Ζ ζ Zeta (z)
Η η Eta (e)	Θ ϑ Theta (th)	Ι ι Iota (i)	Κ κ Kappa (k)	Λ λ Lambda (l)	Μ μ Mu (m)
Ν ν Nu (n)	Ξ ξ Xi (x)	Ο ο Omicron (o)	Π π Pi (p)	Ρ ρ Rho (r)	Σ σ, ς Sigma (s)
Τ τ Tau (t)	Υ υ Upsilon (ü)	Φ φ, ϕ Phi (f)	Χ χ Chi (ch)	Ψ ψ Psi (ps)	Ω ω Omega (o)

Mathematics

General symbols, numbers, definitions

Mathematical symbols The following table shows a selection of the most important mathematical symbols.

Symbol	Designation
$+$	Plus
$-$	Minus
$\cdot$ or x	Multiplied by
/ or :	Divided by
$=$	Equals
$\neq$	Not equal to
$<$	Less than
$\leqq$	Less than or equal to
$>$	Greater than
$\geqq$	Greater than or equal to
$\approx$	Approximately equal to
$\ll$	Much less than
$\gg$	Much greater than
$\triangleq$	Corresponds to
$\ldots$	And so forth, to
$\sim$	Proportional
$\circ$	Interlinking, composition
Σ	Summation of
Π	Product

Symbol	Designation
$\sqrt{}$	Root of ($\sqrt[n]{}$ = nth root of)
n!	n factorial (example: $3! = 1 \cdot 2 \cdot 3 = 6$)
$\|x\|$	Absolute value of x
$\to$	Approaches
∞	Infinity
i or j	Imaginary unit, $i^2 = -1$
$\perp$	Perpendicular to
$\parallel$	Parallel to
$\measuredangle$	Angle
$\triangle$	Triangle
lim	Limit
Δ	Delta (difference between two values)
d	Total differential
∂	Partial differential
$\int$	Integral
log	Logarithm
ln	Logarithm to base e, $e = 1 + 1/1! + 1/2! + 1/3! + \ldots$
lg	Logarithm to base 10

Frequently used numbers The following table shows a selection of numbers or constants which are frequently used in mathematics.

Symbol	Value	Symbol	Value	Symbol	Value
e	2,718 282	ln 10	2,302 585	π	3,141 593
e^2	7,389 056	1/(ln 10)	0,434 294	$\sqrt{\pi}$	1,772 454
$1/e$	0,367 879	$\sqrt{2}$	1,414 214	$1/\pi$	0,318 310
lg e	0,434 294	$1/\sqrt{2}$	0,707 107	π^2	9,869 604
$\sqrt{e}$	1,648 721	$\sqrt{3}$	1,732 051	$180/\pi$	57,295 780
1/(lg e)	2,302 585			$\pi/180$	0,017 453

Golden section

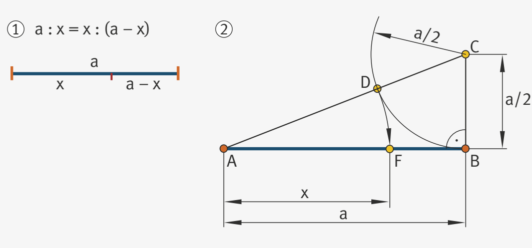
The golden section (division in extreme and mean ratio) is defined as the ratio a/x, where:

Equation 1

$$a : x = x : (a - x) \qquad \text{where} \quad \frac{a}{x} = \frac{(\sqrt{5}+1)}{2} \approx 1{,}618$$

Figure 1
Golden section

① Division in extreme and mean ratio
② Construction, general formulation



Pythagorean numbers

Pythagorean numbers are integers x, y, z, to which the following equation applies:

Equation 2

$$x^2 + y^2 = z^2$$

Triangles formed from the sides x, y, z in any unit of length are right-angled. If we apply:

Equation 3

$$x = 2pq \qquad y = p^2 - q^2 \qquad z = p^2 + q^2$$

and where p and q are random integers, this gives the Pythagorean numbers:

p	q	x	y	z	p	q	x	y	z
2	1	4	3	5	4	2	16	12	20
3	1	6	8	10	5	2	20	21	29
4	1	8	15	17	4	3	24	7	25
5	1	10	24	26	5	3	30	16	34
3	2	12	5	13	5	4	40	9	41

The following should apply:

Equation 4

$0 < q < p$	Natural numbers
p, q	Co-prime
$p + q$	Odd

Prime numbers The following table shows prime numbers and compound numbers that are not divisible by 2, 3 or 5, with their smallest factors. The numbers listed are below 1 000.

7		107		209	11	311		409		511	7	613		713	23	817	19	917	7
11		109		211		313		413	7	517	11	617		719		821		919	
13		113		217	7	317		419		521		619		721	7	823		923	13
17		119	7	221	13	319	11	421		523		623	7	727		827		929	
19		121	11	223		323	17	427	7	527	17	629	17	731	17	829		931	7
23		127		227		329	7	431		529	23	631		733		833	7	937	
29		131		229		331		433		533	13	637	7	737	11	839		941	
31		133	7	233		337		437	19	539	7	641		739		841	29	943	23
37		137		239		341	11	439		541		643		743		847	7	947	
41		139		241		343	7	443		547		647		749	7	851	23	949	13
43		143	11	247	13	347		449		551	19	649	11	751		853		953	
47		149		251		349		451	11	553	7	653		757		857		959	7
49	7	151		253	11	353		457		557		659		761		859		961	31
53		157		257		359		461		559	13	661		763	7	863		967	
59		161	7	259	7	361	19	463		563		667	23	767	13	869	11	971	
61		163		263		367		467		569		671	11	769		871	13	973	7
67		167		269		371	7	469	7	571		673		773		877		977	
71		169	13	271		373		473	11	577		677		779	19	881		979	11
73		173		277		377	13	479		581	7	679	7	781	11	883		983	
77	7	179		281		379		481	13	583	11	683		787		887		989	23
79		181		283		383		487		587		689	13	791	7	889	7	991	
83		187	11	287	7	389		491		589	19	691		793	13	893	19	997	
89		191		289	17	391	17	493	17	593		697	17	797		899	29		
91	7	193		293		397		497	7	599		701		799	17	901	17		
97		197		299	13	401		499		601		703	19	803	11	907			
101		199		301	7	403	13	503		607		707	7	809		911			
103		203	7	307		407	11	509		611	13	709		811		913	11		

Binomial coefficients

The binomial coefficient $\binom{n}{k}$ is defined as:

Equation 5

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad \text{where} \quad n \geqq k$$

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	$\binom{n}{5}$	$\binom{n}{6}$	$\binom{n}{7}$
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	6	4	1			
5	1	5	10	10	5	1		
6	1	6	15	20	15	6	1	
7	1	7	21	35	35	21	7	1
8	1	8	28	56	70	56	28	8
9	1	9	36	84	126	126	84	36
10	1	10	45	120	210	252	210	120
11	1	11	55	165	330	462	462	330
12	1	12	66	220	495	792	924	792
13	1	13	78	286	715	1 287	1716	1716
14	1	14	91	364	1 001	2 002	3 003	3 432
15	1	15	105	455	1 365	3 003	5 005	6 435

n	$\binom{n}{8}$	$\binom{n}{9}$	$\binom{n}{10}$	$\binom{n}{11}$	$\binom{n}{12}$	$\binom{n}{13}$	$\binom{n}{14}$	$\binom{n}{15}$
8	1							
9	9	1						
10	45	10	1					
11	165	55	11	1				
12	495	220	66	12	1			
13	1 287	715	286	78	13	1		
14	3 003	2 002	1 001	364	91	14	1	
15	6 435	5 005	3 003	1 365	455	105	15	1

Arithmetic

Laws and rules The following laws and rules are defined in arithmetic:

Rules of signs The following rules of signs apply:

Equation 6

$$a+(-b) = a-b \qquad a-(-b) = a+b \qquad a\cdot(-b) = -a\,b$$

$$(-a)\cdot(-b) = a\,b \qquad (-a)\cdot b = -a\,b \qquad (-a)/b = -\frac{a}{b}$$

$$a/(-b) = -\frac{a}{b} \qquad (-a)/(-b) = \frac{a}{b}$$

Commutative law The commutative law of addition and multiplication is:

Equation 7

$$a+b = b+a \qquad a\cdot b = b\cdot a$$

Associative law The associative law of addition and multiplication is:

Equation 8

$$(a+b)+c = a+(b+c) \qquad (a\,b)\cdot c = a\cdot(b\,c) = a\cdot b\cdot c$$

Binomials The following equations give examples of products of algebraic sums and binomial formulae.

Equation 9

$$(a+b)\cdot(c+d) = a\,c+a\,d+b\,c+b\,d$$

$$(a\pm b)^2 = a^2 \pm 2\,a\,b+b^2$$

$$(a+b)\cdot(a-b) = a^2-b^2$$

$$(a\pm b)^3 = a^3 \pm 3\,a^2\,b+3\,a\,b^2 \pm b^3$$

$$(a\pm b)^n = \sum_{k=0}^{n} (\pm 1)^k \binom{n}{k} a^k\,b^{n-k}$$

$$(a+b+c)^2 = a^2+b^2+c^2+2\,a\,b+2\,a\,c+2\,b\,c$$

Mean values The arithmetic mean is:

Equation 10

$$\frac{a+b}{2} \;;\; \frac{a+b+c}{3} \;;\; \ldots \qquad A_n = \frac{1}{n}\left(a_1+a_2+\ldots+a_n\right)$$

The geometric mean is:

Equation 11

$$\sqrt{a\cdot b} \;;\; \sqrt[3]{a\cdot b\cdot c} \;;\; \ldots \qquad G_n = \sqrt[n]{a_1\cdot a_2\cdot\ldots\cdot a_n}$$

The harmonic mean is:

Equation 12

$$\frac{2\,a\,b}{a+b} = 2\Big/\left(\frac{1}{a}+\frac{1}{b}\right) \qquad H_n = \frac{n}{1/a_1+1/a_2+\ldots+1/a_n}$$

The following applies:

Equation 13

$$H_n \leqq G_n \leqq A_n$$

Powers
Equation 14

The following equations give examples of how powers are formed:

$$a^n \cdot a^m = a^{n+m} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^n \cdot b^n = (ab)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \qquad (a^m)^n = a^{m \cdot n} \qquad a^{-n} = \frac{1}{a^n}$$

$$0^n = 0 \qquad a^0 = 1 \quad (a \neq 0)$$

Roots
Equation 15

The following equations give examples of how roots are formed:

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \left(\sqrt[n]{a}\right)^n = a \qquad \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \qquad \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad \sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Algebra

Algebraic equation of the 2nd degree

An algebraic equation of the 2nd degree (referred to as a quadratic equation) is described below.

Quadratic equation
Equation 16

The solutions to a quadratic equation are as follows:

Quadratic equation: $a\,x^2 + b\,x + c = 0$

Solutions: $x_1, x_2 = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Discriminant
Equation 17

If we define the discriminant Δ for general instances as:

$$\Delta = b^2 - 4\,a\,c$$

we arrive at the following solutions:

Equation 18

$\Delta > 0$	2 different real solutions
$\Delta = 0$	2 identical real solutions
$\Delta < 0$	2 conjugate complex solutions

Normal form
Equation 19

The normal form of the quadratic equation is:

Normal form: $x^2 + p\,x + q = 0$

Solutions: $x_1, x_2 = -\dfrac{p}{2} \pm \sqrt{(p/2)^2 - q}$

Algebraic equation of the 1st degree
Equation 20

Two equations of the 1st degree with 2 unknown quantities can be calculated with the aid of matrices:

$$a_{11}\,x_1 + a_{12}\,x_2 = k_1$$

$$a_{21}\,x_1 + a_{22}\,x_2 = k_2$$

Determinant

The equations give the corresponding determinant D and the counter determinants D_{x1} and D_{x2}:

Equation 21

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\,a_{22} - a_{21}\,a_{12}$$

$$D_{x1} = \begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix} = k_1\,a_{22} - k_2\,a_{12}$$

$$D_{x2} = \begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix} = a_{11}\,k_2 - a_{21}\,k_1$$

Solution
Equation 22

For $D \neq 0$, this gives the clear solution:

$$x_1 = \frac{D_{x1}}{D} \qquad x_2 = \frac{D_{x2}}{D}$$

Logarithms
Equation 23

The general logarithm is defined as:

$\log_b a = c$ and means that $b^c = a$ $\quad a > 0,\ b > 1$

In this case, b denotes the base, a denotes the anti-logarithm and c denotes the logarithm.

Logarithmic laws
Equation 24

The following relationships are regarded as logarithmic laws:

$$\log_b (a\,c) = \log_b a + \log_b c$$

$$\log_b \frac{a}{c} = \log_b a - \log_b c$$

$$\log_b \left(a^n\right) = n \log_b a$$

$$\log_b \sqrt[n]{a} = \frac{1}{n} \log_b a$$

in addition to the following special cases:

Equation 25

$$\log_b 0 = -\infty \qquad \log_b 1 = 0 \qquad \log_b b = 1 \qquad \log_b \infty = \infty$$

Natural logarithm

Logarithms to the base e = 2,71828128459... are called natural logarithms.
These are written as ln a instead of $\log_e$ a.

The following relationships apply:

Equation 26

$$\ln\left(e^{\pm n}\right) = \pm n$$

$$\ln\left(a \cdot 10^n\right) = \ln a + \ln\left(10^n\right) \qquad \ln\left(a/10^n\right) = \ln a - \ln\left(10^n\right)$$

The module M_b to base b is defined as:

Equation 27

$$M_b = \log_b e = 1/\ln b$$

$$\log_b a = M_b \ln a$$

Common (Briggs') logarithm

Logarithms to the base 10 are called common or Briggs' logarithms.
These are written as lg a instead of $\log_{10}$ a.

The following relationships apply:

Equation 28

$$\lg\left(10^{\pm n}\right) = \pm n$$

$$\lg\left(a \cdot 10^n\right) = \lg a + n \qquad \lg\left(a/10^n\right) = \lg a - n$$

The logarithmic laws which apply to the common logarithm are (see also Equation 24):

Equation 29

$$\lg\left(u \cdot v\right) = \lg u + \lg v$$

$$\lg \frac{u}{v} = \lg u - \lg v \qquad \lg \frac{v}{u} = -\lg \frac{u}{v}$$

$$\lg u^n = n \cdot \lg u \qquad \lg \sqrt[n]{u} = \frac{1}{n} \cdot \lg u$$

Conversion of logarithms

The following relationships apply between natural and common logarithms:

Equation 30

$$M_{10} = 0{,}4342944819 = \lg e = 1/\ln 10 = 1/2{,}3025850930$$

$$\ln x = \ln 10 \lg x = 2{,}3025850930 \lg x$$

$$\lg x = \lg e \ln x = 0{,}4342944819 \ln x$$

$$\ln 10 \lg e = 1$$

Complex numbers
Equation 31

A complex number z consists of a real and an imaginary part:

$$z = x + i\,y$$

The following applies to the imaginary unit i:

Equation 32

$$i = \sqrt{-1} \qquad i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1 \qquad 1/i = -i$$

Conjugation
Equation 33

The complex number conjugated to z is:

$$z^* = x - i\,y$$

Absolute value
Equation 34

The absolute value of z is:

$$r = \sqrt{z \cdot z^*} = \sqrt{(x + i\,y) \cdot (x - i\,y)} = \sqrt{x^2 + y^2}$$

Argument
Equation 35

The argument of z is:

$$\varphi = \arctan\,(y/x)$$

Normal form
Equation 36

The normal form of z is:

$$z = x + i\,y = r\,(\cos\,\varphi + i\,\sin\,\varphi)$$

Exponential form
Equation 37

The following is defined for the exponential form of the complex number z:

$$e^{i\varphi} = \cos\,\varphi + i\,\sin\,\varphi \qquad \text{Euler's equation}$$

$$z = r \cdot (\cos\,\varphi + i\,\sin\,\varphi) = r \cdot e^{i\varphi}$$

$$e^{-i\varphi} = \cos\,\varphi - i\,\sin\,\varphi$$

The trigonometric functions and their relationship with the complex exponential function are presented in the section Circular functions (trigonometric functions) from Page 49 to Page 51.

Power
Equation 38

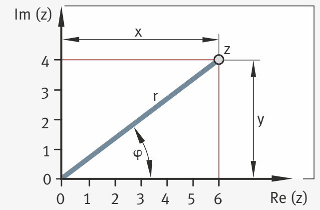
The power of z is:

$$z^n = r^n \cdot (\cos\,n\varphi + i\,\sin\,n\varphi)$$

Representation
Figure 2
Representation of the complex number

Im (z) = imaginary axis
Re (z) = real axis

The complex number z corresponds to a point in a Gaussian plane:



The following is an example of a calculation based on the graph:

Equation 39

$$z = 6 + i\,4 \quad \text{where} \quad x = 6; \quad y = 4$$

$$r = \sqrt{36+16} = \sqrt{52} \approx 7{,}2$$

$$\varphi = \arctan\left(y/x\right) = \arctan 0{,}667 \approx 33{,}7°$$

$$z = 7{,}2 \cdot (\cos 33{,}7° + i \sin 33{,}7°)$$

Sequences and progressions

Arithmetic sequence
Equation 40

In the case of an arithmetic sequence, the difference d between two consecutive terms is constant:

$$a,\ a+d,\ a+2\,d,\ a+3\,d,\ \ldots,\ a+(n-1)\,d$$

Arithmetic progression
Equation 41

In the case of an arithmetic progression, each term is the arithmetic mean of its two adjacent terms:

$$a + (a+d) + (a+2\,d) + (a+3\,d) + \ldots + [a+(n-1)\,d]$$

This gives the following for the k-th term and the final term (n):

Equation 42

$$a_k = a + (k-1)\,d \qquad\qquad a_n = a + (n-1)\,d$$

and for the sum:

Equation 43

$$S = \frac{n}{2}\cdot(a + a_n) = \frac{n}{2}\cdot\left[2\,a + (n-1)\,d\right]$$

Geometric sequence
Equation 44

In the case of a geometric sequence, the quotient q of two consecutive terms is constant:

$$a_1\,,\ a_1 \cdot q\,,\ a_1 \cdot q^2,\ a_1 \cdot q^3,\ \ldots,\ a_1 \cdot q^{n-1}$$

Geometric progression
Equation 45

In the case of a geometric progression, each term is the geometric mean of its two adjacent terms:

$$a_1 + a_1 \cdot q + a_1 \cdot q^2 + a_1 \cdot q^3 + \ldots + a_1 \cdot q^{n-1}$$

This gives the following for the k-th term and the final term (n):

Equation 46

$$a_k = a_1 \cdot q^{k-1} \qquad a_n = a_1 \cdot q^{n-1}$$

and for the sum:

Equation 47

$$S = \frac{a_1 - a_n q}{1 - q} \quad \text{where} \quad q < 1$$

$$S = \frac{a_1 \left(1 - q^n\right)}{1 - q} \quad \text{where} \quad q < 1$$

$$S = \frac{a_n q - a_1}{q - 1} \quad \text{where} \quad q > 1$$

$$S = \frac{a_1 \left(q^n - 1\right)}{q - 1} \quad \text{where} \quad q > 1$$

Analysis

Derivatives and differentials

The differentiation rules specified below apply to the formation of derivatives (differential quotients).

Derivative of sum or difference
Equation 48

The following applies to the derivative of a sum or difference:

$$y = u(x) \pm v(x) \qquad y' = u'(x) \pm v'(x)$$

Derivative of product or quotient
Equation 49

The following applies to the derivative of a product or quotient:

$$y = u(x) \cdot v(x) \qquad y' = v(x) \cdot u'(x) + u(x) \cdot v'(x)$$

$$y = \frac{u(x)}{v(x)} \qquad y' = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{\left[v(x)\right]^2}$$

Chain rule
Equation 50

The following chain rule also applies:

$$y = u \circ v \qquad y'(x) = (u \circ v)'(x) = u'(v(x)) \cdot v'(x)$$

Differential forms of the basic functions

A selection of 1st derivatives of elementary functions can be found below:

Function y(x)	1st derivative y'(x)	Function y(x)	1st derivative y'(x)
$y = a$	$y' = 0$	$y = \sin x$	$y' = \cos x$
$y = x$	$y' = 1$	$y = \sin(ax)$	$y' = a \cdot \cos(a\,x)$
$y = mx + b$	$y' = m$	$y = \cos x$	$y' = -\sin x$
$y = ax^n$	$y' = n \cdot a \cdot x^{n-1}$	$y = \tan x$	$y' = 1/\cos^2 x$
$y = \sqrt{x}$	$y' = 1/\left(2 \cdot \sqrt{x}\right)$	$y = \cot x$	$y' = -1/\sin^2 x$
$y = 1/x$	$y' = -1/x^2$	$y = \ln \sin x$	$y' = \cot x$
$y = a^x$	$y' = a^x \cdot \ln a$	$y = \ln \tan x$	$y' = 2/\sin(2\,x)$
$y = e^x$	$y' = e^x$	$y = \arcsin x$	$y' = 1/\sqrt{1-x^2}$
$y = e^{ax}$	$y' = a \cdot e^{ax}$	$y = \arccos x$	$y' = -1/\sqrt{1-x^2}$
$y = x^x$	$y' = x^x \cdot (1+\ln x)$	$y = \arctan x$	$y' = 1/\left(1+x^2\right)$
$y = \log_a x$	$y' = \frac{1}{x} \cdot \log_a e$	$y = \operatorname{arccot} x$	$y' = -1/\left(1+x^2\right)$
$y = \ln x$	$y' = \frac{1}{x}$	$y = \sinh x$	$y' = \cosh x$

Integration of functions

In contrast to differentiation, there are usually no general algorithms for solving the integrals in the integration of functions. The two methods "integration by substitution" and "integration by parts" provide a possible means of solution.
The antiderivatives for a number of elementary functions are listed in the table Integrals of antiderivatives (basic integrals) on Page 48.

Integration by substitution
Equation 51

In integration by substitution, the following applies:

$$\int_a^b u\left(v(t)\right) \cdot v'(t)\,dt = \int_{v(a)}^{v(b)} u(x)\,dx$$

Integration by parts
Equation 52

In integration by parts, the following applies:

$$\int_a^b u'(x) \cdot v(x)\,dx = \left[u(x) \cdot v(x)\right]_a^b - \int_a^b u(x) \cdot v'(x)\,dx$$

Integrals of antiderivatives (basic integrals)

Integration is the reverse of differentiation.

A selection of basic integrals can be found below:

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $[n \neq -1]$	$\int \cosh x \, dx = \sinh x + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cosh^2 x} = -\tanh x + C$
$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$	$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$
$\int \ln x \, dx = x \ln x - x + C$	$\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arcsinh} x + C = \ln\left(x + \sqrt{x^2+1}\right) + C$
$\int a^{bx} \, dx = \frac{1}{b \ln a} a^{bx} + C$	$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh} x + C = \ln\left(x + \sqrt{x^2-1}\right) + C$
$\int a^x \ln a \, dx = a^x + C$	$\int \frac{dx}{1+x^2} = \arctan x + C = -\operatorname{arccot} x + C$
$\int \sin x \, dx = -\cos x + C$	$\int \frac{dx}{1-x^2} = \operatorname{arctanh} x + C = \frac{1}{2} \ln \frac{1+x}{1-x} + C$ for $[x^2 < 1]$
$\int \cos x \, dx = \sin x + C$	$\int \frac{dx}{1-x^2} = \operatorname{arccoth} x + C = \frac{1}{2} \ln \frac{x+1}{x-1} + C$ for $[x^2 > 1]$
$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{\sqrt{1+x}}{\sqrt{1-x}} \, dx = \arcsin x - \sqrt{1-x^2} + C$
$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{x\sqrt{x^2-1}} = \arccos \frac{1}{x} + C$
$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x\sqrt{1 \pm x^2}} = -\ln \frac{1+\sqrt{1 \pm x^2}}{x} + C$

Geometry

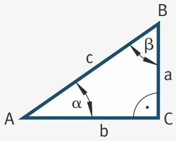
Circular functions (trigonometric functions)

Circular or trigonometric functions are defined as:

Equation 53

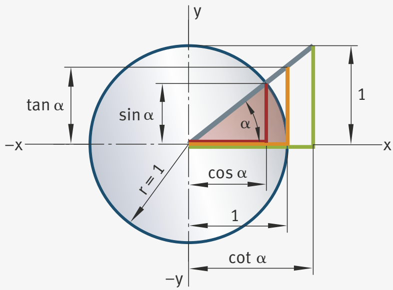
$$\sin\alpha = \frac{a}{c} \qquad \cos\alpha = \frac{b}{c} \qquad \tan\alpha = \frac{a}{b} \qquad \cot\alpha = \frac{b}{a}$$

Figure 3
Right-angled triangle



This is represented on the unit circle as follows:

Figure 4
Trigonometric functions on the unit circle



φ =	±α	90 ± α	180 ± α	270 ± α
sin φ =	±sin α	cos α	∓sin α	–cos α
cos φ =	cos α	∓sin α	–cos α	±sin α
tan φ =	±tan α	∓cot α	±tan α	∓cot α
cot φ =	±cot α	∓tan α	±cot α	∓tan α

Conversion from degree size to arc size:

Equation 54

$$\widehat{\alpha} = \operatorname{arc}\alpha = \frac{\pi\cdot\alpha}{180^\circ}\,\text{rad} = \frac{\alpha}{57{,}3^\circ}$$

$$\widehat{1^\circ} = \operatorname{arc} 1^\circ = \frac{\pi}{180} = 0{,}017\,435$$

$$\operatorname{arc} 57{,}3^\circ = 1$$

Relationships between trigonometric functions

Equation 55

The relationships described below also apply between the trigonometric functions:

$$\cos^2\alpha + \sin^2\alpha = 1 \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cot\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$$

$$\sin 2\alpha = 2\sin\alpha\,\cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan 2\alpha = \frac{2}{\cot\alpha - \tan\alpha} \qquad \cot 2\alpha = \frac{\cot\alpha - \tan\alpha}{2}$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \qquad \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\sin\frac{\alpha}{2} = \frac{1}{2}\cdot\sqrt{2 - 2\cos\alpha} \qquad \cos\frac{\alpha}{2} = \frac{1}{2}\cdot\sqrt{2 + 2\cos\alpha}$$

Addition theorems for trigonometric functions

Equation 56

The addition theorems for trigonometric functions are:

$$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\cdot\tan\beta}$$

$$\cot(\alpha \pm \beta) = \frac{\cot\alpha\cdot\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha \pm \beta}{2}\cdot\cos\frac{\alpha \mp \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cdot\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\cdot\sin\frac{\alpha - \beta}{2}$$

$$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cdot\cos\beta}$$

$$\cot\alpha \pm \cot\beta = \frac{\sin(\beta \pm \alpha)}{\sin\alpha\cdot\sin\beta}$$

Euler's equation
Equation 57

With the aid of Euler's equation $e^{\pm i x} = \cos x \pm i \sin x$ we arrive at:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad ; \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \text{where} \quad i = \sqrt{-1}$$

Hyperbolic functions

The following relationships apply to hyperbolic functions (where, for example, "sinh" is read as "hyperbolic sine"):

Equation 58

$$\sinh x = \left(e^{x} - e^{-x}\right)/2 \qquad \cosh x = \left(e^{x} + e^{-x}\right)/2$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\sinh x + \cosh x = e^{x} \qquad \sinh 2x = 2 \sinh x \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \cosh 2x = \cosh^2 x + \sinh^2 x$$

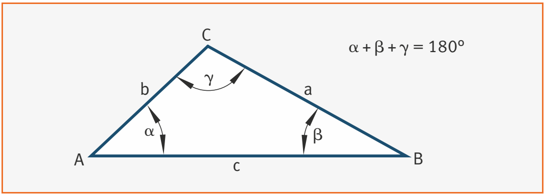
Ultimately arriving at (where "ar" is read as "Area"):

Equation 59

$$y = \sinh x \qquad x = \operatorname{ar} \sinh y$$

Plane triangle
Figure 5
Plane triangle

The plane triangle is marked as follows:



Sine law
Equation 60

This gives the sine law:

$$a : b : c = \sin \alpha : \sin \beta : \sin \gamma$$

Cosine law
Equation 61

and the cosine law:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$$

The following applies on the same basis to the remaining angles:

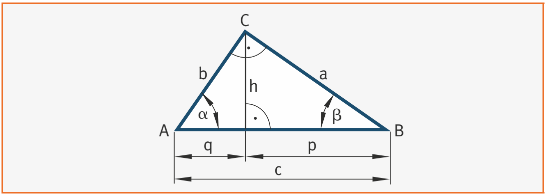
Equation 62

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$

Right-angled triangle
Figure 6
Right-angled triangle

The right-angled triangle is represented as follows:



The following rules apply to right-angled triangles:

Pythagorean theorem
Equation 63

The Pythagorean theorem:

$$c^2 = a^2 + b^2 \quad ; \qquad c = \sqrt{a^2 + b^2}$$

Altitude theorem
Equation 64

The altitude theorem:

$$h^2 = p \cdot q \quad ; \qquad h = \sqrt{p \cdot q}$$

Cathetus theorem
Equation 65

The cathetus theorem:

$$a^2 = p \cdot c \quad ; \quad a = \sqrt{p \cdot c} \qquad \text{and} \qquad b^2 = q \cdot c \quad ; \quad b = \sqrt{q \cdot c}$$

Trigonometric functions

The following aspect ratios apply to the right-angled triangle:

$\sin\alpha = a : c$	Opposite side : hypotenuse	$\sin\beta = b : c$
$\cos\alpha = b : c$	Adjacent side : hypotenuse	$\cos\beta = a : c$
$\tan\alpha = a : b$	Opposite side : adjacent side	$\tan\beta = b : a$
$\cot\alpha = b : a$	Adjacent side : opposite side	$\cot\beta = a : b$

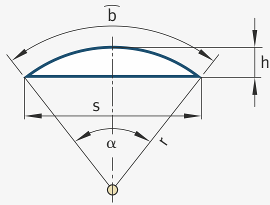
This gives the trigonometric functions:

Cathetus	a =	$\sqrt{c^2 - b^2}$	$b \cdot \tan\alpha$	$b \cdot \cot\beta$	$c \cdot \sin\alpha$	$c \cdot \cos\beta$
	b =	$\sqrt{c^2 - a^2}$	$a \cdot \tan\beta$	$a \cdot \cot\alpha$	$c \cdot \sin\beta$	$c \cdot \cos\alpha$
Hypotenuse	c =	$\sqrt{a^2 + b^2}$	$\frac{a}{\sin\alpha}$	$\frac{a}{\cos\beta}$	$\frac{b}{\sin\beta}$	$\frac{b}{\cos\alpha}$
Angle	α =	$90^\circ - \beta$	$\sin\alpha = \frac{a}{c}$	$\tan\alpha = \frac{a}{b}$	$\cos\alpha = \frac{b}{c}$	$\cot\alpha = \frac{b}{a}$
	β =	$90^\circ - \alpha$	$\sin\beta = \frac{b}{c}$	$\tan\beta = \frac{b}{a}$	$\cos\beta = \frac{a}{c}$	$\cot\beta = \frac{a}{b}$
Area	A =	$\frac{a \cdot b}{2}$	$\frac{a \cdot c \cdot \sin\beta}{2}$	$\frac{a^2 \cdot \tan\beta}{2}$	$\frac{b \cdot c \cdot \cos\beta}{2}$	$\frac{b^2 \cdot \cot\beta}{2}$
		$\frac{c^2 \cdot \sin\alpha \cdot \cos\alpha}{2}$	$\frac{b \cdot c \cdot \sin\alpha}{2}$	$\frac{b^2 \cdot \tan\alpha}{2}$	$\frac{a \cdot c \cdot \cos\alpha}{2}$	$\frac{a^2 \cdot \cot\alpha}{2}$

Arc lengths, arc heights, chord lengths, circular segments

Arc length, arc height, chord length and circular segment are calculated as follows:

Figure 7
Arc of circle



Chord length
Equation 66

Thus, the chord length is:

$$s = 2r\sin\frac{\alpha}{2}$$

Arc height
Equation 67

the arc height is:

$$h = r\left(1-\cos\frac{\alpha}{2}\right) = \frac{s}{2}\tan\frac{\alpha}{4} = 2r\sin^2\frac{\alpha}{4}$$

Arc length
Equation 68

the arc length is:

$$b = \pi r\frac{\alpha}{180°} = \sqrt{s^2+\frac{16}{3}h^2} \approx 0{,}017\,453\ r\cdot\alpha$$

Content of the circular segment
Equation 69

the content of the circular segment is:

$$A = \frac{r^2}{2}\left(\frac{\pi}{180°}\alpha - \sin\alpha\right)$$

Content of the circular sector
Equation 70

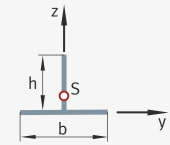
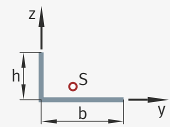
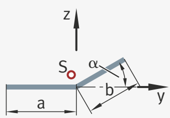
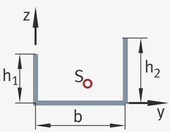
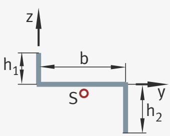
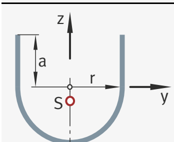
and the content of the circular sector is:

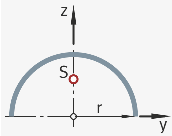
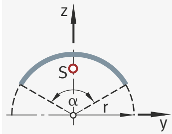
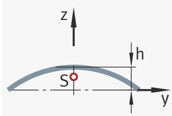
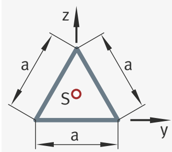
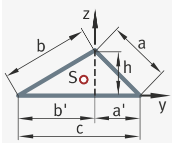
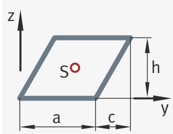
$$A = \frac{\alpha}{360°}\pi r^2 \approx 0{,}008\,726\,65\ \alpha r^2$$

If b = r, then
$\alpha = 57°17'44{,}86'' = 57{,}295\,779\,5° = 206\,264{,}86'' = 1$ rad

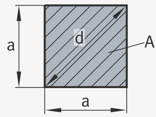
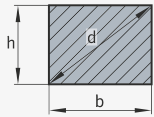
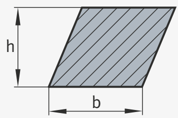
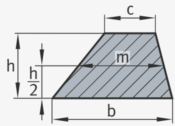
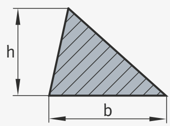
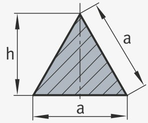
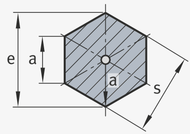
Centre of gravity of lines

The centre of gravity of lines is calculated using:

	$z_S = \frac{h^2}{2(b+h)}$
	$y_S = \frac{b^2}{2(b+h)}$ $z_S = \frac{h^2}{2(b+h)}$
	$y_S = \frac{a^2 + b^2 \cos\alpha}{2(a+b)}$ $z_S = \frac{b^2 \sin\alpha}{2(a+b)}$
	$y_S = \frac{b(b/2 + h_2)}{b + h_1 + h_2}$ $z_S = \frac{h_1^2 + h_2^2}{2(b + h_1 + h_2)}$
	$y_S = \frac{b(b/2 + h_2)}{b + h_1 + h_2}$ $z_S = \frac{h_1^2 - h_2^2}{2(b + h_1 + h_2)}$
	$z_S = \frac{a^2 - 2r^2}{2a + \pi r}$

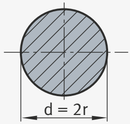
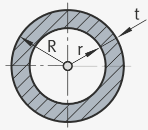
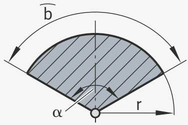
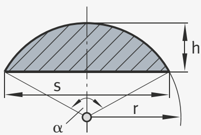
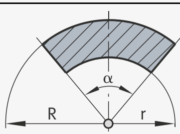
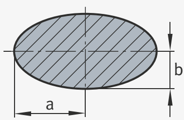
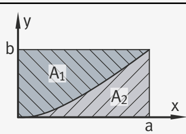
	$z_S = \frac{2r}{\pi}$
	$z_S = \frac{2r \sin\left(\frac{\alpha}{2}\right)}{\alpha}$ α in radians
	$z_S \approx 2h/3$
	$z_S = \frac{a}{6}\sqrt{3}$
	$y_S = \frac{a'(a+a') - b'(b+b')}{2(a+b+c)}$ $z_S = \frac{h(a+b)}{2(a+b+c)}$
	$y_S = \frac{a+c}{2}$ $z_S = \frac{h}{2}$

Calculating surfaces Geometric surfaces are calculated using:

Square		$A = a^2$ $a = \sqrt{A}$ $d = a\sqrt{2}$
Rectangle		$A = b\,h$ $d = \sqrt{b^2 + h^2}$
Parallelogram		$A = b\,h$ $b = \frac{A}{h}$
Trapezoid		$m = \frac{b+c}{2}$ $A = m\,h$
Triangle		$A = \frac{b\,h}{2}$ $b = \frac{2\,A}{h}$
Equilateral triangle		$A = \frac{a^2}{4}\sqrt{3}$ $h = \frac{a}{2}\sqrt{3}$
Regular hexagon		$A = \frac{3\,a^2\sqrt{3}}{2}$ $e = 2\,a$ $e \approx 1{,}155\ s$ $s = a\sqrt{3} = e\frac{\sqrt{3}}{2}$ $s \approx 0{,}866\ e$

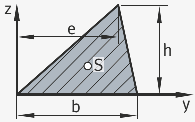
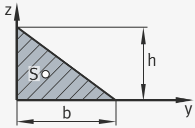
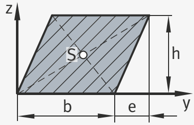
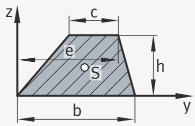
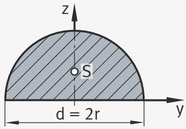
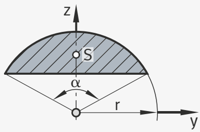
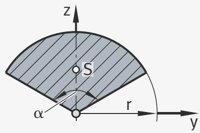
Continuation of table, see Page 56.

Continuation of table, Calculating surfaces, from Page 55.

Circle		$A = \frac{d^2\pi}{4} = r^2\pi \approx 0{,}785\,d^2$ $U = 2r\pi = d\pi$
Annulus		$A = \pi\left(R^2 - r^2\right) = \left(2r + t\right)\pi t$ $t = R - r$
Circular sector		$A = r^2\pi\frac{\alpha}{360°} = \frac{b\,r}{2}$ $b = r\pi\frac{\alpha}{180°}$
Circular segment		$A = r^2\pi\frac{\alpha}{360°} - \frac{r^2}{2}\sin\alpha$ $\approx \frac{h}{6s}\left(3h^2 + 4s^2\right)$ $t = r - h$ $\qquad r = \frac{h}{2} + \frac{s^2}{8h}$ $s = 2r\sin\frac{\alpha}{2} = \frac{r^2}{t}\sin\alpha$ $h = r\left(1 - \cos\frac{\alpha}{2}\right)$
Annular sector		$A = \frac{\pi\left(R^2 - r^2\right)\alpha}{360°}$
Ellipse		$A = a\,b\,\pi$ $U \approx \left(a + b\right)\pi$
Polynomial surface		$y = b\left(x/a\right)^n$ $A_1 = \frac{n}{n+1}a\,b$ $\qquad A_2 = \frac{1}{n+1}a\,b$

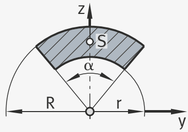
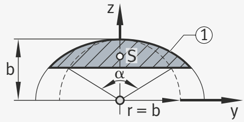
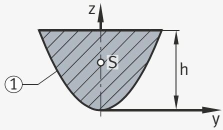
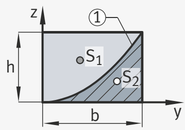
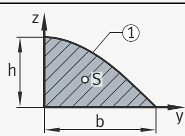
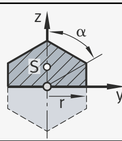
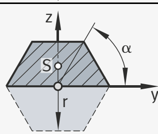
Centre of gravity of plane surfaces

The centre of gravity of plane surfaces is calculated using:

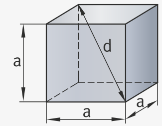
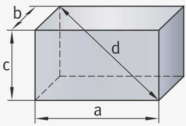
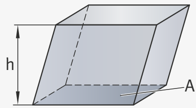
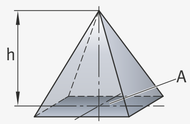
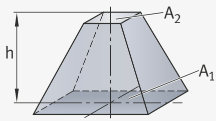
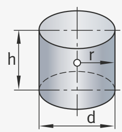
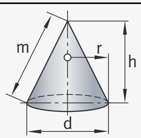
Triangle		$y_S = \frac{b+e}{3}$ $z_S = \frac{h}{3}$
Right-angled triangle		$y_S = \frac{b}{3}$ $z_S = \frac{h}{3}$
Parallelogram		$y_S = \frac{b+e}{2}$ $z_S = \frac{h}{2}$
Trapezoid		$y_S = \frac{b^2 - c^2 + e\left(b+2c\right)}{3\left(b+c\right)}$ $z_S = \frac{h\left(b+2c\right)}{3\left(b+c\right)}$
Semicircle		$z_S = \frac{4\,r}{3\,\pi}$
Circular segment		$z_S = \frac{4\,r\,\sin^3\left(\frac{\alpha}{2}\right)}{3\left(\alpha - \sin\alpha\right)}$ α in radians
Circular sector		$z_S = \frac{4\,r\,\sin\left(\frac{\alpha}{2}\right)}{3\,\alpha}$ α in radians

Continuation of table, see Page 58.

Continuation of table, Centre of gravity of plane surfaces, from Page 57.

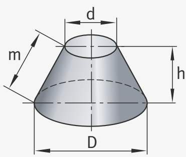
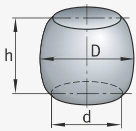
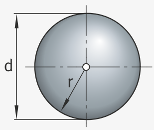
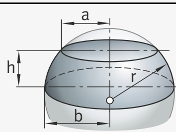
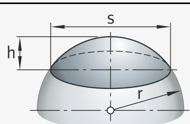
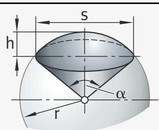
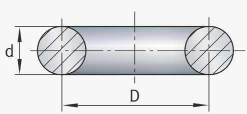
Annular sector		$z_S = \frac{4\left(R^3-r^3\right)\sin\left(\frac{\alpha}{2}\right)}{3\left(R^2-r^2\right)\alpha}$ α in radians
Elliptical segment ① Ellipse		$z_S = \frac{4\,r\sin^3\left(\frac{\alpha}{2}\right)}{3\left(\alpha-\sin\alpha\right)}$ α in radians
Parabola segment 1 ① Parabola		$z_S = \frac{3}{5}h$
Parabola segment 2 ① Parabola		$y_{S_1} = \frac{3}{8}b$ $y_{S_2} = \frac{3}{4}b$ $z_{S_1} = \frac{3}{5}h$ $z_{S_2} = \frac{3}{10}h$
Cosine segment ① Cosine line		$y_S = \left(1-\frac{2}{\pi}\right)b$ $z_S = \frac{\pi}{8}h$
Hexagon half 1		$z_S = \frac{4\,r}{3\,\pi}\cdot\frac{\alpha\left(3+\cos\alpha\right)}{4\sin\alpha}$ α in radians
Hexagon half 2		$z_S = \frac{4\,r}{3\,\pi}\cdot\frac{\alpha}{2\sin\left(\frac{\alpha}{2}\right)}$ α in radians

Calculating solids The volume of solids (volume V, surface area O, lateral area M) is calculated using:

Cube		$V = a^3$ $O = 6\,a^2$ $d = a\sqrt{3}$
Cuboid		$V = a\,b\,c$ $O = 2\left(a\,b + a\,c + b\,c\right)$ $d = \sqrt{a^2 + b^2 + c^2}$
Oblique cuboid		$V = A\,h$ (Cavalieri's principle)
Pyramid		$V = \frac{A\,h}{3}$
Truncated pyramid		$V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1\,A_2}\right) \approx h\,\frac{A_1 + A_2}{2}$
Cylinder		$V = \frac{d^2\pi}{4}\,h$ $O = 2\,\pi\,r\,(r + h)$ $M = 2\,\pi\,r\,h$
Cone		$V = \frac{r^2\pi\,h}{3} \qquad m = \sqrt{h^2 + \left(\frac{d}{2}\right)^2}$ $O = \pi\,r\,(r + m)$ $M = \pi\,r\,m$

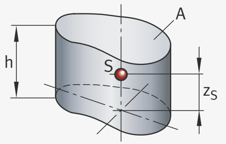
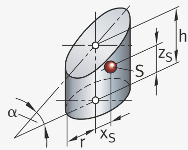
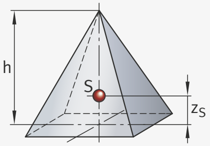
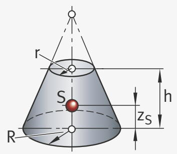
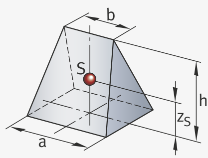
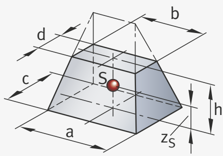
Continuation of table, see Page 60.

Continuation of table, Calculating solids, from Page 59.

Truncated cone		$V = \frac{\pi h}{12}\left(D^2 + D d + d^2\right)$ $m = \sqrt{\left(\frac{D-d}{2}\right)^2 + h^2}$ $M = \frac{\pi m}{2}(D+d)$
Barrel		$V = \frac{h \pi}{12}\left(2 D^2 + d^2\right)$ (barrel)
Sphere		$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3 \approx 4{,}189\, r^3$ $O = 4\pi r^2 = \pi d^2$
Spherical zone		$V = \frac{\pi h}{6}\left(3a^2 + 3b^2 + h^2\right)$ $M = 2\pi r h$ $a = f(h)$
Spherical segment		$V = \frac{\pi h}{6}\left(\frac{3}{4}s^2 + h^2\right) = \pi h^2\left(r - \frac{h}{3}\right)$ $M = 2\pi r h = \frac{\pi}{4}\left(s^2 + 4h^2\right)$
Spherical sector		$V = \frac{2}{3}\pi r^2 h$ $O = \frac{\pi r}{2}(4h + s)$
Circular torus (cylindrical ring)		$V = \frac{D \pi^2 d^2}{4}$ $O = D d \pi^2$

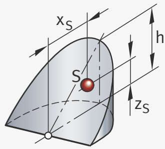
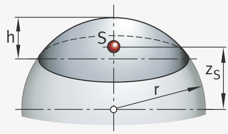
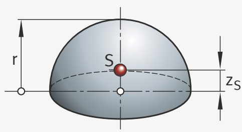
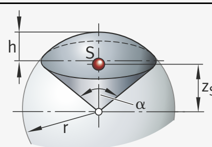
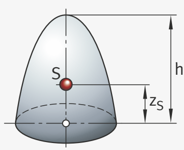
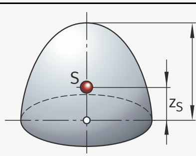
Centre of gravity of homogeneous solids

The centre of gravity of homogeneous solids is calculated using:

Cylinder with random cross section		$z_S = \frac{h}{2}$
Chamfered cylinder		$x_S = \frac{r^2 \tan\alpha}{4h}$ $z_S = \frac{h}{2} + \frac{r^2 \tan^2\alpha}{8h}$
Pyramid, cone		$z_S = \frac{h}{4}$
Truncated cone		$z_S = \frac{h}{4} \cdot \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}$
Wedge		$z_S = \frac{h}{2} \cdot \frac{a+b}{2a+b}$
Truncated wedge		$z_S = \frac{h}{2} \cdot \frac{ac+ad+bc+3bd}{2ac+ad+bc+2bd}$

Continuation of table, see Page 62.

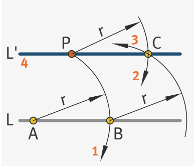
Continuation of table, Centre of gravity of homogeneous solids, from Page 61.

Section of a cylinder		$x_S = \frac{3\,\pi\,r}{16}$ $z_S = \frac{3\,\pi\,h}{32}$
Spherical segment		$z_S = \frac{3}{4} \cdot \frac{(2\,r - h)^2}{(3\,r - h)}$
Hemisphere		$z_S = \frac{3}{8}\,r$
Spherical sector		$z_S = \frac{3\,r\left(1 + \cos\frac{\alpha}{2}\right)}{8} = \frac{3\,(2\,r - h)}{8}$
Paraboloid		$z_S = \frac{h}{3}$
Ellipsoid		$z_S = \frac{3}{8}\,h$

Basic geometric constructions

The following examples show the basic methods used in geometric construction.

Drawing parallel lines



Given: L and P

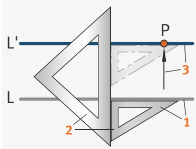
1 Draw an arc about any point A with radius $r = \overline{AP}$ (intersecting point B).

2 Draw an arc with radius r about P.

3 Draw an arc with radius r about B (intersecting point C).

4 Draw the required parallel line L' through the two points P and C.

Drawing parallel lines: Variant using set square



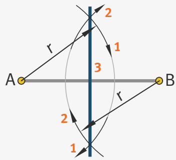
Given: L and P

1 Apply an angle to L.

2 Apply a second angle to the first.

3 Move the first angle to point P and draw the required parallel line L'.

Creating mid-perpendiculars



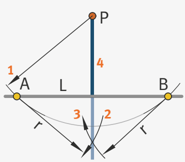
Given: $\overline{AB}$

1 Draw arcs with radius r about A (r must be greater than $1/2\ \overline{AB}$).

2 Draw arcs with radius r about B.

3 The connecting line between the intersecting points is the required mid-perpendicular.

Drawing perpendicular lines



Given: L and P

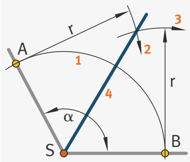
1 Draw any arc about P (intersecting points A and B).

2 Draw an arc with radius r about B.

3 Draw an arc with radius r about A (r must be greater than $1/2\ \overline{AB}$).

4 The connecting line of the intersecting point with P is the required perpendicular.

Bisecting angles



Given: angle α

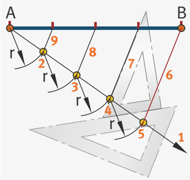
1 Draw any arc about S (intersecting points A and B).

2 Draw an arc with radius r about A (r must be greater than 1/2 $\overline{AB}$).

3 Draw an arc with radius r about B.

4 The connecting line of the intersecting point with S is the required angle bisecting line.

Dividing lines (equal division)



Given: $\overline{AB}$

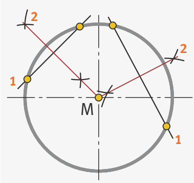
1 From A, draw a half-line at any angle.

2 to 5
Mark off divisions of equal length and any size r to the required number (in this instance 4) on the half-line of A.

6 Connect the last end point (5) with B.

7 to 9
Draw parallel lines to $\overline{B5}$: this gives the required subdivisions of the length $\overline{AB}$.

Finding the centre of a circle



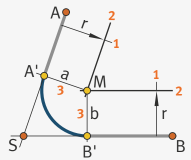
Given: circle without a centre

1 Draw any 2 chords.

2 Create the two mid-perpendiculars. The point of intersection M is the required centre.

(The chords should be at less than 90° to each other where possible, as this will increase the accuracy of the design)

Rounding an angle



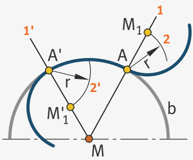
Given: angle ASB and rounding radius r

1 Apply r so that it is perpendicular to the sides.

2 Draw the parallel lines to AS and BS.
The point of intersection M is the required rounding centre.

3 Draw the perpendiculars a and b from M to the sides.
The intersecting points A' and B' are the transition points.

Rounding a circle

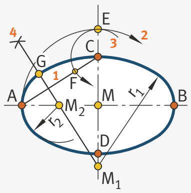


Given: arc b and rounding radius r

1 From M, draw a half-line.

2 From intersecting point A, mark off from radius r.
The end point is the required rounding centre M_1,
A is the transition point.

Ellipse approximation (false ellipse)



Given: axes $\overline{AB}$ and $\overline{CD}$

1 Draw $\overline{AC}$.

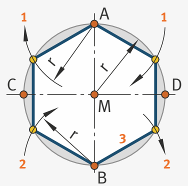
2 Draw an arc with radius $\overline{MA}$ about M to give E.

3 Draw an arc with radius $\overline{CE}$ about C to give F.

4 Create mid-perpendicular to $\overline{AF}$.

5 The intersecting points M_1 and M_2 are the required foci of the ellipse for r_1 and r_2,
A is the transition point.

Hexagon and dodecagon



Given: circle

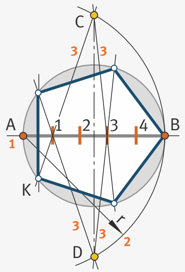
1 Draw arcs with radius r about A.

2 Draw arcs with radius r about B.

3 Draw hexagonal lines.

For the dodecagon, it is necessary to define the intermediate points. Recess in C and D

Regular polygon in the circle, approximation construction



Given: circle

Taking a pentagon as an example:

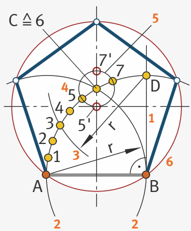
1 Divide $\overline{AB}$ into 5 sections (for example using equal division).

2 Draw an arc with radius r = $\overline{AB}$ about A.

3 Join C and D using all odd divisions (1, 3, 5). Joining the intersecting points with the circle gives the required pentagon.

For polygons with an even number of corners, C and D must be joined with all of the even divisions (2, 4, 6, ...).

Regular polygon with a given side length, approximation construction



Given: $\overline{AB}$

Taking a pentagon as an example:

1 Create perpendicular at B.

2 Draw arcs with radius r = $\overline{AB}$ about A and B to give intersecting points C and D.

3 Draw an arc with radius r about D to give point 3.

4 Using trial and error and a divider, search for points 1, 2, 4 and 5 (equal distances).

5 Draw an arc with radius $\overline{C5}$ about C to give 5' and 7',
5' is the centre of a pentagon,
7' is the centre of a heptagon.

6 Draw a circle around 5' and plot $\overline{AB}$ 5 times. This gives the required pentagon.

Calculating interest

Compound interest

If b = the initial amount, p = the interest rate in % and b_n = the final amount after n years, the outcome for the interest factor q and the final amount b_n after n years at compound interest is:

Equation 71

$$q = 1 + \frac{p}{100} \qquad b_n = b \cdot q^n = b \cdot \left(1 + \frac{p}{100}\right)^n$$

Sample calculation

What final amount will 30 000 € increase to in 5 years at an interest rate of 5,5%?

Solution:

Equation 72

$$b_5 = 30\,000\ € \cdot \left(1 + \frac{5,5}{100}\right)^5 = 39\,209\ €$$

Growth

The growth of a basic amount b_0 based on compound interest as the result of regular **additional payments** r at the end of any given year can be calculated as a final amount b_n after n years:

Equation 73

$$b_n = b_0 \cdot q^n + \frac{r\left(q^n - 1\right)}{q - 1}$$

Reduction

The reduction of a basic amount b_0 based on compound interest as the result of regular **repayments** r (e.g. pension) at the end of any given year can be calculated as a final amount b_n after n years:

Equation 74

$$b_n = b_0 \cdot q^n - \frac{r\left(q^n - 1\right)}{q - 1}$$

Repayment formula

The repayment formula for $b_n = 0$ is:

Equation 75

$$b_0 \cdot q^n = \frac{r\left(q^n - 1\right)}{q - 1}$$

Set theory

Set theory symbols

The following table shows a selection of the most important symbols used in set theory.

Symbol	Use	Definition
$\in$	$x \in M$	x is an element of M
$\notin$	$x \notin M$	x is not an element of M
	$\{x_1, ..., x_n\} \in A$	$x_1, ..., x_n$ are elements of A
$\{\mid\}$	$\{x \mid \varphi\}$	The set (class) of all x with φ
$\{, ...,\}$	$\{x_1, ..., x_n\}$	The set with the elements $x_1, ..., x_n$
$\subseteq$	$A \subseteq B$	A is the subset of B, A sub B, B is the superset of A (contains A = B)
$\subset$	$A \subset B$	A is properly contained in B (with $A \neq B$)
$\cap$	$A \cap B$	A intersected with B, A intersection B
$\cup$	$A \cup B$	A united with B, A union B
$\complement$	$\bar{A}$ or $\complement A$ or $A^{\complement}$	Complement of A
$\setminus$ or $\complement$	$A \setminus B$ or $\complement_A B$	A minus B, A reduced by B, difference between sets A and B, relative complement of B with respect to A
Δ	$A \Delta B$	Symmetrical difference of A and B
$\emptyset$	$A \cap B = \emptyset$	Empty set, A and B are disjoint
$<\mid,>$	$<x, y>$	Pair of x and y
$\{,\mid\}$	$\{x, y \mid \varphi\}$	Relationship between x, y with φ
x	A x B	Cartesian product of A and B, A cross B
$^{-1}$	R^{-1}	Inverse relation of R, inverse relation to R
$\circ$	$R \circ S$	Relation product of R and S, R linked to S
D	D (f)	Definition range of f
W	W (f)	Value range of f
$\mid$	$f \mid A$	Restriction of f to A
equ	A equ B	A is equivalent to B
card	card A	Cardinal number of A
$\mathbb{N}$ or N		Set of natural numbers
$\mathbb{Z}$ or Z		Set of integers
$\mathbb{Q}$ or Q		Set of rational numbers
$\mathbb{R}$ or R		Set of real numbers
$\mathbb{C}$ or C		Set of complex numbers

Numerics – number systems in data processing

Numeric and alphanumeric data and commands are represented by a combination of binary characters in digital computers. These are generally condensed into words of fixed length, producing standard word lengths of 4, 8, 16, 32, 48 and 64 bits.

Code

Alphanumeric data are usually strung together, character by character, in coded form and in one word. The most frequently used code is composed of 8 bits = 1 byte.

Fixed-point number and floating-point number

Numeric data can be represented in positional (fixed-point number) or floating-point (floating-point number) notation:

Fixed-point representation:

Sign	2^{n-1}	2^{n-2}	...	2^2	2^1	2^0

Floating-point representation:

Sign	Exponent	Mantissa

In the case of fixed-point notation, the highest representable value in terms of its amount is limited by the word length. In the case of a computer in 16-bit format for example, this is $2^{15} - 1 = 32\,767$. Double words can be formed if a larger number range is required.

In the case of a floating-point number, the number of bits of the mantissa defines the relative accuracy of the number and those of the exponent define the magnitude of the number range.

Positional notation systems and representation

The characteristics of a number system are characterised by the digit repertoire and the positional notation. Here, the value of the digit Z depends on its position within the row of digits (positional notation system).

Integers

Positive integers N_B can be represented on selecting a base B (base number) in the following general form:

Equation 76

$$N_B = \sum_{i=0}^{n-1} Z_i \cdot B^i = Z_{n-1} \cdot B^{n-1} + \ldots + Z_1 \cdot B^1 + Z_0 \cdot B^0$$

with Z_i from {0, 1, 2, ..., (B – 1)} as the digit repertoire for base B.

Decimal system

For the decimal system with digit repertoire $Z_i = 0, 1, 2, ..., 9$ and digit number $n = 3$, the representation is:

Equation 77

$$N_{10} = 257 = 2 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0$$

Binary system

For the binary system with digit repertoire $Z_i = 0, 1$ and digit number $n = 5$, the representation is:

Equation 78

$$N_2 = 10\,101 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

Hexadecimal system

As the decimal digits from 0 to 9 do not suffice in the hexadecimal system, the missing digits from 10 to 15 are replaced by the capital letters A to F.

Fractional numbers

The general representation of fractional numbers is:

Equation 79

$$R_B = \sum_{i=0}^{m} Z_i \cdot B^{-i} = Z_1 \cdot B^{-1} + Z_2 \cdot B^{-2} + \ldots + Z_m \cdot B^{-m}$$

Table: Number systems

The following table shows examples of number systems with various bases.

Decimal system	Hexadecimal system	Octal system	Binary system	Tetrad representation	BCD representation	Excess-3 or Stibitz code	Aiken code	1 out of 10 code
0	0	0	0	0 000	0 000	0 011	0 000	0 000 000 001
1	1	1	1	0 001	0 001	0100	0 001	0 000 000 010
2	2	2	10	0 010	0 010	0101	0 010	0 000 000 100
3	3	3	11	0 011	0 011	0110	0 011	0 000 001 000
4	4	4	100	0100	0100	0111	0100	0 000 010 000
5	5	5	101	0101	0101	1000	1011	0 000 100 000
6	6	6	110	0110	0110	1001	1100	0 001 000 000
7	7	7	111	0111	0111	1010	1101	0 010 000 000
8	8	10	1000	1000	1000	1011	1110	0100 000 000
9	9	11	1001	1001	1001	1100	1111	1000 000 000
10	A	12	1010	1010	00 010 000	01 000 011	00 010 000	00 000 000 100 000 000 001
11	B	13	1011	1011	00 010 001	01 000 100	00 010 001	00 000 000 100 000 000 010
12	C	14	1100	1100	00 010 010	01 000 101	00 010 010	00 000 000 100 000 000 100
13	D	15	1101	1101	00 010 011	01 000 110	00 010 011	00 000 000 100 000 001 000
14	E	16	1110	1110	00 010 100	01 000 111	00 010 100	00 000 000 100 000 010 000
15	F	17	1111	1111	00 010 101	01 001 000	00 011 011	00 000 000 100 000 100 000

Conversion between number systems

Numbers can be converted from one number system (source system N_Q) to another (target system N_Z).

The following always applies:

Equation 80

$$N_Q = N_Z$$

The following methods are used for converting to a different number system:

Division method

The division method works with numbers from the source notation only. It is based on the division of the number belonging to the source system N_Q by the largest possible powers of the target base while simultaneously splitting the relevant integral quotient that is generated in the division step. The remainder is divided by the next lowest power. This continues until the zero power has been obtained.

This means:

Equation 81

$$N_Q = N_Z$$

$$N_Q = Z_{n-1} \cdot B_Z{}^{n-1} + Z_{n-2} \cdot B_Z{}^{n-2} + \ldots + Z_1 \cdot B_Z{}^1 + Z_0 \cdot B_Z{}^0$$

1st step:

Equation 82

$$N_Q / B_Z{}^{n-1} = Z_{n-1} + \text{Remainder}_1 \qquad Z_{n-1} = \text{1st digit of } N_Z$$

2nd step:

Equation 83

$$\text{Remainder}_1 / B_Z{}^{n-2} = Z_{n-2} + \text{Remainder}_2$$

$$Z_{n-2} = \text{2nd digit of } N_Z$$

etc.

Example

The decimal number $6\,345_{10}$ is to be converted to an octal number:

$6\,345 : 8^4 =$	1	↓	Remainder	2 249
$2\,249 : 8^3 =$	4		Remainder	201
$201 : 8^2 =$	3		Remainder	9
$9 : 8^1 =$	1		Remainder	1
$1 : 8^0 =$	1		Remainder	0

The required octal number is $14\,311_8$ ($6\,345_{10} = 14\,311_8$).

Summand method

The summand method is based on a source number with summands of the form $\sum_{i=1}^{n-1} Z_i \cdot B_Z{}^i$, each of which contains B_Z as the factor:

Equation 84

$$N_Q = N_Z$$

$$N_Q = \sum_{i=0}^{n-1} Z_i \cdot B_Z{}^i = \sum_{i=1}^{n-1} Z_1 \cdot B_Z{}^i + Z_0 \cdot B_Z{}^0 \quad \text{where} \quad B_Z{}^0 = 1$$

This gives the following:

Equation 85

$$N_Q = B_Z \cdot \sum_{i=1}^{n-1} Z_1 \cdot B_Z{}^{i-1} + Z_0$$

If we divide N_Q by B_Z, we arrive at the integral share:

Equation 86

$$N_i = \sum_{i=1}^{n-1} Z_i \cdot B_Z{}^{i-1} + \text{Remainder } Z_0 \qquad \text{where } Z_0 = \text{last digit of } N_Z$$

The integral share can now be represented as:

Equation 87

$$N_1 = \sum_{i=2}^{n-1} Z_i \cdot B_Z{}^{i-1} + Z_1 \cdot B_Z{}^0 \qquad \text{where} \quad B_Z{}^0 = 1$$

If we divide again by B_Z, we again arrive at an integral share N_2 and the remainder Z_1 (penultimate digit of the number in the target system) etc.

Example

The decimal number $6\,345_{10}$ is to be converted to an octal number:

6 345 : 8 =	793	Remainder	1	↑
793 : 8 =	99	Remainder	1	
99 : 8 =	12	Remainder	3	
12 : 8 =	1	Remainder	4	
1 : 8 =	0	Remainder	1	

The required octal number is $14\,311_8$ ($6\,345_{10} = 14\,311_8$).

This conversion method is particularly suitable for a computing program.

Conversion of binary numbers

For number systems to base 2n, for example number notations based on 2, 8, 16, simpler methods of conversion exist between them. These are based on the fact that the source and target bases are related in a ratio of powers of two. The digit repertoire of the octal system is covered by a three-digit binary number, while that of the hexadecimal system is covered by a four-digit binary number. Conversion of a binary number to an octal or a hexadecimal number is achieved easily by combining groups of three or four of the binary number.

Example

Converting the binary number 110101111_2 to an octal or a hexadecimal number:

	6	5	7	Octal number
	110	101	111	Binary number
1	A	F		Hexadecimal number

Fundamental arithmetic operations in the binary system

The following arithmetic rules apply to the arithmetic operations of addition, subtraction and multiplication:

Addition	Result	Carry (bit)	Sub-traction	Result	Carry (bit)	Multi-plication	Result	Carry (bit)
0 + 0	0	0	0 – 0	0	0	0 · 0	0	0
0 + 1	1	0	0 – 1	1	–1[1)]	0 · 1	0	0
1 + 0	1	0	1 – 0	1	0	1 · 0	0	0
1 + 1	0	+1[1)]	1 – 1	0	0	1 · 1	1	0

Source: Koch, G.; Reinhold, U.: Einführung in die Informatik für Ingenieure und Naturwissenschaftler, Teil 1, München: Hanser-Verlag 1977.

[1)] If, when applying the operations to multiple-digit numbers, the carry amount ("borrowing") is included in the calculation, the same rules apply as in the decimal system.

Technical statistics

Functions and areas of application

Functions

The function of technical statistics is to describe sets of elements of the same type having different attribute values by means of statistical parameters. As a result, it is then possible to make objective comparisons and assessments.

In addition, it provides estimates of the statistical parameters of larger sets (the population) through the evaluation of a relatively small number of individual data (random samples).

Areas of application

The most important areas of application of technical statistics are:

- statistical quality control
- evaluation of test results
- calculation of errors

Terms, values and definitions

Population

The population is the set comprising all the units or events that are to be considered in statistical analysis (measurement, observation).

The attribute value that is of interest is described by means of statistical parameters.

Random sample

A random sample is a set taken from the population for the purpose of determining certain attribute values. Evaluation of this sample facilitates estimates of statistical parameters of the population.

Raw data list

The raw data list is the term for the original attribute values (for example measurement values) from a random sample.

Terms and values The following table gives descriptions of some important terms and values in statistics.

Value	Definition	Explanations, relationships
N	Population size	The size of the total population is also designated simply as the population
n	Number of attribute values in the random sample	Attribute values are recorded in the raw data list
x_i	Individual attribute value, such as a measurement value	Ordinal number of the attribute values i = 1, 2, 3, ..., n
$\bar{x}$	Arithmetic mean value of the attribute values in the random sample	$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
R	Range of the attribute values	$R = x_{max} - x_{min}$
k	Number of classes into which R is subdivided	Guide value $k = \sqrt{n}$ $k \geqq 10$ for $n \leqq 100$ $k \geqq 20$ for $n \leqq 10^5$
Δx	Class interval	$\Delta x = R/k$
x_j	Values of the class midpoints, arithmetic mean value of the class limits	Ordinal number of the classes j = 1, 2, 3, ..., k
n_j	Population density of the individual classes, absolute frequency	The population density n_j indicates how many values in the raw data list fall into the j-th class: $\sum_{j=1}^{k} n_j = n$
h_j	Relative frequency in the j-th class	$h_j = \frac{n_j}{n}$ $\sum_{j=1}^{k} h_j = 1$
f	Relative frequency density	$f = h_j/\Delta x$ The relative frequency density corresponds to the relative frequency divided by the class width
G_j	Cumulative population density	G_j is the population density added up to the j-th class: $G_j = \sum_{i=1}^{j} n_i$
H_j	Cumulative frequency	$H_j = \frac{G_j}{n} = \sum_{i=1}^{j} h_i$
x_0	Reference value of the population	Normally the approximated, rounded mean value or midpoint of class with the greatest frequency: $x_0 \approx \bar{x}$ $d_i = x_i - x_0$ $\bar{x} = x_0 + \frac{1}{n} \sum_{i=1}^{n} (x_i - x_0)$ $\bar{x} = x_0 + \bar{d}$

Continuation of table, see Page 76.

Continuation of table, Terms and values, from Page 75.

Value	Definition	Explanations, relationships
s^2	Variance of the random sample (mean square deviation)	$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - x_0)^2$ $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} d_i^2$ $s^2 = \frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - x_0 \sum_{i=1}^{n} x_i \right)$
s	Standard deviation of the random sample (scatter), square root of variance	$s = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - x_0 \sum_{i=1}^{n} x_i \right)}$ s approximates σ for high values of n
μ	Mean value of the population, expected value	The arithmetic mean value $\overline{x}$ of the random sample is an estimated value true to expectancy for the expected value μ of the population
σ	Standard deviation of the population	A measure of the variation of the individual values about the mean value
u	Scatter factor	Certain ranges $\mu \pm u \cdot \sigma$ can be defined in which P% of the measurement values lie
F(x)	Distribution function, cumulative (distribution) function	The distribution function describes the relationship between the random variables x and the cumulative frequency or probability for values $\leqq$ x, while in empirical distributions it corresponds to the cumulative curve
f(x)	Frequency density function	$f(x) = \frac{dF(x)}{dx}$ As a **continuous** function, the frequency density function corresponds to the representation of the relative frequency density of the random sample by means of a stepped curve (histogram)
R(x)	Reliability function	R(x) = 1 – F(x) The reliability function is sometimes also referred to as the survival function

Statistical evaluation (example)

Function The nominal diameter D of 8 mm and the deviations from this value is to be checked for a batch of rolling bearing balls (the population).

Solution A random sample of 200 balls is taken from the batch. It is assumed that the sample will give a sufficiently accurate representation of the population. The diameter x (the attribute value) is measured to an accuracy of 0,001 mm.

The data obtained from this study of a particular investigation feature are initially present unsorted in the so-called raw data list.

Determining the mean value

Within the measurement accuracy, the random sample mean value determined from the raw data list of 200 measurement values is the nominal value:

Equation 1

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{200} \sum_{i=1}^{200} x_i = 8{,}000 \text{ mm}$$

Of the 200 measured values, the largest diameter x_{max} = 8,013 mm and the smallest diameter x_{min} = 7,987 mm.

As a result, the range of the attribute values is:

Equation 2

$$R = x_{max} - x_{min} = 0{,}026 \text{ mm}$$

The attribute values are divided into k = 13 classes with a class interval Δx = 0,002 mm.

Based on the raw data list of 200 measurements, classification can be carried out and presented in tabular form. The relative frequencies and the cumulative frequency H_j are also determined, as defined in the table Terms and values, Page 75.

Classification and frequency

Based on the raw data list of 200 measurements, the following classification and the values for the relative frequencies and the cumulative frequency H_j are determined.

j	Classification x_{lower} to under mm	x_{upper} mm	x_j mm	n_j	h_j	H_j
1	7,987	7,989	7,988	1	0,005	0,005
2	7,989	7,991	7,990	5	0,025	0,030
3	7,991	7,993	7,992	7	0,035	0,065
4	7,993	7,995	7,994	16	0,080	0,145
5	7,995	7,997	7,996	25	0,125	0,270
6	7,997	7,999	7,998	29	0,145	0,415
7	7,999	8,001	8,000	34	0,170	0,585
8	8,001	8,003	8,002	32	0,160	0,745
9	8,003	8,005	8,004	22	0,110	0,855
10	8,005	8,007	8,006	14	0,070	0,925
11	8,007	8,009	8,008	9	0,045	0,970
12	8,009	8,011	8,010	4	0,020	0,990
13	8,011	8,013	8,012	2	0,010	1,000
				Σ 200	Σ 1,000	

Confirmation of the mean value

In order to confirm the mean value $\overline{x}$ where the class interval is identical, the class midpoints x_j are multiplied by their frequencies h_j as weighting factors:

Equation 3

$$\overline{x} = \frac{1}{n} \sum_{j=1}^{k} n_j \cdot x_j = \sum_{j=1}^{k} h_j \cdot x_j$$

$$\overline{x} = 8{,}000 \text{ mm}$$

Representation of relative frequency

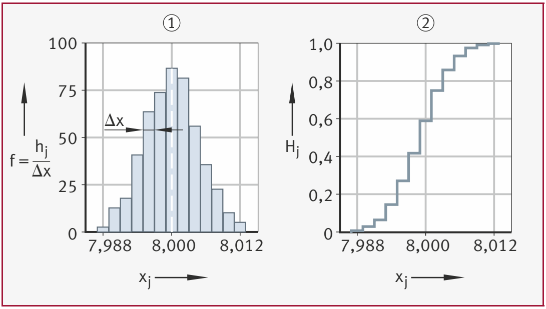
The relative frequency as a function of the class midpoints is represented by means of a bar chart showing the frequency density of the random sample (frequency diagram). The relative frequency density f corresponds to the relative frequency divided by the class width.
This representation of the frequency diagram is also known as a histogram, see Figure 1, ①. It provides a representation of the frequency distribution as an approximation for the distribution function.

In the representation of the cumulative frequency as a function of the class midpoints, the staircase curve shows the so-called cumulative curve or empirical distribution function, see Figure 1, ②.

In comparison with the frequency diagram, the cumulative curve has the advantage that it is possible to easily read off the percentage of the measurements that lie within any interval.

Figure 1
Histogram and cumulative curve

① Histogram/frequency diagram
② Cumulative curve



Variance and standard deviation

Equation 4

The variance s^2 of the random sample is determined as follows:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \sum_{j=1}^{k} n_j \cdot (x_j - \bar{x})^2$$

This can be used to determine the standard deviation s of the random sample:

Equation 5

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \sum_{j=1}^{k} n_j \cdot (x_j - \bar{x})^2}$$

The calculation scheme for the standard deviation can be used to determine the variance:

j	x_j	n_i	$x_j - \bar{x}$	$n_j (x_j - \bar{x})^2$
1	7,988	1	−0,012	0,000144
2	7,990	5	−0,010	0,000500
3	7,992	7	−0,008	0,000448
4	7,994	16	−0,006	0,000576
5	7,996	25	−0,004	0,004000
6	7,998	29	−0,002	0,000116
7	8,000	34	0,000	0,0
8	8,002	32	+0,002	0,000128
9	8,004	22	+0,004	0,000352
10	8,006	14	+0,006	0,000504
11	8,008	9	+0,008	0,000576
12	8,010	4	+0,010	0,000400
13	8,012	2	+0,012	0,000288
		Σ 200		**Σ 0,004432**

This gives the random sample variance s^2 as follows:

Equation 6

$$s^2 = \frac{1}{n-1} \sum_{j=1}^{k} n_j (x_j - \bar{x})^2$$

$$s^2 = \frac{1}{199} \cdot 0,00\,4432 = 22,27 \cdot 10^{-6}$$

The variance ultimately gives the standard deviation s = 0,0047 mm.

Representation in a probability plot

The values of the cumulative frequency H_j are entered in the probability plot as ordinates against the upper class limits. If a normal distribution is present, the ordinates of the cumulative frequency in the probability plot are distorted such that the S-shaped cumulative curve becomes a straight line, see Figure 2.

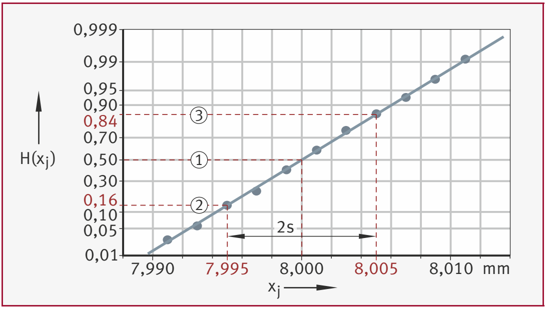


Figure 2
Normal distribution in a probability plot

① 50% of cumulative frequency
② 16% of cumulative frequency
③ 84% of cumulative frequency

The following values can be derived from this representation:

mean value $\overline{x}$ at 50% of the cumulative frequency:

Equation 7

$$\overline{x} = 8{,}000\ \text{mm}$$

and the standard deviation s, determined as double the standard deviation from the abscissa values at 16% and 84% of the cumulative frequency:

Equation 8

$$2s = x_{(H\,=\,0{,}84)} - x_{(H\,=\,0{,}16)}$$

$$2s = 8{,}005\ \text{mm} - 7{,}995\ \text{mm} = 0{,}010\ \text{mm}$$

$$s = 0{,}005\ \text{mm}$$

Within the scope of the reading accuracy, this value shows good agreement with the calculated value. The coefficient of variation indicates the standard deviation related to the mean value:

Equation 9

$$V_x = \frac{s}{\overline{x}}$$

$$V_x = \frac{0{,}005\ \text{mm}}{8{,}000\ \text{mm}} = 0{,}000\,63$$

Note on the evaluation of measurement series (confidence interval)

If a large number of random samples each comprising n values is taken from one and the same population with the mean value μ and the standard deviation σ, the mean values $\overline{x}_1$; $\overline{x}_2$; ... of the random samples will show scatter about the true value of μ:

Equation 10

$$\overline{x} = \mu \pm u \cdot \frac{\sigma}{\sqrt{n}}$$

The values for the factor u are listed in the table Value frequency, Page 83.

If only the values $\overline{x}$ and s of a random sample are known and a statement is to be made about the true mean value μ of the population, a so-called confidence interval can be indicated.

The mean value μ should lie within the confidence interval with P% probability:

Equation 11

$$\mu = \overline{x} \pm t \cdot \frac{s}{\sqrt{n}}$$

The values for the factor t are given in the following table.

n		2	3	5	10	20	50	...
t values for P =	90%	6,31	2,92	2,13	1,83	1,73	1,68	1,65
	95%	12,7	4,30	2,78	2,26	2,09	2,01	1,96
	99%	63,7	9,92	4,60	3,25	2,86	2,68	2,58

The true mean value μ of the population, at a probability of 90%, is:

$$\mu = 8{,}000\,\text{mm} \pm 1{,}65\,\frac{0{,}0047\,\text{mm}}{\sqrt{200}}$$

$$\mu = 8{,}000\,\text{mm} \pm 0{,}0005\,\text{mm}$$

Gaussian normal distribution

A Gaussian normal distribution generally occurs when a large number of random influences that are independent of each other act on a single attribute value of a population (collective) while none of the influences plays a dominant role.

If a normal distribution is present, this results in a straight line for the cumulative frequency in the probability plot, see Figure 2, Page 80.

Frequency density function and cumulative function

The staircase curves shown in Figure 1, Page 78, and Figure 2, Page 80, for the relative frequency density and cumulative frequency are incorporated in this case in the continuous trends of the frequency density function f(x) and the cumulative function F(x) with a mean value μ and standard deviation σ.

The frequency density function f(x) is described as follows:

Equation 12

$$f(x;\ \mu,\ \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}}$$

The cumulative function F(x) is determined as follows:

Equation 13

$$F(x;\ \mu,\ \sigma) = \int_{-\infty}^{x} f(x;\ \mu,\ \sigma)\, dx$$

Representation of the Gaussian normal distribution

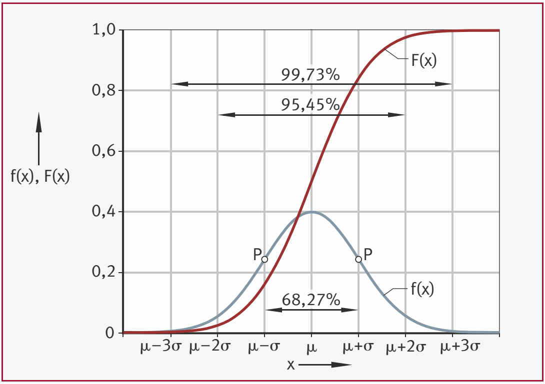
The normal distribution is symmetrical to the mean value μ of the population and shows an inflection point in each case at $x = \mu \pm \sigma$. The greater the value σ, the greater the distance between these two points. This starts at $x = -\infty$ and ends at $x = +\infty$. The total area under this "bell curve" corresponds to 1 = 100%.

Multiple values for the standard deviation can be used to define intervals $x = \mu \pm u \cdot \sigma$ in which P% of the x values lie. It can be seen from the table of value frequency that 99,73% of all values lie in the interval $\pm 3 \cdot \sigma$.

The frequency density function f(x) and the cumulative function F(x) are clearly defined by the mean value μ and the standard deviation σ of the distribution.

Figure 3
Gaussian normal distribution

f(x) = frequency density function
F(x) = cumulative function
P = inflection point



Value frequency

A value frequency within $\pm u \cdot \sigma$ gives the following values:

u	1,00	1,28	1,64	1,96	2,00	2,33	2,58	3,00	3,29
P%	68,27	80	90	95	95,45	98	99	99,73	99,9

Standard normal distribution

The integral of the cumulative function cannot be evaluated on an elementary basis. It is therefore necessary to present the function F(x; μ, σ) in tabular form.

The standard normal distribution is considered on the basis of the mean value $\mu = 0$ and the standard deviation $\sigma = 1$:

Equation 14

$$F(x; 0, 1) = F(x)$$

In many evaluations, it is only the deviations from a stated or known mean value that are of interest, so it is appropriate in these cases to calculate the function $\Phi^*(x)$:

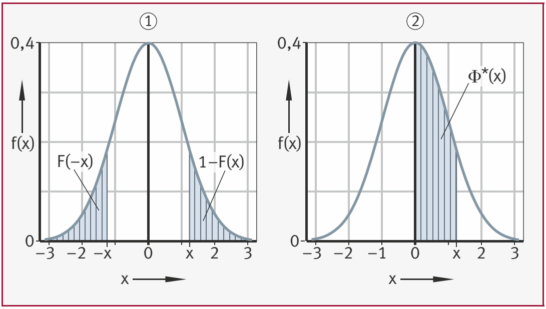
Equation 15

$$\Phi^*(x) = F(x) - \frac{1}{2}$$

The functions F(x) and $\Phi^*(x)$ appear in graphic form as an area under the bell curve:

Figure 4
Standard normal distribution, area under the bell curve

① Function F(x)
② Function $\Phi^*(x)$



The complete area under the bell curve is defined as:

Equation 16

$$F\left(x = +\infty\right) = 1$$

The standard normal distribution is symmetrical to the mean value $\mu = 0$. It is then sufficient to present the function $\Phi^*(x)$ in tabular form for positive values of x only.

The cumulative frequency between the values $\pm x$ is then:

Equation 17

$$\int_{-x}^{+x} F\left(x;\, 0,\, 1\right) dx = 2 \cdot \Phi^*\left(x\right)$$

First approximation

If the size of the random sample n taken is very large in relation to the population N, the following simplification is permitted:

- mean value $\overline{x}$ of the random sample = estimated and approximately actual parameter for mean value μ
- standard deviation s of the random sample = parameter for standard deviation σ of the population

Scatter in a process

If the standard deviation σ is known, this can also be used to determine the natural scatter in a process, in other words to define an interval that contains almost the complete distribution.
In practice, the value frequently selected in this case is $\mu \pm 3\sigma$ (99,73%).

Weibull distribution

The Weibull distribution has proved effective in practice for evaluation of the lifetime of engineering products. It has been applied as a standard procedure in rolling bearing technology.

Weibull cumulative function

The Weibull cumulative function is as follows:

Equation 18

$$F(t) = 1 - e^{-(t/\eta)^{\beta}}$$

The associated reliability function R(t) is also known as the survival function and is determined as follows:

Equation 19

$$R(t) = 1 - F(t)$$

$F(t)$ = cumulative function
probability that a specimen from a random sample or a collective will fail by the time t

$R(t)$ = survival function;
reliability function

t = attribute value, failure time

β = measure of the scatter of the failure times, failure gradient

η = characteristic rating life;
the time by which 63,2% of the specimens in a test procedure have failed

The characteristic rating life η is determined by using $t = \eta$ in the Weibull cumulative function:

Equation 20

$$F(\eta) = 1 - e^{-1^{\beta}} = 1 - \frac{1}{e}$$

$$F(\eta) = 0{,}632 \triangleq 63{,}2\%$$

Evaluation of a rating life test

For evaluation of tests, a linear representation of the cumulative function is not suitable.

Double logarithmisation gives the following:

Equation 21

$$\left(\frac{t}{\eta}\right)^{\beta} = \ln \frac{1}{1-F(t)}$$

$$\beta\left(\lg t - \lg \eta\right) = \lg \ln \frac{1}{1-F(t)}$$

This relationship appears in the Weibull paper with an abscissa graduation lg t and an ordinate graduation lg ln $1/(1 - F(t))$ as a straight line for $F(t)$.

Cumulative failure frequency

For the evaluation of a rating life test comprising n tests, the cumulative frequency H_i is plotted against the rating life values t arranged according to magnitude in accordance with the median ranking method:

Equation 22

$$H_i = \frac{i-0{,}3}{n+0{,}4}$$

i = ordinal number of the failure times of the specimens

In order to achieve both statistical authoritativeness of test results and an acceptable test duration, it is necessary to carry out a rating life test with a large random sample n up to a cumulative failure frequency of at least $H_i = 0{,}5$.

η and β are random values, on a similar basis to $\overline{x}$ and s in the Gaussian normal distribution.

For random samples $n \geqq 50$, the following relationship gives confidence intervals for the values to be expected of the population:

Equation 23

$$\eta \pm \left(\frac{u}{\sqrt{n}}\right) \cdot 1{,}052 \cdot \left(\frac{\eta}{\beta}\right)$$

$$\beta \pm \left(\frac{u}{\sqrt{n}}\right) \cdot 0{,}78 \cdot \beta$$

The values for u are given in the table Value frequency, Page 83.

Basic rating life

In rolling bearing technology, the definition states that 10% of the bearings in a large collective are permitted to have failed by the time the basic rating life is reached. A relationship is obtained between η and L_{10} if the values $t = L_{10}$ and $F(t) = 0{,}10$ are used in the Weibull cumulative function:

Equation 24

$$L_{10} = \eta \cdot \ln\left(\frac{1}{1-0{,}1}\right)^{\frac{1}{\beta}} = \eta \cdot 0{,}10536^{\frac{1}{\beta}}$$

Design of experiments

Introduction to the design of experiments – terms

Design of experiments is an approach used in the targeted planning and evaluation of experimental or simulation tests, which aims to identify the impact of various influencing factors on one or more target values or quality characteristics of a defined system, resulting in the following relationships:

- The system boundaries correspond to the investigation boundaries.
- The quality characteristics correspond to the object of the investigation.
- The parameters correspond to the set of all input variables.
- The factors correspond to the reproducible parameters taken into account in the experimental design (often differentiated into categorical and numerical factors).
- The factor levels correspond to the factor settings.
- The effect corresponds to the quantifiable effect of a factor on the system.

Conflict is typically encountered in the design of experiments when it comes to choosing the number of experiments: A large number of experiments permits more precise statements about the effects of the various factors on the quality characteristics. A smaller number of experiments allows the experimental procedure to be carried out more quickly and cost-effectively.

Types of statistical experimental designs

A distinction is often made in the design of experiments between screening designs and designs for detailed experiments.

Screening design of experiments

Screening designs are used to examine a larger number of factors and obtain a general overview of the relevant factors.
In **full factorial** designs, all possible combinations of the various factor levels are tested. In most cases, each factor has two levels.
Generally speaking, the level spacing of a factor can have a bearing on the magnitude of the identified effect.
By contrast, in **fractional factorial** designs, only a subset of a full factorial design is used in order to identify the most influential factors with minimal experimental work. As a result, these designs, in the same way as Plackett-Burman design of experiments, are suitable for screening.

The following example shows a full factorial design with three factors (A, B, C), each with two levels:

A	0	1	0	1	0	1	0	1
B	0	0	1	1	0	0	1	1
C	0	0	0	0	1	1	1	1
Y	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8

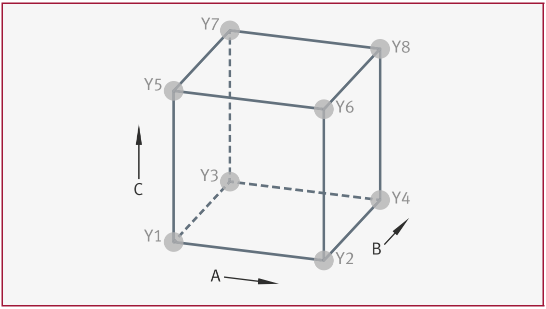


Figure 5
Full factorial design with three factors (A, B, C), each with two levels

Response surface design of experiments

In contrast to screening designs, response surface designs allow a more precise analysis of the effects of individual factors, taking into account non-linear relationships and interactions with other factors.
Examples of response surface designs include the central composite design and the Box-Behnken design.

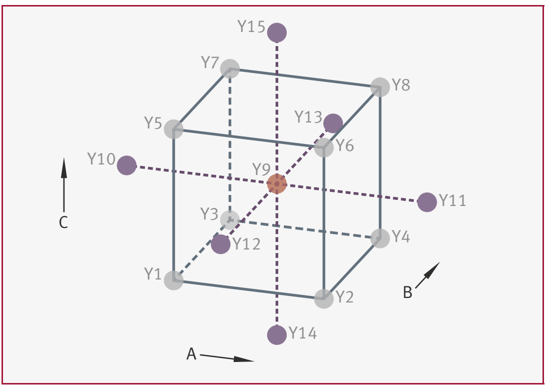


Figure 6
Central composite design for three factors (A, B, C)

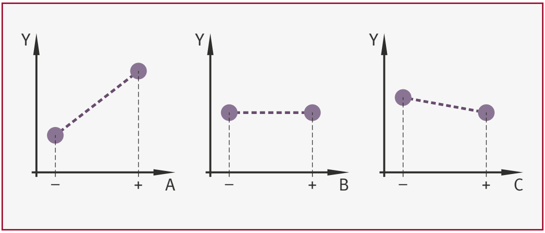
OFAT design of experiments

A popular design of experiments used in practice for investigating simple relationships and the effects of individual factors is the OFAT (one-factor-at-a-time) design. With this design, the factors are varied one at a time in succession while holding all other factors constant at their initial value. Interactions between the individual factors cannot, however, be identified using this method.

The following example shows an OFAT design for three factors and a sample visualisation of the effects:

A	+	–	0	0	0	0
B	0	0	+	–	0	0
C	0	0	0	0	+	–
Y	Y1	Y2	Y3	Y4	Y5	Y6

Figure 7
OFAT design for three factors and sample visualisation of the effects



The selection of a suitable design of experiments is normally dependent on the number of factors to be investigated and the maximum number of possible experiments. The maximum number of possible experiments may be limited, however, for reasons of time or cost constraints governing experimentation.

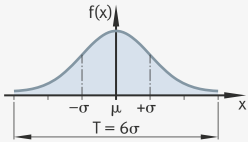
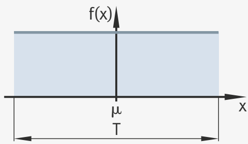
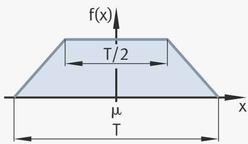
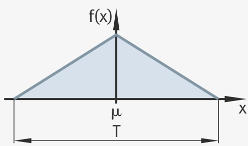
Planning of random experiments

If the aim is not to examine the influence of deterministic factors on various quality characteristics, but rather the influence of statistically fluctuating variables on the resulting distribution of quality characteristics, then random experiments must be conducted and evaluated. This requires the statistical distributions underlying the fluctuating factors (e.g. normal distribution, uniform distribution) to be defined and described first.

The first four statistical moments are among other methods used to describe these distributions in more detail:

- expected value
- variance
- skewness
- kurtosis

A number of statistical distribution forms with sample applications can be visualised as follows:

Normal distribution		Series production in large quantities
Uniform distribution		Production with systematic influencing factors, e.g. tooling wear
Trapezoidal distribution		Simulation distribution
Triangular distribution		Small-scale production

Pseudo-random numbers are then taken as samples from these distributions and used as input variables for the relevant (computer) experiment.

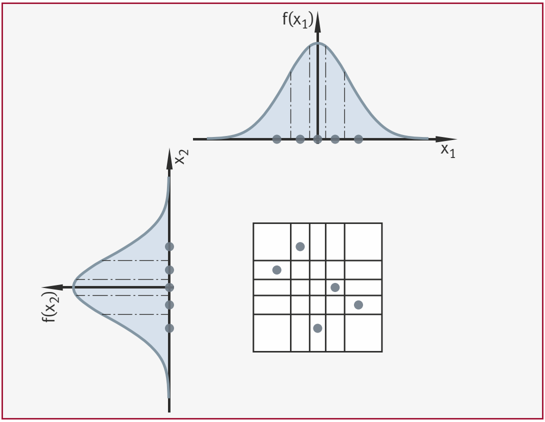
Monte Carlo method

In this context, the Monte Carlo method is a simple sampling method in which realisations are determined on the basis of distribution parameters for random input parameters. The major disadvantage of this method, however, lies in the weak convergence, which means that a relatively large number of calculation runs have to be carried out in order to obtain reliable information about the statistics of the system response. The reason for this is that rare input parameters are only considered statistically when samples are relatively large.

Latin Hypercube sampling

In contrast to the traditional Monte Carlo method, Latin Hypercube sampling divides the distributions of the input variables into N classes with equal probability mass, where N corresponds to the specified sample size and the probability mass corresponds to a proportional stratification. The random values of the input variables generated by this method are linked to each other by a Latin square.

Figure 8
Latin Hypercube sampling visualisation of two normally distributed factors and sample size N = 5



Evaluation of experimental results

In addition to its use in the planning of experimental and simulation studies, design of experiments is also applied in the evaluation of experimental results.
In this context, assuming the simple variation of only one factor with only two levels, the effect of factor variations can be evaluated in a very simple way by dividing the difference between the two factor levels by the difference between the corresponding results of the upper and lower factor level.
By contrast, particularly in detailed studies, the experimental results are commonly evaluated using regression and correlation analyses, in which the regression or correlation coefficients can be interpreted as effects of the factors.

Regression and correlation

The role of regression

The regression calculation aims to determine the relationship between an independent variable x and a dependent variable y from the value pairs (x_i, y_i) where i = 1, 2, ..., n of a random sample of size n.

The precondition is that the value pairs were each determined on the same i-th element.

Theoretical regression function

For the theoretical regression function, a polynomial of a k-th degree is generally selected whose coefficients α_j with j = 0,1, ..., k are to be determined:

Equation 25

$$f(x) = \alpha_k \cdot x^k + \alpha_{k-1} \cdot x^{k-1} + \ldots + \alpha_j \cdot x^j + \ldots + \alpha_1 \cdot x^1 + \alpha_0 \cdot x^0$$

If there is a linear relationship between x and f(x), the regression line gives a good approximation.

Determining the coefficients

Equation 26

The coefficients α_j are determined according to the Gaussian method of least squares:

$$\sum_{i=1}^{n} \left(y_i - f(x_j)\right)^2 = \sum_{j=1}^{n} \left(y_i - \sum_{j=0}^{k} \alpha_j \cdot x_i{}^j\right)^2 = g$$

with (α_0, α_1, ..., α_n) = minimum.

Linear approach

The partial derivations $\partial g / \partial \alpha_j = 0$ give (k + 1) linear equations that can be solved using the methods for linear equation systems.

For the linear case:

Equation 27

$$y = \alpha_0 + \alpha_1 \cdot x$$

and with the mean values:

Equation 28

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

the relationship is as follows:

Equation 29

$$\alpha_0 = \bar{y} - \alpha_1 \cdot \bar{x}$$

If the following is used:

Equation 30

$$y - \bar{y} = \alpha_1 (x - \bar{x})$$

the relationship is as follows:

Equation 31

$$\alpha_1 = \frac{\left(\sum x_i\, y_i - n\bar{x}\,\bar{y}\right)}{\left(\sum x_i{}^2 - n\bar{x}^2\right)}$$

Variances

The variances s^2 of the random sample are determined as follows:

Equation 32

$$s_x^2 = \frac{1}{n-1}\left[\sum x_i{}^2 - \left(\left(\sum x_i\right)^2 \cdot \frac{1}{n}\right)\right]$$

$$s_y^2 = \frac{1}{n-1}\left[\sum y_i{}^2 - \left(\left(\sum y_i\right)^2 \cdot \frac{1}{n}\right)\right]$$

The covariance s of the random sample is determined as follows:

Equation 33

$$s_{xy} = \frac{1}{n-1}\sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}\sum (x_i\, y_i - n\,\bar{x}\,\bar{y})$$

Coefficient

The coefficient α_1 is determined as:

Equation 34

$$\alpha_1 = \frac{s_{xy}}{s_x^2}$$

If all the measurement points lie on the straight line, the variances s^2 are as follows:

Equation 35

$$s_{xy}^2 = s_x^2 \cdot s_y^2$$

The role of correlation

If there are no recognisable reasons for a functional dependence of the random variables y on the independently applied variables x, the correlation calculation (correlation interdependence) is used to check the quality of a subordinate relationship.

A linear dependence can be stated using the correlation coefficient r_{xy}. This is in the following relationship with the values determined in the section Theoretical regression function, Page 92:

Equation 36

$$r_{xy} = \frac{s_{xy}}{\sqrt{s_x^2 \cdot s_y^2}}$$

$$-1 \leqq r_{xy} \leqq 1$$

If $r_{xy} < 0$ this is known as a negative correlation: large values for x are associated with small values for y and vice versa.

The value $B = r_{xy}^2$ is known as the coefficient of determination.

Chemistry

Elements and values

The periodic system of elements

In the periodic system, chemical elements are arranged according to their atomic weight. The atomic number, symbol, name and relative atomic mass (or rather the atomic mass of the most stable isotope in [ ]) is displayed in each case.

Peri-od	Group								
	Main groups		Subgroups						
	1	2	3	4	5	6	7	8	9
1	1 **H** Hydrogen 1,0079								
2	3 **Li** Lithium 6,941	4 **Be** Beryllium 9,0122							
3	11 **Na** Sodium 22,99	12 **Mg** Mag-nesium 24,305							
4	19 **K** Potassium 39,098	20 **Ca** Calcium 40,078	21 **Sc** Scandium 44,956	22 **Ti** Titanium 47,867	23 **V** Vanadium 50,942	24 **Cr** Chromium 51,996	25 **Mn** Manga-nese 54,938	26 **Fe** Iron 55,845	27 **Co** Cobalt 58,933
5	37 **Rb** Rubidium 85,468	38 **Sr** Strontium 87,62	39 **Y** Yttrium 88,906	40 **Zr** Zirconium 91,224	41 **Nb** Niobium 92,906	42 **Mo** Molyb-denum 95,94	43 **Tc** Tech-netium [97,907]	44 **Ru** Ru-thenium 101,07	45 **Rh** Rhodium 102,906
6	55 **Cs** Caesium 132,905	56 **Ba** Barium 137,327	57 – 71 Lantha-nides, see Page 96	72 **Hf** Hafnium 178,49	73 **Ta** Tantalum 180,948	74 **W** Tungsten 183,84	75 **Re** Rhenium 186,207	76 **Os** Osmium 190,23	77 **Ir** Iridium 192,217
7	87 **Fr** Francium [223,02]	88 **Ra** Radium [226,03]	89 – 103 Actinides, see Page 96	104 **Rf** Ruther-fordium [267]	105 **Db** Dubnium [270]	106 **Sg** Sea-borgium [269]	107 **Bh** Bohrium [270]	108 **Hs** Hassium [277]	109 **Mt** Meitner-ium [278]

			Main groups					
10	11	12	13	14	15	16	17	18
								2 **He** Helium 4,0026
			5 **B** Boron 10,811	6 **C** Carbon 12,011	7 **N** Nitrogen 14,007	8 **O** Oxygen 15,999	9 **F** Fluorine 18,998	10 **Ne** Neon 20,18
			13 **Al** Aluminium 26,982	14 **Si** Silicon 28,086	15 **P** Phos- phorus 30,974	16 **S** Sulphur 32,065	17 **Cl** Chlorine 35,453	18 **Ar** Argon 39,948
28 **Ni** Nickel 58,693	29 **Cu** Copper 63,546	30 **Zn** Zinc 65,38	31 **Ga** Gallium 69,723	32 **Ge** Ger- manium 72,64	33 **As** Arsenic 74,922	34 **Se** Selenium 78,96	35 **Br** Bromine 79,904	36 **Kr** Krypton 83,798
46 **Pd** Palladium 106,42	47 **Ag** Silver 107,868	48 **Cd** Cadmium 112,411	49 **In** Indium 114,818	50 **Sn** Tin 118,71	51 **Sb** Antimony 121,76	52 **Te** Tellurium 127,6	53 **I** Iodine 126,9	54 **Xe** Xenon 131,293
78 **Pt** Platinum 195,078	79 **Au** Gold 196,967	80 **Hg** Mercury 200,59	81 **Tl** Thallium 204,383	82 **Pb** Lead 207,2	83 **Bi** Bismuth 208,98	84 **Po** Polonium [208,98]	85 **At** Astatine [209,99]	86 **Rn** Radon [222,02]
110 **Ds** Darm- stadtium [281]	111 **Rg** Roentge- nium [282]	112 **Cn** Coper- nicium [285]	113 **Nh** Nihonium [286]	114 **Fl** Flerovium [289]	115 **Mc** Mosco- vium [289]	116 **Lv** Liver- morium [293]	117 **Ts** Tennes- sine [294]	118 **Og** Oganes- son [294]

Lanthanides Addition of lanthanides to table, The periodic system of elements, from Page 94.

57 **La** Lantha- num 138,905	58 **Ce** Cerium 140,116	59 **Pr** Praseo- dymium 140,908	60 **Nd** Neodymium 144,242	61 **Pm** Promethium [144,91]	62 **Sm** Samarium 150,36	63 **Eu** Europium 151,964	64 **Gd** Gadolinium 157,25
	65 **Tb** Terbium 158,925	66 **Dy** Dysprosium 162,5	67 **Ho** Holmium 164,93	68 **Er** Erbium 167,259	69 **Tm** Thulium 168,934	70 **Yb** Ytterbium 173,04	71 **Lu** Lutetium 174,967

Actinides Addition of actinides to table, The periodic system of elements, from Page 94.

89 **Ac** Actinium [227,03]	90 **Th** Thorium [232,04]	91 **Pa** Protactinium [231,04]	92 **U** Uranium [238,03]	93 **Np** Neptunium [237,05]	94 **Pu** Plutonium [244,06]	95 **Am** Americium [243,06]	96 **Cm** Curium [247,07]
	97 **Bk** Berkelium [247,07]	98 **Cf** Californium [251,08]	99 **Es** Einsteinium [252,08]	100 **Fm** Fermium [257,1]	101 **Md** Mendelevium [258,1]	102 **No** Nobelium [259,1]	103 **Lr** Lawrencium [262,11]

Physical properties: Chemical elements The physical properties of the chemical elements are listed in the following table.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm³ [1]	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Actinium	Ac	89	(227)	–	1 050	3 200	–	0,12
Aluminium	Al	13	26,98	2,70	660	2 450	238	0,88
Americium	Am	95	(243)	11,7	>850	2 600	–	0,14
Antimony	Sb	51	121,75	6,68	631	1380	19	0,21
Argon	Ar	18	39,95	1,40[1]	–189	–186	0,02	0,52
Arsenic	As	33	74,92	5,72	817[2]	613	–	0,33
Astatine	At	85	(209,99)	–	302	335	–	0,14
Barium	Ba	56	137,34	3,50	714	1640	–	0,29
Berkelium	Bk	97	(247)	–	–	–	–	–
Beryllium	Be	4	9,01	1,85	1 280	2 480	168	1,02
Bismuth	Bi	83	208,98	9,8	271	1 560	8,1	0,12
Boron	B	5	10,81	2,34	(2 030)	3 900	–	1,04
Bromine	Br	35	79,90	3,12	–7	58	–	0,45
Cadmium	Cd	48	112,40	8,65	321	765	96	0,23
Caesium	Cs	55	132,91	1,87	29	690	–	0,22

Continuation of table, see Page 97.
Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

[1] Gas: density in kg/m³ (at +25 °C and 1013 hPa).
[2] In sealed tube at 27,5 bar.

Continuation of table, Physical properties: Chemical elements, from Page 96.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm³ 1)	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Calcium	Ca	20	40,08	1,55	838	1490	130	0,66
Californium	Cf	98	(251)	–	–	–	–	–
Carbon	C	6	12,01	2,26	3730	4830	168	0,65
Cerium	Ce	58	140,12	6,78	795	3470	10,9	0,18
Chlorine	Cl	17	35,45	1,56 1)	−101	−35	0,008	0,47
Chromium	Cr	24	52,00	7,19	1900	2642	69	0,44
Cobalt	Co	27	58,93	8,90	1490	2900	96	0,43
Copper	Cu	29	63,55	8,96	1083	2600	398	0,38
Curium	Cm	96	(247)	7	–	–	–	–
Dysprosium	Dy	66	162,50	8,54	1410	2600	10	0,17
Einsteinium	Es	99	(254)	–	–	–	–	–
Erbium	Er	68	167,26	9,05	1500	2900	9,6	0,17
Europium	Eu	63	151,96	5,26	826	1440	–	0,17
Fermium	Fm	100	(257)	–	–	–	–	–
Fluorine	F	9	19,00	1,51 1)	−220	−188	0,02	0,83
Francium	Fr	87	(223,02)	–	(27)	(680)	–	0,14
Gadolinium	Gd	64	157,25	7,89	1310	3000	8,8	0,23
Gallium	Ga	31	69,72	5,91	30	2400	40	0,37
Germanium	Ge	32	72,59	5,32	937	2830	62	0,31
Gold	Au	79	196,97	19,3	1063	2970	314	0,13
Hafnium	Hf	72	178,49	13,1	2000	5400	93	0,14
Helium	He	2	4,003	0,15 1)	−270	−269	0,16	5,23
Holmium	Ho	67	164,93	8,80	1460	2600	–	0,16
Hydrogen	H	1	1,008	0,07 1)	−259	−253	0,17	14,14
Indium	In	49	114,82	7,31	156	2000	24	0,23
Iodine	I	53	126,90	4,94	114	183	0,43	0,22
Iridium	Ir	77	192,22	22,5	2450	4500	58	0,13
Iron	Fe	26	55,85	7,86	1540	3000	72	0,44
Krypton	Kr	36	83,80	2,16 1)	−157	−152	0,01	0,25
Lanthanum	La	57	138,91	6,17	920	3470	13,8	0,20

Continuation of table, see Page 98.

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Gas: density in kg/m³ (at +25 °C and 1013 hPa).

Continuation of table, Physical properties: Chemical elements, from Page 97.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm^{3} [1]	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Lawrencium	Lr	103	(256)	–	–	–	–	–
Lead	Pb	82	207,2	11,4	327	1740	35	0,13
Lithium	Li	3	6,94	0,53	180	1330	71	3,6
Lutetium	Lu	71	174,97	9,84	1 650	3 330	–	–
Magnesium	Mg	12	24,31	1,74	650	1110	171	1,01
Manganese	Mn	25	54,94	7,43	1 250	2100	30	0,47
Mendelevium	Md	101	(258)	–	–	–	–	–
Mercury	Hg	80	200,59	13,53	−39	357	8,1	0,14
Molybdenum	Mo	42	95,94	10,2	2 610	5 560	142	0,24
Neodymium	Nd	60	144,24	7,00	1020	3 030	16	0,19
Neon	Ne	10	20,18	1,20[1]	−249	−246	0,05	1,03
Neptunium	Np	93	237,05	20,4	640	–	57	–
Nickel	Ni	28	58,71	8,90	1450	2 730	61	0,43
Niobium	Nb	41	92,91	8,55	2 420	4 900	52	0,27
Nitrogen	N	7	14,01	0,81[1]	−210	−196	0,02	1,04
Nobelium	No	102	(256)	–	–	–	–	–
Osmium	Os	76	190,2	22,4	3 000	5 500	87	0,13
Oxygen	O	8	16,00	1,15[1]	−219	−183	0,03	0,92
Palladium	Pd	46	106,4	12,0	1550	3125	69	0,25
Phosphorus	P	15	30,97	1,82	44	280	–	0,67
Platinum	Pt	78	195,09	21,4	1770	3 825	71	0,13
Plutonium	Pu	94	(244)	19,8	640	3 230	9	–
Polonium	Po	84	(208,98)	9,4	254	962	–	0,13
Potassium	K	19	39,10	0,86	64	760	97	0,76
Praseodymium	Pr	59	140,91	6,77	935	3130	12	0,19
Promethium	Pm	61	(145)	–	(1030)	(2 730)	–	0,19
Protactinium	Pa	91	231,04	15,4	(1230)	–	–	0,12
Radium	Ra	88	226,03	5	700	1530	–	0,12
Radon	Rn	86	(222,02)	4,4[1]	−71	−62	–	0,09
Rhenium	Re	75	186,2	21,0	3180	5 630	48	0,14

Continuation of table, see Page 99.
Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

[1] Gas: density in kg/m^3 (at +25 °C and 1013 hPa).

Continuation of table, Physical properties: Chemical elements, from Page 98.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm³ 1)	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Rhodium	Rh	45	102,91	12,4	1970	3730	88	0,24
Rubidium	Rb	37	85,47	1,53	39	688	58	0,33
Ruthenium	Ru	44	101,07	12,2	2300	3900	106	0,25
Samarium	Sm	62	150,4	7,54	1070	1900	–	0,20
Scandium	Sc	21	44,96	3,0	1540	2730	63	0,56
Selenium	Se	34	78,96	4,80	217	685	0,2	0,33
Silicon	Si	14	28,09	2,33	1410	2680	80	0,68
Silver	Ag	47	107,87	10,5	961	2210	418	0,23
Sodium	Na	11	22,99	0,97	98	892	138	1,22
Strontium	Sr	38	87,62	2,6	770	1380	–	0,29
Sulphur	S	16	32,06	2,07	113	–	0,26	0,68
Tantalum	Ta	73	180,95	16,6	3000	5430	55	0,12
Technetium	Tc	43	97,907	11,5	2140	(4600)	–	0,25
Tellurium	Te	52	127,60	6,24	450	1390	1,2	0,21
Terbium	Tb	65	158,93	8,27	1360	2800	–	0,18
Thallium	Tl	81	204,37	11,85	303	1460	50	0,13
Thorium	Th	90	232,04	11,7	1700	4200	38	0,14
Thulium	Tm	69	168,93	9,33	1550	1730	–	0,16
Tin	Sn	50	118,69	7,30	232	2270	63	0,22
Titanium	Ti	22	47,90	4,50	1670	3260	16	0,24
Tungsten	W	74	183,85	19,3	3410	5930	130	0,14
Uranium	U	92	238,03	18,90	1130	3820	24	0,12
Vanadium	V	23	50,94	5,8	1900	3450	32	0,51
Xenon	Xe	54	131,30	3,5 1)	–112	–108	0,005	0,16
Ytterbium	Yb	70	173,04	6,98	824	1430	–	0,14
Yttrium	Y	39	88,91	4,5	1500	2930	14	0,29
Zinc	Zn	30	65,37	7,14	419	906	113	0,39
Zirconium	Zr	40	91,22	6,49	1850	3580	21	0,28

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Gas: density in kg/m³ (at +25 °C and 1013 hPa).

Physical properties: Liquids The following table shows the physical properties for a selection of (pure and anhydrous) liquids.

Substance	Density ρ		Melting point	Boiling point	Thermal conductivity λ	Thermal capacity c_p
	kg/dm³	at °C	°C	°C	W/(m · K)	kJ/(kg · K)
Acetone	0,791	20	−95,35	56,35	0,16	1,21
Acetic acid	1,05	20	16,7	118	–	2,03
Anthracene oil	1,05	15	−20	270 ... 400	0,47	1,33
Benzene	0,83	15	5,4	80	0,14	1,7
Carbon tetrachloride	1,598	18	−22,8	46,3	–	0,845
Diesel fuel	0,83	15	−30	210 ... 380	0,15	2,05
Diethyl ether	0,72	20	−116	35	0,14	2,3
Ethyl acetate	0,975	20	−83,6	77,1	–	2,0
Ethyl alcohol, ≈ 98%	0,80	15	−114	78,5	0,17 ... 23	2,33
Ethyl chloride	0,92	15	−139	12,5	0,16	1,79
Ethylene glycol	1,114	20	−17,4	197,2	0,25	2,4
Glycerine	1,26	20	19	290	0,29	2,43
Heating oil, extra light	<0,86	20	−10	>175	0,14	2,07
Hydrochloric acid, 10%	1,05	15	−14	102	0,50	3,14
Linseed oil	0,93	20	−15	316	0,17	1,88
Machine oil	0,91	15	−5	380 ... 400	0,125	1,80
Mercury	13,55	15	−38,9	357,25	10	0,14
Methyl alcohol	0,80	15	−98	65	0,211	2,55
Methylene chloride	1,335	20	−97	40,1	–	–
Nitric acid	1,51	15	−41,3	86	0,26	1,72
Paraffin oil	0,81	15	−70	150 ... 300	0,13	2,1
Petrol	0,72 ... 0,73	15	−20 ... −50	40 ... 200	0,13	2,1
Petroleum ether	0,66	20	−160	40 ... 70	0,138	1,76
2-propanol (iso...)	0,79	20	−88	83	0,26	2,49
Rapeseed oil	0,91	20	0	300	0,17	1,97
Rectified spirit, 95 perc. by vol.[1)]	0,811	20	−90	78	0,16	2,43
Resin oil	0,96	20	−20	150 ... 300	0,15	–
Silicone oil	0,94	20	–	–	0,22	1,09
Sulphuric acid	1,84	15	10,5	338	0,47	1,42
Table salt solution, 20%	1,15	15	−18	108,8	0,59	3,43
Tar	1,2	20	−15	300	0,19	1,58
Toluene	0,87	15	−97	110	0,14	1,48
Transformer oil	0,87	15	−5	170	0,13	1,88
Trichloroethylene	1,47	18	−83	86,8	–	0,95
Turpentine oil	0,87	15	−10	160	0,10	1,80
Water, distilled	1,0	4	0	100	0,60	4,19

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Ethyl alcohol, denatured.

Physical properties: Solids The following table shows the physical properties for a selection of solids.

Substance	Density ρ kg/dm³	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Agate	2,5 ... 2,8	≈ 1600	≈ 2 590	10,68	0,79
Asphalt	1,1 ... 1,5	80 ... 100	≈ 300	0,69	0,92
Barium chloride ($BaCl_2$)	3,10	956	1830	–	0,37
Basalt	2,9	–	–	1,67	0,86
Boiler scale	≈ 2,5	≈ 1200	–	0,12 ... 2,3	0,79
Borax, anhydrous	1,72	741	–	–	0,99
Brass (63 Cu, 37 Zn)	8,5	900	–	116	0,38
Bronze (94 Cu, 6 Sn)	8,73	910	2 300	64	0,37
Charcoal	0,3 ... 0,5	–	≈ 3 540	0,08	1,0
Chromium(III) oxide (Cr_2O_3)	5,22	2 330	–	0,4 (powder)	0,75
Coke	1,6 ... 1,9	–	–	0,183	0,84
Concrete	1,8 ... 2,45	–	–	0,8 ... 1,4	0,87
Corundum (Al_2O_3)	3,9 ... 4,0	2 050	2 700	12 ... 23	0,96
Diamond	3,51	–	–	–	0,52
Flake graphite cast iron	7,25	1150 ... 1250	2 500	≈ 52	≈ 0,5
Glass fibre mats	0,03 ... 0,2	≈ 700	–	0,04	0,84
Glass (window)	2,4 ... 2,7	≈ 700	–	0,58 ... 1,0	0,84
Granite	2,6 ... 2,8	–	–	3,5	0,82
Graphite, pure	2,26	≈ 3 830	≈ 4 200	168	0,71
Greases	0,92 ... 0,94	30 ... 175	≈ 300	0,2	0,62 ... 0,79
Gypsum ($CaSO_4$)	2,3	1200	–	0,34 ... 0,46	1,1
Hard metal K20	14,8	≈ 2 000	≈ 4 000	81,4	0,80
Heat conducting alloy (80 Ni, 20 Cr)	8,3	1400	2 300	14,6	0,50
Hydrated ferric oxide (rust)	5,1	1565	–	0,58 (powder)	0,67
Ice	0,92	–	100	2,3	2,1
Leather, dry	0,85 ... 1,02	–	–	≈ 0,17	≈ 1,5
Limestone ($CaCO_2$)	2,6 ... 2,8	decomposes into CaO & CO_2		2,2	0,91
Litharge (lead monoxide)	9,53	888	1580	–	0,21
Magnesium alloys	≈ 1,8	≈ 630	1500	46 ... 140	–
Marble ($CaCO_3$)	2,6 ... 1,8	1290	decomposes	2,1 ... 3,5	0,88
Mica	2,6 ... 3,2	≈ 1300	≈ 700	0,34	0,87
Monel metal	8,8	1240 ... 1330	–	19,7	0,43
Porcelain	2,3 ... 2,5	≈ 1600	–	0,8 ... 1,0	0,80
Quartz	2,5 ... 2,8	≈ 1400	2 230	9,9	0,80
Red bronze ($CuSn_5$ ZnPb)	8,8	950	2 300	38	0,67
Sand, dry	1,2 ... 1,6	1480	2 230	0,6	0,80
Sandstone	2,2 ... 2,5	≈ 1500	–	2,3	0,71
Soot	1,7 ... 1,8	–	–	0,07	0,84
Steel (18 Cr, 8 Ni)	7,9	1450	–	14	0,51
Steel (18 W)	8,7	1450	–	26	0,42
Steel, low alloy	7,8 ... 7,86	1450 ... 1530	2 500	46 ... 58	0,49
Table salt (NaCl)	2,15	802	1440	–	0,92
Wood	0,5 ... 0,8	–	–	0,17 ... 0,34	2,1 ... 2,9

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

Physical properties: Gases and vapours The following table shows the physical properties for a selection of gases and vapours.

Substance	Density ρ kg/m^3	Relative density η_{rel} (air = 1)	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)	$\kappa = c_p/c_x$
Acetylene (ethine)	1,17	0,91	−81	0,019	1,68	1,26
Air	1,29	1	−192	0,026	1,00	1,40
Ammonia	0,77	0,60	−33,4	0,024	2,22	1,32
Blast furnace gas	1,28	0,99	−170	0,023	1,05	1,40
Carbon dioxide	1,98	1,52	−78,5	0,015 3	0,88	1,30
Carbon disulphide	3,41	2,64	46	0,007 2	0,67	1,19
Carbon monoxide	1,25	0,97	−191	0,024	1,05	1,40
Cyanogen $(CN)_2$	2,33	1,80	−21,2	–	1,72	1,27
Ethane	1,356	1,049	−88	0,021	–	1,13
Ethyl alcohol vapour	2,07	1,60	78,5	0,032	–	1,13
Ethylene, ethene	1,26	0,98	−102	0,037	1,55	1,25
Freon 12 (Cl_2F_2)	5,08	3,93	−30	–	–	1,14
Generator gas	1,22	0,94	−170	0,023	1,05	1,40
Hydrogen chloride	1,939	1,27	−85	0,014	0,79	1,41
Hydrogen fluoride	0,893	0,713	19,5	–	–	–
Hydrogen sulphide	1,539	1,191	−60,2	–	1,34	–
Isobutane	2,67	2,06	−10	–	–	1,11
Methyl choride	1,545	1,2	−24,0	–	0,74	1,20
Natural gas (methane)	0,718	0,64	−162	–	–	–
n-butane	2,703	2,09	1	–	–	–
Ozone	2,14	1,65	−112	–	–	1,29
Propane	2,019	1,562	−45	–	–	1,14
Propylene (propene)	1,915	1,481	−47	–	–	–
Sulphur dioxide	2,93	2,26	−10	0,010	0,63	1,40
Town gas (illuminating gas)	0,56 ... 0,61	0,47	−210	0,064	2,13	1,40
Water vapour at 100 °C	0,598	0,62	100	0,0191	2,00	1,32

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

Melting point of salts

The following table lists the melting points of a number of salts for salt baths.

Salt	Melting point °C	Salt	Melting point °C
Aluminium chloride	192	Potassium chloride	770
Ferric chloride	304	Calcium chloride	772
Potassium nitrate	308	Sodium chloride (table salt)	800
Sodium nitrate	310	Lithium fluoride	848
Zinc chloride	313	Sodium carbonate (soda)	852
Cuprous chloride	432	Potassium fluoride	857
Lithium carbonate	461	Potassium carbonate	897
Lead chloride	498	Barium chloride	955
Lithium chloride	614	Sodium fluoride	992
Cupric chloride	630	Calcium fluoride	1392

Metal salts in water – solubility

The solubility of inorganic salts in water is stated in g/100 g of water (in extracts).

Compound	Formula	At 0 °C	At 20 °C	At 100 °C
Cadmium sulphate	$CdSO_4$	75,4	76,6	60,8
Calcium bicarbonate	$Ca(HCO_3)_2$	16,1	16,6	18,4
Ferrous chloride	$FeCl_2$	49,7	62,5	94,9
Potassium ferrocyanide	$K_4Fe(CN)_6$	14,3	28,2	74,2
Copper(II) sulphate pentahydrate	$CuSO_4\ 5H_2O$	23,1	32	114
Magnesium formate	$Mg(HCO_2)_2$	14	14,4	22,9
Sodium dichromate	$Na_2Cr_2O_7$	163	183	415
Nickel(II) bromide	$NiBr_2$	113	131	155
Mercury(II) bromide	$HgBr_2$	0,3	0,56	4,9
Silver nitrate	$AgNO_3$	122	216	733
Zinc chloride	$ZnCl_2$	342	395	614

Source: Internetchemie.info, updated 06.07.2022, viewed 14.07.2023, https://www.internetchemie.info/chemie-lexikon/daten/l/loeslichkeits-produkte.php.

Series

Electrolytic series

If two metals come into contact in the presence of water, acids, bases or salts, electrolytic decomposition of the less noble metal will occur. The less noble metal (has a lower position in the electrolytic series) corrodes and the more noble metal is protected.

Voltage values are stated against a hydrogen electrode.

Substance	Voltage V	Substance	Voltage V
Gold	+1,50	Indium, thallium	−0,34
Chlorine	+1,36	Cadmium	−0,40
Bromine	+1,09	Iron	−0,40
Platinum	+0,87	Chromium	−0,56
Mercury	+0,86	Zinc	−0,76
Silver	+0,80	Aluminium, oxidised	−0,70 ... 1,3
Iodine	+0,58	Manganese	−1,1
Copper	+0,51	Aluminium, bright	−1,45
Arsenic	+0,30	Magnesium	−1,55
Bismuth	+0,23	Beryllium	−1,96
Antimony	+0,20	Calcium	−2,50
Hydrogen	0,00	Sodium	−2,72
Lead	−0,13	Barium	−2,80
Tin	−0,15	Potassium	−2,95
Nickel	−0,22	Lithium	−3,02
Cobalt	−0,29	Fluorine	−4,0

Thermoelectric series

Voltage values for a temperature difference of +100 °C are stated against copper as a reference material (0 °C).

Substance	Voltage mV	Substance	Voltage mV
Chrome nickel	+1,44	Manganin	−0,04
Iron	+1,04	Aluminium	−0,36
Tungsten	+0,05	Platinum	−0,76
Copper	0,00	Nickel	−2,26
Silver	−0,04	Constantan	−4,16

Technically important chemical substances

Commercial names and formulae The following table shows a selection of technically important, chemical substances with the corresponding commercial names and formulae.

Commercial name	Chemical designation	Formula
Acetone	Acetone (propanone)	$(CH_3)_2 \cdot CO$
Acetylene	Acetylene	C_2H_2
Alum	Potassium aluminium sulphate	$KAl(SO_4)_2 \cdot 12H_2O$
Alumina	Aluminium oxide	Al_2O_3
Ammonia	Ammonia	NH_3
Ammonia solution	Ammonia in aqueous solution	NH_3 in H_2O
Aqua fortis	(see Nitric acid)	
Bauxite	Aluminium hydroxide	$Al_2O_3 \cdot 2H_2O$
Benzene	Benzene	C_6H_6
Blue vitriol	Copper(II) sulphate	$CuSO_4 \cdot 5H_2O$
Borax	Sodium tetraborate	$Na_2B_4O_7 \cdot 10H_2O$
Boric acid	Boric acid	H_3BO_3
Brownstone	Manganese dioxide	MnO_2
Calcined soda	Sodium carbonate, anhydrous	Na_2CO_3
Calcium carbide	Calcium carbide	CaC_2
Calcium chloride	Calcium chloride	$CaCl_2$
Carbon dioxide	Carbon dioxide	CO_2
Carbon monoxide	Carbon monoxide	CO
Carborundum	Silicon carbide	SiC
Caustic lime	Calcium hydroxide	$Ca(OH)_2$
Caustic potash	Potassium hydroxide	KOH
Caustic potash solution	Potassium hydroxide in aqueous solution	KOH
Caustic soda	Sodium hydroxide	$NaOH$
Chalk	Calcium carbonate	$CaCO_3$
Chile saltpetre	Sodium nitrate	$NaNO_3$

Continuation of table, see Page 106.

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Continuation of table, Commercial names and formulae, from Page 105.

Commercial name	Chemical designation	Formula
China clay	Kaolin	$Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$
Chlorinated lime	Chlorinated lime	$CaCl(OCl)$
Cinnabar	Mercury sulphide	HgS
Corundum (emery)	Aluminium oxide	Al_2O_3
Dolomite	Calcium magnesium carbonate	$CaMg(CO_3)_2$
Emery	(see Corundum)	
English red	Iron oxide	Fe_2O_3
Epsom salt	Magnesium sulphate	$MgSO_4 \cdot 7H_2O$
Ether	Ethyl ether (diethyl ether)	$(C_2H_3)_2O$
Fixing salt	Sodium thiosulphate	$Na_2S_2O_3 \cdot 5H_2O$
Glauber's salt	Sodium sulphate	Na_2SO_4
Glycerine	Propanetriol	$C_3H_8O_3$
Green vitriol	Iron(II) sulphate	$FeSO_4$
Gypsum	Calcium sulphate	$CaSO_4 \cdot 2H_2O$
Hydrochloric acid	Hydrochloric acid	HCl
Hydrogen sulphide	Hydrogen sulphide	H_2S
Iron oxide	Iron oxide	Fe_2O_3
Lime, burnt	Calcium oxide	CaO
Lime, slaked	(see Caustic lime)	
Lime, with phosphoric acid	Calcium phosphate	$Ca_3(PO_4)_2$
Limestone	Calcium carbonate	$CaCO_3$
Litharge	Lead monoxide	PbO
Lithopone	Mixture of zinc sulphide and barium sulphate	ZnS and $BaSO_4$
Lye	(see Caustic soda)	
Magnesia	Magnesium oxide	MgO
Marble	(see Limestone)	
Mine gas	Methane	CH_4
Minium	(see Red lead)	
Nitric acid	Nitric acid	HNO_3
Oil of vitriol	Concentrated sulphuric acid	H_2SO_4

Continuation of table, see Page 107.

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Continuation of table, Commercial names and formulae, from Page 106.

Commercial name	Chemical designation	Formula
Petrol	Petrol	(C_nH_{2n+2})
Phosphoric lime	Calcium phosphate	$Ca_3(PO_4)_2$
Potash	Potassium carbonate	K_2CO_3
Prussiate of potash, red	Potassium ferricyanide	$K_3Fe(CN)_6$
Prussiate of potash, yellow	Potassium ferrocyanide	$K_4Fe(CN)_6$
Red lead	Red lead	Pb_3O_4
Rust	Hydrated ferric oxide	$Fe_2O_3 \cdot xH_2O$
Sal ammoniac	Ammonium chloride	NH_4Cl
Silver bromide	Silver bromide	$AgBr$
Soda, crystalline	Sodium carbonate, anhydrous	Na_2CO_3
Soda lye	Sodium hydroxide in aqueous solution	$NaOH$
Sodium bicarbonate	Sodium hydrogen carbonate	$NaHCO_3$
Soldering flux	Aqueous solution of zinc chloride	$ZnCl_2$
Sulphuric acid	Sulphuric acid	H_2SO_4
Sulphurous acid	Dihydrogen sulphite	H_2SO_3
Table salt	Sodium chloride	$NaCl$
Tetra	Tetrachloromethane	CCl_4
Tin chloride	Tin(IV) chloride	$SnCl_4$
Trilene	Trichloroethylene	C_2HCl_3
Vinegar	Acetic acid	$C_2H_4O_2$
Water glass	Sodium silicate or potassium silicate in aqueous solution	Na_4SiO_4 or Na_2SiO_2 K_4SiO_4 or K_2SiO_3
White lead	Basic lead carbonate	$Pb(OH)_2 \cdot 2PbCO_3$
Zinc, with hydrochloric acid	Zinc chloride	$ZnCl_2$

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Physics

Definitions, values and constants

Atomic building blocks A number of important atomic building blocks of matter are described as follows:

Name		Explanation			
Atom		Smallest, chemically uniform particle of an element, consisting of a nucleus and an electron shell; order of magnitude of the diameter 10^{-10} m; the atomic nuclei are smaller by a factor of 10^4 to 10^5; the principal mass of the atom is located in the nucleus (density approximately 10^{14} g/cm^3); all chemical processes (as well as many electrical, magnetic and optical processes) take place in the atomic shell; atoms consist of elementary particles, around 300 are known			
Elementary particles		Elementary particles are the smallest known building blocks of matter. The particles included in the standard model of particle physics are: 6 quarks, 6 leptons, gauge bosons (mediators) and the Higgs boson			
	Fermions	Particles with a **half-integral spin**			
			Generations I	**Generations II**	**Generations III**
		Quarks	Up Down	Charm Strange	Top Bottom
		Leptons	Electron Electron neutrino	Muon Muon neutrino	Tau Tau neutrino
	Bosons (interactions)	Particles with **integral spin**			
		Gauge bosons	Gluon Photon Z boson W boson		
		Scalar bosons	Higgs		
Composite particles		Particles composed of elementary particles			
	Hadrons	Subatomic particles held together by the strong interaction **Baryons** (for example nucleons), also refer to table entry on Page 109 **Mesons** (for example pion, kaon), also refer to table entry on Page 109			

Continuation of table, see Page 109.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Atomic building blocks, from Page 108.

Name		Explanation
Photons		Quanta of the electromagnetic radiation field
	Light quantum	Charge = 0 Mass = 0 Half-life = ∞
Leptons		Particles **extraneous to the nucleus** with **a half-integral spin** (l = 1/2)
	Electron neutrino	Mass, theoretical = 0 (< 0,2 keV) Charge = 0 Half-life = ∞
	Electron	Smallest elementary particle with negative charge Charge = –e Rest mass = $9{,}1093826 \cdot 10^{-31}$ kg Half-life = ∞
	Positron	Smallest elementary particle with positive charge Charge = +e Mass = $9{,}1093826 \cdot 10^{-31}$ kg
Baryons		**Nuclear-active** particles with **a half-integral spin** (l = 1/2, 3/2, ...)
	Nucleons	Collective term for protons and neutrons, which are constantly transforming into one another in the atomic nucleus; at the same time, the π-meson field produces the charge transfer
	Proton	Positively charged nuclear building block Charge = +e Rest mass = $1{,}67262171 \cdot 10^{-27}$ kg ≈ 1840 electron masses
	Neutron	Uncharged nuclear building block Charge = 0 Rest mass = $1{,}674927498 \cdot 10^{-27}$ kg
Mesons		**Nuclear-active** particles with **integral spin** (l = 0, 1, 2, ...); example: π- and K-mesons
Molecule		Two or polyatomic particles; composed of atoms, held together by chemical bonds

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Atomic and mass numbers, nuclear and atomic radii

The following table lists the atomic numbers, mass numbers, nuclear radii and atomic radii, complete with corresponding ratios, for a number of selected elements.

Element	Atomic number Z	Mass number M (most common isotope)	Nuclear radius r_K 10^{-15} m	Atomic radius r_A 10^{-10} m	Radii ratio r_A/r_K
Li	3	7	2,3	1,5	65 217
Ne	10	20	3,3	0,5	15 152
Na	11	23	3,4	1,8	52 941
Ar	18	40	4,1	0,9	21 951
K	19	39	4,1	2,2	53 659
Kr	36	84	5,3	1,1	20 755
Rb	37	85	5,3	2,4	45 283
Xe	54	132	6,1	1,3	21 311
Cs	55	133	6,2	2,6	41 935
Rn	86	222	7,3	1,9	26 027

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Values used in nuclear physics and other fields

The following table shows a selection of values used in nuclear physics and other fields.

Name	Unit	Relationship/ formula symbols	Definition
Atomic mass	$u = 1{,}660\,538\,86 \cdot 10^{-27}$ kg Particle number	$u = m_{C12}/M_{C12} = 1/N_A$ $N = \frac{m}{M} N_A$	The relative mass of the nuclide ^{12}C is the unit M = molar mass
Half-life	s, min, d, a	$T_{1/2} = \ln 2/\lambda$ (λ = decay constant)	Time required for half of the original quantity of atoms to decay
Atomic energy	Electron volt $1\ eV = 1{,}602\,176\,634 \cdot 10^{-19}$ J $1\ MeV = 10^6$ eV	$W = e\,U$	The energy gained by an electron on travelling through a potential of 1 V is the unit

Continuation of table, see Page 111.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Values used in nuclear physics and other fields, from Page 110.

Name	Unit	Relationship/ formula symbols	Definition
Mass	1 MeV	$m = \frac{E}{{c_0}^2}$ $m = \frac{m_0}{\sqrt{1-(c/c_0)^2}}$	From the equivalence of energy and mass (according to Einstein)
Absorbed dose	Gray[1)] 1 Gy = 1 J/kg	D = W/m	Energy absorbed per unit mass of irradiated material; 1 rem = 10^{-2} Gy (obsolete)
Activity of a radioactive substance	Becquerel 1 Bq = 1/s	A	A measure of the intensity of radioactive radiation; 1 Ci (Curie) = $3{,}7 \cdot 10^{10}$ Bq
Dose equivalent	Sievert[1)] 1 Sv = 1 J/kg	$H = D\,w_R$	A measure of the relative biological effectiveness of the radiation effect; the energy absorbed in the human body as a result of exposure to a specific type of radiation; radiation weighting factor $w_R = 1$ (γ-radiation up to 20, α-radiation, hard neutron radiation)
Energy dose rate	W/kg	$\dot{D}$	–
Ion dose	C/kg	J = Q/m	Charge/mass; 1 R (Roentgen) = $258 \cdot 10^{-6}$ C/kg (obsolete)
Ion dose rate	A/kg	$\dot{j} = \frac{I}{m} = \frac{Q}{m \cdot t}$	Current/mass or charge/(mass · time)
Effective cross-section	m^2	σ	A measure of the yield of nuclear reactions; imaginary cross-section through the irradiated atoms
Amount of substance	Mole	$n = N/N_A = m/M$	Amount of substance = particle number/Avogadro's constant

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

1) The units Gray (Gy) and Sievert (Sv) are both equivalent to the unit J/kg.
Gy is used to express the pure (physical) absorbed dose of radiation. Sv is used if a factor has been included to take account of the biological effectiveness of the absorbed dose.
Until 1985, the unit rem was used to measure dose equivalent; today, the unit Sv is used.

Physical constants A selection of important physical constants is described below.

Name	Value	Explanation
Gravitational constant	$G = 6{,}674\,28 \cdot 10^{-11}\ m^3/(kg \cdot s^2)$	Force in N which attracts 2 bodies weighing 1 kg each and set 1 m apart
Standard gravitational acceleration	$g_n = 9{,}806\,65\ m/s^2$	Standard value defined by the 3rd General Conference on Measures and Weights in 1901
Molar gas constant	$R = 8{,}314\,472\ J/(mol \cdot K)$	The work that must be done to heat 1 mol of an ideal gas by 1 K under constant pressure; same value for all sufficiently ideal gases
Standard molar volume	$V_m = 22{,}413\,996 \cdot 10^{-3}\ m^3/mol$	Volume occupied by 1 mol of an ideal gas under standard conditions
Avogadro's constant	$N_A = 6{,}022\,1415 \cdot 10^{23}\ mol^{-1}$	Number of atoms or molecules in 1 mol of a substance
Loschmidt's constant	$N_L = 2{,}686\,8 \cdot 10^{25}\ m^{-3}$	Number of atoms or molecules in 1 m^3 of a gas under standard conditions (0 °C and 1 013,25 hPa)
Boltzmann's constant	$k = R/N_A$ $= 1{,}380\,650\,5 \cdot 10^{-23}\ J/K$	Average energy increase of a molecule or atom when heated by 1 K
Faraday's constant	$F = N_A \cdot e$ $= 9{,}648\,533\,83 \cdot 10^4\ C/mol$	The charge quantity transported by 1 mol of singly charged ions
Elementary charge	$e = F/N_A$ $= 1{,}602\,176\,53 \cdot 10^{-19}\ C$	The smallest possible charge (charge of an electron)
Permittivity of free space (electric constant)	$\epsilon_0 = \dfrac{1}{\mu_0 \cdot c^2}$ $= 8{,}854\,2 \cdot 10^{-12}\ F/m$	Proportionality factor between the charge density and the electric field strength
Permeability of free space (magnetic constant)	$\mu_0 = 1{,}256\,6 \cdot 10^{-6}\ H/m$ $= 4 \cdot \pi \cdot 10^{-7}\ H/m$	Proportionality factor between the induction and the magnetic field strength
Speed of light in a vacuum	$c_0 = 2{,}997\,9 \cdot 10^8\ m/s$	Propagation rate of electromagnetic waves
Planck's constant (action quantum)	$h = 6{,}626 \cdot 10^{-34}\ J \cdot s$	Combines the energy and frequency of a light quantum as a proportionality factor
Characteristic impedance of a vacuum (impedance of free space)	$Z_0 = 376{,}730\ \Omega$	Propagation resistance for electromagnetic waves in a vacuum
Stefan-Boltzmann's constant	$\sigma = 5{,}670\,40 \cdot 10^{-8}\ W/(m^2 \cdot K^4)$	Combines the radiation energy and temperature of a radiating body

Continuation of table, see Page 113.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Physical constants, from Page 112.

Name	Value	Explanation
Planck's radiation constants	$c_1 = 3{,}741\,771\,38 \cdot 10^{-16}\ \mathrm{W \cdot m^2}$ $c_2 = 1{,}438\,775\,2 \cdot 10^{-2}\ \mathrm{m \cdot K}$	Constants of Planck's radiation law in the original wavelength-dependent formulation
Wien's constant	$K = 2{,}897\,8 \cdot 10^{-3}\ \mathrm{m \cdot K}$	Combines the wavelength of the radiation maximum with the absolute temperature of a radiating body
Rydberg's constant	$R_\infty = 1{,}097\,37 \cdot 10^{7}\ \mathrm{m^{-1}}$	Fundamental, nuclear physical constant occurring in standard formulae for spectral lines
Rest mass of an electron	$m_e = 9{,}109\,382\,6 \cdot 10^{-31}\ \mathrm{kg}$	Mass of a stationary electron
Electron radius	$r_e = 2{,}817\,9403\,25 \cdot 10^{-15}\ \mathrm{m}$	Radius of an electron (spherical formation)
Bohr's radius	$r_1 = 5{,}291\,772\,108 \cdot 10^{-11}\ \mathrm{m}$	Radius of the innermost electron path in Bohr's atom model
Atomic unit of mass	$u = 1{,}660\,538\,86 \cdot 10^{-27}\ \mathrm{kg}$	Unified atomic mass unit (one twelfth of the mass of an atom of the nuclide ^{12}C)
Unit of mass	$1\ \mathrm{UM} = 931{,}494\ \mathrm{MeV/c^2}$ [1)]	Used for energy conversions
Solar constant	$S \approx 1\,365\ \mathrm{J/m^2 s} = 1\,365\ \mathrm{W/m^2}$	Radiation energy of the Sun which arrives vertically at the upper limits of the Earth's atmosphere

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

[1)] Electron volt (eV) and mega electron volt (MeV) are measures of energy in nuclear physics. 1 eV is the energy gained by an electron when it is accelerated in an electrical field of 1 volt ($1\ \mathrm{eV} = 1{,}602\,176\,53 \cdot 10^{-19}\ \mathrm{J}$).

Electromagnetic radiation

The characteristics of electromagnetic radiation are:

- propagation at the speed of light
- wave nature
- no deflection by electric or magnetic fields
- wavelength $\lambda = c/f = c \cdot T$
  c = speed of light = $2{,}997\,92 \cdot 10^{8}$ m/s (in a vacuum)
  f = frequency in Hz
  T = period of oscillation in s

Other definitions and values used in relation to electromagnetic radiation are described as follows:

Radiation type Wavelength λ		Occurs in the event of energy changes in	Is generated by	Is absorbed by (examples)
Cosmic rays				
0,000 2 ... 0,02 pm		Nucleons (nuclear building blocks)	High-energy nuclear reactions	Around 10 cm of lead
Gamma rays				
0,5 ... 27 pm		Atomic nuclei	Atomic nuclear reactions and radioactive decay	Around 1 cm of lead
X-radiation				
Hard 5,7 ... 80 pm (0,057 ... 0,8 Å)		Internal electron shells	High-vacuum and gas-discharge tubes at high operating voltages	Around 3 ... 0,04 cm of aluminium
Soft 0,08 ... 2 nm (0,8 ... 20 Å)				Around 400 ... 1 μm of aluminium, bone, glass
Ultrasoft 2 ... 37,5 nm (20 ... 375 Å)				Less than 1 μm of aluminium, air
Light rays				
Ultraviolet (short-wave) 0,014 ... 0,18 μm		External electron shells	Spark, arc, glow discharge in vacuum, quartz lamp etc.	Air
Ultraviolet (long-wave) 0,18 ... 0,36 μm				Quartz ($\lambda < 0{,}15$ μm) Glass ($\lambda < 0{,}31$ μm)
Violet Blue Green Yellow Red	0,36 ... 0,42 μm 0,42 ... 0,49 μm 0,49 ... 0,53 μm 0,53 ... 0,65 μm 0,65 ... 0,81 μm		Sun, glowing substances etc.	Opaque substances
Infrared (heat rays) 0,81 ... 400 μm			Heated bodies[1]	Glass

Continuation of table, see Page 115.

[1] The following applies to "black body" radiation:
average wavelength λ (in μm) = 2 898/(absolute temperature in K).
Example: at +20 °C (= 293 K) λ = 9,89 μm,
i.e. at +20 °C the maximum heat radiation intensity is at λ = 9,89 μm.
Source: Krist, Handbuch für Techniker und Ingenieure.

Continuation of table, Electromagnetic radiation, from Page 114.

Radiation type	Wavelength λ	Occurs in the event of energy changes in	Is generated by	Is absorbed by (examples)
Hertzian waves				
0,01 ... 30 cm		Atoms or molecules	Spark transmitters, velocity-modulated tubes	Metals
Broadcasting waves				
Ultrashort	0,3 ... 10 m	Resonant circuits with capacitance and inductance	Transistor senders	Metals
Short	10 ... 100 m		Propagation of these waves is no longer ray-like and this is why there are no “wave shadows” in valleys and behind mountains; waves are diffracted on the Heaviside layer (ionosphere) and deflected back to Earth; with increasing wavelength, the space wave steps behind the ground wave	
Medium	100 ... 600 m			
Long	600 ... 3000 m			
Telegraphic waves	3 ... 30 km			

Temperature points

General, important temperature points are:

Triple point[1] of water	+0,01 °C
Boiling point[2] of water	+100,00 °C
Boiling point[2] of oxygen	−182,97 °C
Boiling point[2] of nitrogen	−196,00 °C
Boiling point[2] of air	−191,0 °C
Boiling point[2] of sulphur	+444,6 °C
Freezing point of silver	+960,8 °C
Freezing point of gold	+1063,0 °C

[1] At 611,657 Pa.

[2] At normal pressure (1013,25 hPa).

Thermal expansion of solids and gases

Almost all solids expand when their temperature is increased and shrink when the temperature decreases. Water does not follow this rule. It exhibits its greatest density at +4 °C and expands irrespective of whether it rises above or falls below this temperature.

Homogeneous solids expand uniformly in all directions (volume expansion). In many cases, we are only interested in expansion in a specific direction (superficial or linear expansion). If a solid’s linear expansion or volume change is impeded in the event of a temperature change, stresses occur within the solid.

Linear thermal expansion coefficient

In the case of solids, the linear thermal expansion coefficient (coefficient of linear expansion) is the relative change in length per degree of temperature increase.

Thus, the change in length Δl of a solid is described in terms of:

Equation 1

$$\Delta l = l_0 \cdot \alpha \cdot \Delta T$$

Δl = length change
l_0 = initial length
α = coefficient of linear thermal expansion
ΔT = temperature increase

A temperature increase ΔT produces the following in the solid:

Equation 2

$$\epsilon_{\Delta T} = \frac{\Delta l}{l_0} = \alpha \cdot \Delta T$$ in the case of unimpeded expansion

$$\sigma_{\Delta T} = E \cdot \epsilon_{\Delta T} = E \cdot \alpha \cdot \Delta T$$ in the case of impeded expansion

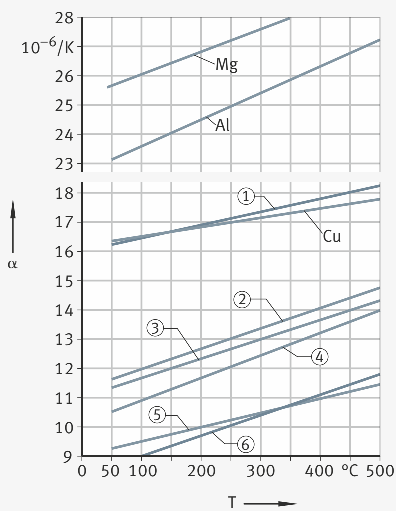
The following diagram shows the influence of temperature on the coefficient of linear expansion α.

Figure 1

Influence of temperature on α in the case of steels and non-ferrous metals

T = temperature
α = coefficient of linear thermal expansion
Mg = magnesium
Al = aluminium
Cu = copper

1 18% Cr + 9% Ni steel
2 0,4% Mo steel
3 Cr-Mo steel
4 Unalloyed steel: (0,2 – 0,6% C)
5 13% Cr steel
6 Cast iron



The following table lists examples of values for coefficients of linear thermal expansion α at +20 °C.

Material	α 10^{-6}/K	Material	α 10^{-6}/K	Material	α 10^{-6}/K
Cast iron	9 ... 10	Copper	16 ... 17	Thermoplastics	70 ... 250
Unalloyed steel	11 ... 12	Aluminium	23 ... 24	Brickwork	5 ... 8
Cr-Mo steel	12 ... 13	Magnesium	25,5	Rubble	3
Cr-Ni steel	16 ... 17	Thermosets	10 ... 80	Glass	8 ... 10

Coefficient of cubical expansion

The coefficient of cubical expansion (volume expansion coefficient) of a solid, liquid or gaseous body is the relative volume change per degree of temperature increase.

Thus, the change in volume ΔV is described in terms of:

Equation 3

$$\Delta V = V_0 \cdot \beta \cdot \Delta T$$

ΔV = volume change
V_0 = initial volume
β = coefficient of thermal volume expansion
ΔT = temperature increase

In the case of homogeneous, solid bodies:

Equation 4

$$\beta = 3 \cdot \alpha$$

In the case of ideal gases, the coefficient of cubical expansion is the same value for all gases and temperatures at constant pressure and relative to the volume V_0 at 0 °C:

Equation 5

$$\beta = \frac{1}{V_0} \cdot \frac{\Delta V}{\Delta T} = \frac{1}{273{,}15} \frac{1}{K}$$

Coefficient of superficial thermal expansion

Superficial thermal expansion can be described by the thermal volume expansion coefficient:

Equation 6

$$\Delta A = A_0 \cdot \frac{2}{3} \cdot \beta \cdot \Delta T$$

ΔA = surface change
A_0 = initial surface
β = coefficient of thermal volume expansion
ΔT = temperature increase

Astronomical and terrestrial definitions and values

Astronomical units The following table shows a selection of important astronomical units.

Name	Value	Explanation
Speed of light in a vacuum	c_0 = $2,9979 \cdot 10^8$ m/s	Propagation rate of electromagnetic waves
Light-year	L_y = $9,46073 \cdot 10^{15}$ m	Distance covered by electromagnetic waves in space in 1 year
Sidereal year (stellar year)	S_y = 365,2564 mean solar days = 365 d 6 h 9 min 9,54 s	The sidereal year is based on the position of the Sun relative to the fixed stars[1)]. However, this period is not constant. The stated value corresponds to the reference point of 01.01.2000.
Tropical year (solar year)	T_y = 365,2422 mean solar days = 365 d 5 h 48 min 45 s	The mean vernal equinox is the reference point
Sidereal month (stellar month)	S_m = 27,32166 d mean solar time = 27 d 7 h 43 min 11,5 s	–
Tropical month (solar month)	T_m = 27,32158 d mean solar time = 27 d 7 h 43 min 4,7 s	–
Synodic month (lunar month)	S_{ym} = 29,53059 d mean solar time = 29 d 12 h 44 min 2,9 s	Time between two identical moon phases: new moon to new moon
Orbital period of the Moon around the Earth	$t_{M\ sid}$ = 27,32166 d $t_{M\ tro}$ = 27,32158 d	Sidereal year Tropical year
Sidereal day	d_{Sy} = 0,9972696 mean solar days = 23 h 56 min 4,091 s	–
Mean solar day	d_{Ty} = 1,0027379 sidereal days	–
Day	d = 24 h = 1440 min = 86400 s	The day is 3 min 56 s longer than the sidereal day
Astronomical unit	AU = $1,496 \cdot 10^{11}$ m	Mean distance between the Earth and the Sun

[1)] Time interval between two successive passages of the Sun through the same point along the apparent trajectory of the Sun (ecliptic).
The point of the ecliptic is measured in relation to a fixed star.
Average obliquity of the ecliptic is currently ≈ 23° 27′ 15″.

Our Solar System A number of interesting values relating to our Solar System are:

Solar planets	Equator diameter km	Mass (Earth = 1[1])	Mean density kg/m^3	Sidereal rotation period	Distance from the Sun 10^6 km	Sidereal orbital period years
Sun	1 391 016	333 062	1 409	25,23 d 5,52 h	–	–
Earth	12 742	1,000	5 513,4	23,93 h	149,60	1,00
Moon[2]	3 475	0,012	3 344	27 d 7,1 h	0,384 00 (from the Earth)	
Mercury	4 879	0,055	5 428,9	58 d 15,51 d	58	0,24
Venus	12 104	0,815	5 243,0	243 d 0,43 d	108	0,62
Mars	6 779	0,107	3 934,0	1 d 0,62 h	228	1,88
Jupiter	139 822	317,828	1 326,2	9,92 h	778	11,86
Saturn	116 464	95,161	687,1	10,66 h	1 428	29,46
Uranus	50 724	14,536	1 270	17,24 h	2 872	84,02
Neptune	49 244	17,148	1 638	16,11 h	4 498	164,79
Pluto[3]	2 377	0,002 2	–	6 d 9,29 d	5 910	249,17

Source: Solar System Dynamics, Jet Propulsion Laboratory, California Institute of Technology, December 2019 and Solar System Exploration Website, NASA: Sun data October 2021, moon data December 2019, https://ssd.jpl.nasa.gov/ and https://solarsystem.nasa.gov/

[1] Earth's mass = $5{,}97217 \cdot 10^{24}$ kg, volume of the globe = 1 083 206,9 million km^3, average density of the Earth = 5 513,4 kg/m^3, circumference of the Earth's trajectory = 939 120 000 km.

[2] Satellite of the Earth.

[3] Reclassified as a dwarf planet in 2006 on the orders of the International Astronomical Union.

The Earth – fundamental figures A number of important, fundamental figures relating to our Earth are (rounded values):

Earth's surface	510,1 million km^2
Of which total surface area of land	147,9 million km^2 (29%)
Of which total surface area of water	362,2 million km^2 (71%)
Length of the Equator[1]	40 076 km
Equatorial radius a	6 378 km
Length of the meridian[2]	40 000 km
Polar radius b	6 356 km
Length of the tropic	36 778 km
Length of a polar circle	15 996 km
Flattening (a – b)/a	1 : 297
Mean longitude between two great circles at a distance of 1°	111,120 km (corresponds to 60 nautical miles)

[1] The Equator is the largest circle of latitude (great circle). Circles of latitude run parallel to the Equator in an east-west direction.

[2] Meridians, also known as lines of longitude, are great circles, which run in a north-south direction and intersect at the poles.

Legend

a m Equatorial radius	b m Polar radius.

Interesting speeds The following table shows a number of interesting speeds (rounded values).

Name	Speed	
	m/s	km/h
Gulf Stream	1,8 ... 2,5	6,5 ... 9
Wind force 6 (strong breeze)	11 ... 14	39 ... 49
Wind force 12 (hurricane)	$>$32	$>$118
Sound in air (at +20 °C)	340	1 200
Point on the Equator	464	1 670
Earthquake waves	3 000 ... 7 000	180 ... 420
Orbital speed of a satellite	7 800 (LEO[1]) ... 3 000 (GEO[2])	28 080 (LEO[1]) ... 10 800 (GEO[2])
Speed required to leave the Earth's gravitational field (point on the Equator)	11 200	40 320
Speed required to leave the Solar System (directly from the Sun)	617 400	2 222 640
Mean orbital path of the Earth around the Sun	29 800	107 280
Lightening (approx. 1/10 ... 1/3 of the speed of light)	30 000 000 ... 100 000 000	108 000 000 ... 360 000 000
Cathode rays (electrons, 50 kV)	100 000 000	360 000 000
Light in a vacuum	299 792 458[3]	1 079 252 849

[1] Low Earth Orbit, from an altitude of around 200 km.
[2] Geostationary Orbit, at an altitude of 35 786 km.
[3] Also refer to Table Astronomical units, Page 118.

Dimensionless characteristic values

Dimensionless characteristic values The definitions for similarity characteristics are described in the following table.

Type of similarity	Scale factors (invariant)	Name of characteristic	Definition	Similarity of physical subject matter
Geometric Length	$\varphi_L = \dfrac{L_1}{L_0}$	–	–	All lengths are similar if the scale is the same (pantograph)
Kinematic Length Time	φ_L, φ_t	Mach	$Ma = \dfrac{v}{c_s}$	Relationship between velocity v and sound velocity c_s of the fluid
Static Length Force	$\varphi_F = \dfrac{\rho_1}{\rho_0} \cdot \varphi_L{}^3$	–	–	Similarity of weights (constant gravitational acceleration)
	φ_L, φ_F	Hooke	$H_0 = \dfrac{F}{E \cdot L^2}$	Sole effect of elastic forces (similarity of expansions)

Continuation of table, see Page 121.

Continuation of table, Dimensionless characteristic values, from Page 120.

Type of similarity	Scale factors (invariant)	Name of characteristic	Definition	Similarity of physical subject matter
Dynamic Length Time Force	$\varphi_L, \varphi_t, \varphi_F$	Newton	$Ne = \frac{F}{\rho \cdot v^2 \cdot L^2}$	Ratio of resistance force to flow force
		Cauchy	$Ca = \frac{v}{\sqrt{E/\rho}}$	Ratio of inertia forces to elastic forces
		Froude	$Fr = \sqrt{\frac{v^2}{g \cdot L}}$	Ratio of inertia forces to gravitational forces
		Reynolds	$Re = \frac{v \cdot L}{\nu}$	Ratio of inertia forces to viscous forces
		Weber	$We = \frac{\rho \cdot v^2 \cdot L}{\sigma}$	Ratio of inertia force to surface force
		Euler	$Eu = \frac{\Delta p}{\rho \cdot v^2}$	Ratio of compressive forces to inertia forces
Thermal Length Time Temperature	$\varphi_L, \varphi_t, \varphi_q$	Péclet	$Pe = v \cdot L \left(\frac{\rho \cdot c_p}{\lambda}\right)$	Ratio of transported to conducted heat quantity
		Prandtl	$Pr = \frac{P_e}{R_e} = \nu \cdot \left(\frac{\rho \cdot c_p}{\lambda}\right)$	Ratio between kinematic viscosity and thermal diffusivity
		Nußelt	$Nu = \frac{\alpha \cdot L}{\lambda}$	Heat transfer ratio with moving and stagnant layer
		Fourier	$Fo = \left(\frac{\lambda}{\rho \cdot c_p}\right) \cdot \frac{t}{L^2}$	Ratio of conducted to stored heat

Legend

L Length	m	g Gravitational acceleration	m/s^2
t Time	s	α Heat transfer coefficient	$W/(m^2K)$
F Force	N	λ Thermal conductivity	$W/(m \cdot K)$
ρ Density	kg/m^3	ν Kinematic viscosity	m^2/s
v Velocity	m/s	c_p Specific heat capacity.	$J/(kg \cdot K)$
E Modulus of elasticity	N/m^2		

Mechanics

Definitions

Mechanics Mechanics is a branch of physics which describes the motion events occurring in nature and engineering. It deals with the plane and spatial motion of bodies and the effect of forces.

Dynamics Dynamics deals with the forces (general: interactions) which cause bodies to move. It describes the association of these motions with mass and acting forces.

The branches of dynamics are:

- **Kinematics**
  Kinematics describes the occurrence of motions in space and time, without consideration of forces. Important terms used in dynamics are the motion values displacement, velocity and acceleration.
- **Kinetics**
  Kinetics describes the change in the above-mentioned motion values which occurs under the influence of forces in space.

Newton's fundamental law All dynamic motion events are based on Newton's fundamental physical laws:

- **Principle of inertia**
  A body will remain at rest or in uniform motion in a straight line unless compelled to change its state of motion by the action of external forces.
- **Action principle/principle of linear momentum**
  The resulting force needed to change the motion of a body is equal to the change in the quantity of motion over time, i.e. the linear momentum or angular momentum.
  Alternatively, a resulting force is exerted on a body, causing it to accelerate in the direction of this force.
  See also section Principle of angular momentum, Page 136.
- **Principle of action and reaction**
  Forces always act reciprocally.
  If a body A exerts a force F on body B, then body B will exert an equal but opposite force on body A.

Values and units

Values and units The following table shows the names, values and units used in mechanics.

Designation	Value	Unit	Explanation
Acceleration	$\vec{a}$	m/s^2	$\vec{a} = d\vec{v} / dt$; $(\ddot{x}, \ddot{y}, \ddot{z})$
Angle, angle of rotation	φ	rad[1]	1 rad = 57°17′45″
Angular acceleration	$\vec{\alpha}, \dot{\omega}, \ddot{\varphi}$	rad/s^2	$\vec{\alpha} = \dot{\vec{\omega}} = d\dot{\vec{\omega}} / dt = \ddot{\vec{\varphi}}$
Angular frequency	$\vec{\omega}$	1/s	$f = \vec{\omega} / (2\pi)$
Angular impulse	$\vec{H}$	Nms	$\vec{H} = \int \vec{M} \cdot dt$
Angular velocity	$\vec{\omega}, \dot{\varphi}$	rad/s	$\vec{\omega} = d\vec{\varphi} / dt = \dot{\vec{\varphi}}$
Area	A	m^2	$1\ m^2 = 10^6\ mm^2$
Cartesian coordinates	x, y, z	m	Right-handed coordinate system
Coefficient of friction	μ	1	$\mu = \vec{F}_R / \vec{F}_N$ (Coulomb)
Density, mass density	ρ	kg/m^3	$\rho = m/V$
Dynamic viscosity	η	$N \cdot s/m^2$	$\vec{\tau} = \eta \cdot d\vec{v} / d\vec{h}$ (Newton)
Efficiency	η	1	$\eta = P_{eff}/P_{the}$
Force	$\vec{F}$	N	$\vec{F} = m \cdot \vec{a}$ (Newton)
Frequency	f	1/s	–
Friction energy	W_f	Nm	$W_f = \int \vec{F}_f \cdot d\vec{s}$
Gravitational acceleration[2]	$\vec{g}$	m/s^2	$\vec{g} = 9{,}80665\ m/s^2$
Impulse	$\vec{I}$	Ns	$\vec{I} = \int \vec{F} \cdot dt$
Kinematic viscosity	ν	m^2/s	$\nu = \eta/\rho$
Kinetic energy	E_k	J = Nm	$E_k = (m/2) \cdot \vec{v}^2$
Linear momentum	$\vec{p}$	$kg \cdot m/s$	$\vec{p} = m \cdot \vec{v}$
Mass	m	kg	SI base unit
Moment of force, torque	$\vec{M}$	Nm	$\vec{M}$ = force · lever arm
Mass moment of inertia	J	$kg\ m^2$	Second moment of mass
Moment of momentum, angular momentum	$\vec{L}$	$kg \cdot m^2/s$	$\vec{L} = J \cdot \vec{\omega}$
Path length, curve length	s	m	SI base unit
Potential energy (in the Earth's gravitational field or on the Earth's surface)	E_p	J = Nm	$E_p = m \cdot \vec{g} \cdot \vec{h}$ (position)
Power	P	W = Nm/s	P = W/t
Pressure	p	N/m^2	$p = \vec{F} / \vec{A}$
Rotational speed	n	1/s	$n = \vec{\omega} / (2\pi)$

Continuation of table, see Page 124.

[1] The unit can be replaced by the number “1”.

[2] For a more detailed explanation, refer to Table Conservative forces, Page 128

Continuation of table, Values and units, from Page 123.

Designation	Value	Unit	Explanation
Time, timespan, duration	t	s	SI base unit
Velocity	$\vec{v}$	m/s	$\vec{v} = d\vec{s} / dt \; ; \left(\dot{\vec{x}}, \dot{\vec{y}}, \dot{\vec{z}}\right)$
Volume	V	m^3	$1\ m^3 = 10^9\ mm^3$
Weight force[1)]	$\vec{F}_G$	N	$\vec{F}_G = m \cdot \vec{g}$
Work	W	Nm = J	$W = \int \vec{F} \cdot d\vec{s} \; ; \; W = \int \vec{M} \cdot d\vec{\varphi}$

1) For a more detailed explanation, refer to Table Conservative forces, Page 128

Motion equations

Fundamental law of accelerated motion

Newton's second fundamental law governing a constant mass is:
in translation of the centre of gravity "S":

Equation 1

$$\sum \vec{F}_S = m \cdot \vec{a}$$

in rotation about the centre of gravity "S":

Equation 2

$$\sum \vec{M}_S = J_S \cdot \dot{\vec{\omega}}$$

in rotation about the instantaneous centre of rotation "MP":

Equation 3

$$\sum \vec{M}_{MP} = J_{MP} \cdot \dot{\vec{\omega}}$$

Dynamic equilibrium according to d'Alembert

If the inertia forces ($m \cdot a$) and the moments resulting from the inertia effect ($J \cdot \omega$) are interpreted as a kinetic reaction (externally impressed forces or moments) during the accelerated motion of a body, this gives:

Equation 4

$$\sum \vec{F}_S + \left(-m\vec{a}\right) = 0 \qquad \sum \vec{F}_S + \vec{F}_K = 0$$

or:

Equation 5

$$\sum \vec{M}_S + \left(J_S \cdot \dot{\vec{\omega}}\right) = 0 \qquad \sum \vec{M}_S + \vec{M}_K = 0$$

Reduction to static equilibrium problem

The kinetic reaction is always opposing the (positively defined) direction of acceleration. Thus, the kinetic problem can be reduced to a static equilibrium problem.

The fundamental equations of statics will then apply:

Equation 6

$$\sum \overrightarrow{F^*} = 0 \qquad \sum \overrightarrow{M^*} = 0$$

The force sums $\sum \overrightarrow{F^*}$ and moment sums $\sum \overrightarrow{M^*}$ contain the respective kinetic reactions.

Solving motion equations

The following guidelines apply to the formulation of equilibrium relationships and solving of motion equations.

Defining co-ordinates

Define the co-ordinates at the centre of gravity of the body and in the expected direction of motion:

- for the translational motion of the centre of gravity
- for the rotational motion about the centre of gravity

Plotting kinetic reactions

Plot kinetic reactions against the positively defined directions of acceleration:

- inertia forces
- moments resulting from the inertia effect (where present)

Plotting the remaining forces

Plot all other impressed forces (external forces, weight forces) and reaction forces (friction forces and support reactions).

Generating a force equilibrium

During plane motion, generate a force equilibrium for all forces in the direction of motion, including the kinetic reactions:

Equation 7

$$\sum F_x^* = 0 \qquad \sum F_y^* = 0$$

Generating a moment equilibrium

Generate a moment equilibrium about the centre of gravity, provided the body performs a rotational motion about its centre of gravity in addition to the translational motion, taking account of the sign in each case:

Equation 8

$$\sum M_{z(S)} = 0$$

Stating geometric relationships

State the geometric relationships between the translational and rotational motions.

Solving equations

Solve the equation in accordance with the required acceleration:

- acceleration = const. or f(t):
  capable of elementary integration, taking into account the boundary conditions
- acceleration = f(dist.): can be integrated once

Equation 9

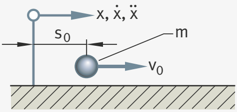
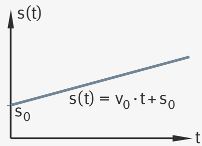
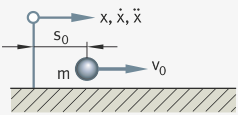
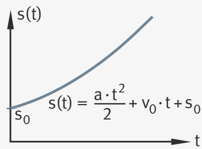
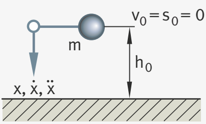
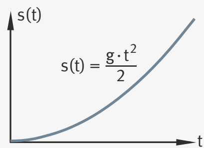
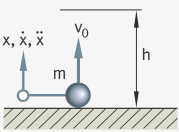
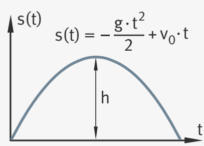
$$\ddot{x} = \frac{\dot{x} \cdot d\dot{x}}{dx} \qquad \ddot{\varphi} = \frac{\dot{\varphi} \cdot d\dot{\varphi}}{d\varphi}$$

- acceleration = f(dist.): an oscillation may be present.

Simple motion events

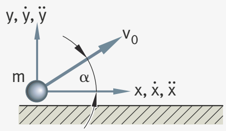
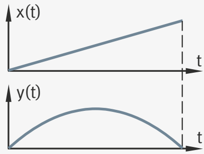
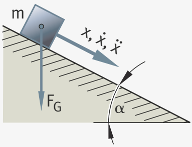
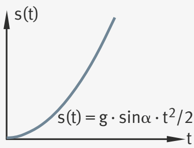
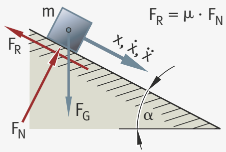
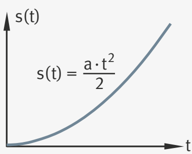
Motions, distance-time diagrams

The following table shows motion events, relationships and the corresponding distance-time diagrams.

Motion event	Relationship	Distance-time diagram
Uniform, rectilinear motion		
Acceleration $\ddot{x} = 0$ Initial conditions s_0, v_0 	$\ddot{x} = 0$ $\dot{x} = v_0 = \text{const.}$ $x = s(t) = v_0 \cdot t + s_0$	
Accelerated, rectilinear motion		
Acceleration $\ddot{x} = a$ Initial conditions s_0, v_0 	$\ddot{x} = a = \text{const.}$ $\dot{x} = v(t) = a \cdot t + v_0$ $x = s(t) = \frac{a \cdot t^2}{2} + v_0 \cdot t + s_0$	
Free fall under acceleration due to gravity		
Acceleration $\ddot{x} = g$ Initial condition, height h_0 	$\ddot{x} = g$ $\dot{x} = v(t) = g \cdot t$ $x = s(t) = \frac{g \cdot t^2}{2}$ $t(h_0) = \sqrt{2 \cdot h_0 / g}$ (time of fall) $v(h_0) = \sqrt{2 \cdot g \cdot h_0}$ (velocity)	
Body thrown vertically upward		
Acceleration $\ddot{x} = -g$ Initial condition v_0, $s_0 = 0$ 	$\ddot{x} = -g$ $\dot{x} = v(t) = -g \cdot t + v_0$ $x = s(t) = -\frac{g \cdot t^2}{2} + v_0 \cdot t$ $h = v_0^2 / (2 \cdot g)$ (height reached) $t(h_0) = v_0 / g$ (time of upward travel)	

Continuation of table, see Page 127.

Continuation of table, Motions, distance-time diagrams, from Page 126.

Motion event	Relationship	Distance-time diagram
Body thrown obliquely at angle α		
	$\ddot{x} = 0$ $\dot{x} = v_0 \cdot \cos\alpha$ $x = v_0 \cdot \cos\alpha \cdot t$ $\ddot{y} = -g$ $\dot{y} = -g \cdot t + v_0 \cdot \sin\alpha$ $y = -g \cdot t^2/2 + v_0 \cdot \sin\alpha \cdot t$	
Motion on an inclined plane, without friction (μ = 0)		
	$\ddot{x} = g \cdot \sin\alpha = a$ $\dot{x} = v(t) = g \cdot \sin\alpha \cdot t$ $x = s(t) = g \cdot \sin\alpha \cdot t^2/2$ $t = 0$ $s_0 = 0$ $v_0 = 0$	
Motion on an inclined plane, with friction, friction coefficient μ		
	$\ddot{x} = g \cdot (\sin\alpha - \mu \cdot \cos\alpha) = a$ $\dot{x} = v(t) = a \cdot t$ $x = s(t) = a \cdot t^2/2$ $t = 0$ $s_0 = 0$ $v_0 = 0$	

Forces in kinetics (selection)

Forces in kinetics The following tables describe a selection of the most important forces in kinetics.

General forces The following general and impressed forces are described in kinetics:

Force	Value	Explanation
Force (general)	F	Muscle power, wind power, driving forces of machines etc.
Impressed forces	Impressed forces are primary forces that act on a body from the outside. They can promote or obstruct motion. As a rule, their magnitude, direction and line of action are known or given.	

Conservative forces In the case of conservative forces, the work (work = force · distance) is independent of the distance curve.

Force	Value	Explanation
Mass attraction force	$F_M = G \cdot \frac{m_1 \cdot m_2}{r^2}$	The mass attraction force between two masses is proportional to the product of the masses and inversely proportional to the square of the distance between their centres of gravity. The proportionality factor is the universal gravitation constant, $G = 6{,}674\,2 \cdot 10^{-11}\ m^3/(kg \cdot s^2)$.
Gravity, weight	$F_G = m \cdot g$	The force of gravity (weight) acts on all bodies in the Earth's proximity. It is based on the mass attraction law and on the centrifugal force (additional force) occurring on the Earth's surface as a result of the Earth's rotation.
Spring force for linear spring rigidities (Hooke's Law)	$F_F = c \cdot w$	The spring force results from the product of the spring constant c and spring deflection w. It acts against the positive deflection of the spring.

Dissipative forces

In the case of dissipative (non-conservative) forces, the work (work = force · distance) is dependent on the distance curve.

Force		Value	Explanation
Sliding friction force		$F_R = \mu \cdot F_N$	The sliding friction force between two contact surfaces (Coulomb friction) is proportional to the acting normal force. The proportionality factor is the sliding friction coefficient μ. The sliding friction force acts against the relative velocity of the surfaces in contact.
Damping force	according to Stokes	$F_D = b \cdot v$	The resistance according to Stokes is valid for low velocities and is proportional to the velocity (damping constant $b = F \cdot t/s$). The resistance according to Newton is valid for high velocities and is proportional to the square of the velocity (constant factor k). The damping force counteracts the positive velocity direction.
	according to Newton	$F_N = k \cdot v^2$	

Constraining or guiding forces

Constraining or guiding forces are caused by the restriction of the freedom of movement of a body or a system of bodies.
The influence of friction-free guides or guiding curves on the body is taken into account by external guiding forces that are perpendicular to the guiding curves.

Force		Value	Explanation
Centripetal force	Curved path	$F_{CP} = m \cdot \frac{v^2}{r}$	When a body moves along a curved path with radius r at velocity v, it is subject to acceleration, known as centripetal acceleration, which is oriented to the centre point of the curvature. The force generating it is referred to as the centripetal force.
	Circular path	$F_{CP} = m \cdot r \cdot \omega^2$	

Kinetic reactions

Kinetic reaction forces represent the action of a moving body as the result of accelerating or decelerating external forces. They always counteract the positive accelerations.
If the kinetic reaction of the body is also interpreted as an external force acting on the body (fictitious force), the kinetic problem can be reduced to a static one and can be handled with the aid of the equilibrium conditions (d'Alembert's principle).

Force		Value	Explanation
Tangential to the trajectory curve		$F_{Kt} = -m \cdot \ddot{s}_t$	The kinetic reaction of a body during accelerated motion counteracts the positive direction of the acceleration that is produced by impressed forces and guiding forces. In the case of guided motion on a trajectory curve, it is advisable to split this into components that are tangential and perpendicular to the trajectory curve. Centrifugal force is one of the kinetic reactions that act perpendicular to the trajectory curve.
Perpendicular to the trajectory curve		$F_{Kn} = -m \cdot \ddot{s}_n$	
Centrifugal force	Curved path	$F_{Kn} = -m \cdot \frac{v^2}{r}$	
	Circular path	$F_{Kn} = -m \cdot r \cdot \omega^2$	

Additional forces in the accelerated reference system

If motion events take place in an accelerated reference system, it is advisable to analyse these as relative motions in relation to the system from the point of view of an observer who is also accelerated.

In the accelerated reference system, all bodies are subject to additional inertia forces (apparent forces) which, from the point of view of the observer who is also accelerated, can be interpreted as external forces acting on the body.

Force		Value	Explanation
Inertia force in the translatory accelerated reference system		$F_{Sys} = -m \cdot a_{Sys}$	In the translatory accelerated reference system (system acceleration a_{Sys}), all bodies are subjected to an inertia force. It acts against the positive direction of system acceleration.
Inertia force in the rotating reference system	Centrifugal force	$F_Z = m \cdot r \cdot \Omega^2$	In the rotating reference system (angular velocity Ω), all bodies are subjected to the centrifugal force. To the observer who is also rotating with the body, this appears like a field force corresponding to gravity (weight) that points away from the rotation centre, and which is intrinsic to the system. For the observer, it has the character of a conservative impressed force, which lends a centrifugal acceleration to all free bodies in the same direction.
	Coriolis force	$\vec{F}_C = -2 \cdot m \cdot \left(\vec{\Omega} \times \vec{v}\right)$	The Coriolis force occurs in relative motions in the rotating reference system. It is perpendicular to the plane spanned by the vectors v and Ω and points in a right-handed screw direction if the vector v is moved over the shortest distance in the direction of Ω. Vectors v and Ω are calculated using the cross product. During guided motion of a body in the rotating reference system, it is subjected to Coriolis acceleration in the direction of the Coriolis force.

Law of conservation of energy

Definition of the energy conservation law

The energy conservation law in mechanics states:

in a self-contained mechanical system, the sum of energies remains constant:

Equation 10

$$E_p + E_k + Q = \text{const.}$$

The energies are composed of potential energy E_p, kinetic energy E_k and heat quantity Q. A mechanical system is referred to as self-contained if no forces are acting on the system from the outside or, if forces are acting on the system from outside, they are negligible.

Frictional energy in the mechanical system

In a mechanical system, only an increase in heat quantity can occur, which is caused by friction losses in the system.

Thus, for the consideration of two points in time 1 and 2 of a motion sequence, the following applies:

Equation 11

$$E_{p1} + E_{k1} + Q_1 = E_{p2} + E_{k2} + Q_2$$

For friction losses occurring along the way between 1 and 2, the following applies:

Equation 12

$$Q_2 - Q_1 = W_{R1,\,2}$$

This gives the following for the energy conservation law:

Equation 13

$$E_{p1} + E_{k1} = E_{p2} + E_{k2} + W_{R1,\,2}$$

Conservative systems

For a **conservative system** in which **no friction losses** occur, the energy conservation law is:

Equation 14

$$E_{p1} + E_{k1} = E_{p2} + E_{k2}$$

The energy conservation law can also be formulated as:

Equation 15

$$E_{k2} - E_{k1} = W_{1,\,2} - W_{R1,\,2}$$

In other words, the change in the kinetic energy between points 1 and 2 of a motion sequence is equal to the work that is done by the impressed forces along the way from 1 to 2, minus the friction losses occurring along the way from 1 to 2.

Forms of energy in kinetics (selection)

Forms of energy in kinetics

The following table describes a selection of the most important forms of energy in kinetics.

Energy	Value	Explanation
Kinetic energies		
Translational energy of the mass centre of gravity	$E_k = \frac{m}{2} \cdot v_S^2$	The total kinetic energy of a moving mass is composed of the translational energy relative to the centre of gravity velocity and the rotational energy about the centre of gravity.
Rotational energy about the mass centre of gravity	$E_k = \frac{J_S}{2} \cdot \omega^2$	
Rotational energy about the instantaneous centre of revolution or rotation	$E_k = \frac{1}{2} \cdot J_{DP} \cdot \omega^2$ $E_k = \frac{1}{2} \cdot J_{MP} \cdot \omega^2$	In the case of a guided rotational motion of the mass, or whenever the motion's instantaneous centre of rotation can be specified, the total kinetic energy can be specified by the rotational energy about the instantaneous centre of revolution or rotation alone.

Continuation of table, see Page 132.

Continuation of table, Forms of energy in kinetics, from Page 131.

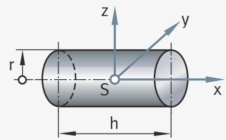
Energy	Value	Explanation
Potential energies		
Energy of the position in the Earth's gravitational field	$E_p = m \cdot g \cdot h$	The energy of the position of a mass in the constant gravitational field (close to the Earth) is the product of the weight and of the altitude of the centre of gravity h above a chosen reference level.
Work in the variable potential field	$W_{1,\,2} = \int_1^2 m \cdot g(h) \cdot dh$	Work in the Earth's gravitational field at greater altitudes: $g(h) = g_0 \cdot R_0{}^2/(R_0 + h)^2$
Elastic deformation energy of a spring:		
Translation spring	$E_p = \frac{1}{2} \cdot c \cdot w^2$	The work done on deformation of a spring is stored in the form of elastic energy in the spring. The energy depends on the spring constant ($c = F/s$ or $c' = F \cdot s$) and the deflection w or torsion φ of the spring.
Torsion spring	$E_p = \frac{1}{2} \cdot c' \cdot \varphi^2$	
Elastic energy of supports on deformation as the result of:		
Normal forces	$E_p = \frac{1}{2} \int \frac{F_N^2}{E \cdot A} dx$	If a support (cross-sectional area A, axial area moment of inertia I_a, polar area moment of inertia I_p) or bar is elastically deformed by external forces, displacements are generated by the internal stresses. The work done along the displacements which are generated by these is equal to the elastic energy stored in the bar or support. Generally, this elastic deformation energy can be described by the spring constant and the deflection or torsion at the point of the deformation (see section on springs).
Bending moments	$E_p = \frac{1}{2} \int \frac{M_b^2}{E \cdot I_a} dx$	
Torsion moments	$E_p = \frac{1}{2} \int \frac{M_t^2}{G \cdot I_p} dx$	
Energy of the position in the centrifugal field	$E_p = \frac{1}{2} \cdot m \cdot \Omega^2 \cdot r^2$	In the rotating system, a force field (centrifugal field) exists for the accelerated observer, the strength of which increases from the rotation centre towards the outside. The energy of the position is relative to the rotation axis.
Energy losses		
Due to sliding friction forces	$W_{R1,\,2} = F_N \cdot \mu \cdot s_{1,\,2}$	If sliding friction forces (resistance forces) are present, these do friction work along the acting distance $s_{1,\,2}$ which manifests itself as heat and is lost for the mechanical motion.
In the case of an incomplete elastic impact	$W_{R1,\,2} = \frac{1}{2}\left(1-e^2\right) \cdot \left(v_1 - v_2\right)^2 \cdot \frac{m_1 \cdot m_2}{m_1 + m_2}$	In the case of an incomplete elastic impact of bodies, energy losses occur as the result of internal material friction. e = distance from centre of gravity – fixing points

Mass moments of inertia of homogeneous solids

Second mass moments of inertia, homogeneous solids

The second mass moments of inertia of homogeneous solids can be calculated as follows:

Solid cylinder

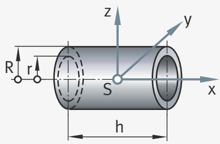


$$m = \rho \pi r^2 h$$

$$J_x = \frac{1}{2} m r^2$$

$$J_y = J_z = \frac{1}{12} m \left(3 r^2 + h^2\right)$$

Hollow cylinder

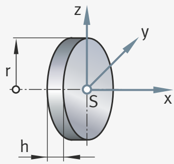


$$m = \rho \pi \left(R^2 - r^2\right) h$$

$$J_x = \frac{1}{2} m \left(R^2 + r^2\right)$$

$$J_y = J_z = \frac{1}{4} m \left(R^2 + r^2 + \frac{h^2}{3}\right)$$

Thin disk

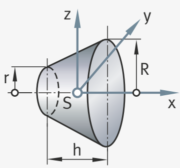


$$m = \rho \pi r^2 h; \; h \ll r$$

$$J_x = \frac{1}{2} m r^2$$

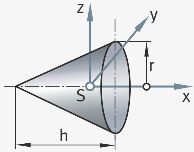
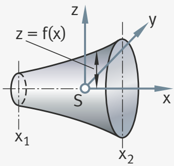
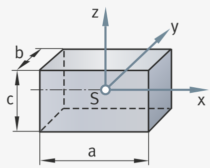
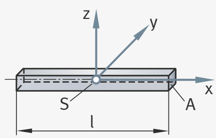
$$J_y = J_z = \frac{1}{4} m r^2$$

Truncated circular cone

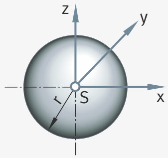


$$m = \frac{1}{3} \rho \pi h \left(R^2 + R r + r^2\right)$$

$$J_x = \frac{3}{10} m \frac{R^5 - r^5}{R^3 - r^3}$$

Circular cone		$m = \frac{1}{3} \rho \pi r^2 h$ $J_x = \frac{3}{10} m r^2$ $J_y = J_z = \frac{3}{80} m \left(4 r^2 + h^2\right)$
Any body of revolution		$m = \rho \pi \int_{x_1}^{x_2} f^2(x)\, dx$ $J_x = \frac{1}{2} \rho \pi \int_{x_1}^{x_2} f^4(x)\, dx$
Cuboid		$m = \rho a b c$ $J_x = \frac{1}{12} m \left(b^2 + c^2\right)$ $J_y = \frac{1}{12} m \left(a^2 + c^2\right)$ $J_z = \frac{1}{12} m \left(a^2 + b^2\right)$
Thin bar		$m = \rho A l$ $J_y = J_z = \frac{1}{12} m l^2$

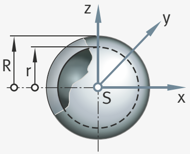
Sphere



$$m = \frac{4}{3}\rho\,\pi\,r^3$$

$$J_x = J_y = J_z = \frac{2}{5}\,m\,r^2$$

Hollow sphere



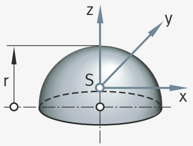
$$m = \frac{4}{3}\rho\,\pi\left(R^3 - r^3\right)$$

$$J_x = J_y = J_z = \frac{2}{5}\,m\,\frac{R^5 - r^5}{R^3 - r^3}$$

for thin-walled hollow spheres:

$$J_x = J_y = J_z = \frac{2}{3}\,m\,r^2$$

Hemisphere

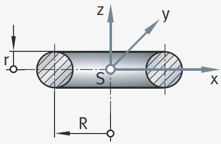


$$m = \frac{2}{3}\rho\,\pi\,r^3$$

$$J_x = J_y = \frac{83}{320}\,m\,r^2$$

$$J_z = \frac{2}{5}\,m\,r^2$$

Circular torus



$$m = 2\rho\,\pi^2\,r^2\,R$$

$$J_x = J_y = \frac{1}{8}\,m\left(4R^2 + 5r^2\right)$$

$$J_z = \frac{1}{4}\,m\left(4R^2 + 3r^2\right)$$

Principle of linear momentum

Definition of the principle of linear momentum

Equation 16

The fundamental dynamic law for motion of a constant mass m under the influence of an external, resulting, constant force $\vec{F}$ is:

$$m \cdot \vec{a} = m \cdot \frac{d\vec{v}}{dt} = \frac{d(m \cdot \vec{v})}{dt} = \frac{d\vec{p}}{dt} \qquad m = \text{const.}$$

Integration over the duration of the effect gives the principle of linear momentum:

Equation 17

$$\int_{t_0}^{t_1} \vec{F} \cdot dt = \vec{F} \cdot (t_1 - t_0) = m \cdot (\vec{v}_1 - \vec{v}_0) = \vec{p}_1 - \vec{p}_0$$

The value $\vec{p} = m \cdot \vec{v}$ is referred to as the linear momentum or momentum of mass m.

Further statements about the principle of linear momentum

The following statements can be made on the basis of the principle of linear momentum:

- The defined time integral over the external resulting force $\vec{F}$ acting on a mass m is equal to the change in the absolute linear momentum of the mass ($\vec{p} = m \cdot \vec{v}$) in the direction of this force.
- If no external resulting force $\vec{F}$ acts on the mass m, its linear momentum $\vec{p}$ remains constant in terms of its magnitude and direction since: $\vec{F} = 0$ and thus $d\vec{p}/dt = 0$, giving $\vec{p} = \text{const.}$
  This statement can be extended to a system of several, individual masses:
  If no external resulting force acts on a system of masses, the system's overall linear momentum remains constant in terms of its magnitude and direction. In other words, the overall centre of gravity of the system either remains at rest or moves uniformly and in a straight line.

Principle of angular momentum

Definition of the principle of angular momentum

On a similar basis to the principle of linear momentum, it follows from the fundamental dynamic law that the motion of a rotating mass with the constant moment of inertia J_0 about its pivotal point "0", under the influence of an external, resulting moment, is:

Equation 18

$$\vec{M}_0 = J_0 \cdot \dot{\vec{\omega}} = J_0 \cdot \frac{d\vec{\omega}}{dt} = \frac{d(J_0 \cdot \vec{\omega})}{dt} = \frac{d\vec{L}}{dt} \qquad J_0 = \text{const.}$$

The following applies:

Equation 19

$$\vec{M}_0 = \vec{r} \times \vec{F}$$

In this instance $\vec{r}$ is the vector from the reference point (pivotal point) "0" to the point of application of the external, resulting force $\vec{F}$.

Integration over the duration of the effect gives the principle of angular momentum:

Equation 20

$$\int_{t_0}^{t_1} \vec{M}_0 \cdot dt = \vec{M}_0 \cdot (t_1 - t_0) = J_0 \cdot (\vec{\omega}_1 - \vec{\omega}_0) = \vec{L}_1 - \vec{L}_0$$

The value $\vec{L} = J_0 \cdot \vec{\omega}$ is referred to as the **angular momentum**, **moment of linear momentum** or **moment of momentum** of the rotating mass J_0 about the reference point "0".

Further statements about the principle of angular momentum

The following statements can be made on the basis of the principle of angular momentum (on a similar basis to the principle of linear momentum):

- The time integral over the resulting moment $\vec{M}_0$ of the external forces $\vec{F}$ acting on a rotating mass J_0 in relation to a reference point "0" is equal to the change in the angular momentum $J_0 \cdot \vec{\omega}$ (moment of momentum $\vec{L}$) of the rotating mass in the direction of the acting moment.
  - If the resulting moment vector points in the direction of the rotation vector $\vec{\omega}$ of the rotating mass, it changes the magnitude of the angular momentum (moment of momentum), i.e. a pure **angular momentum magnitude change**.
  - If the resulting moment vector is perpendicular to the direction of the rotation vector $\vec{\omega}$ of the rotating mass, this results in a change to the direction of the angular momentum (moment of momentum), i.e. a pure **angular momentum direction change**.
    In this case we use the statement of the angular momentum principle in the differential form:

Equation 21

$$\vec{M}_a = \frac{d\vec{L}}{dt}$$

  - The change over time of the moment of momentum is equal to the moment of the external forces in relation to any reference point "0".
- If no external resulting moment $\vec{M}_0$ acts on the rotating mass J_0, then its angular momentum (moment of momentum) $\vec{L}$ remains constant in terms of its magnitude and direction in space since: $\vec{M}_a = 0$ and thus $d\vec{L} = 0$, giving $\vec{L} = \text{const}$.
  This statement can be extended to a system of several, individual masses.

Calculations involving the principles of linear momentum and angular momentum

The following analyses are helpful when studying the motion of bodies under the influence of external forces and moments:

- For the purpose of the calculation it is advisable to
  - choose the centre of gravity as the reference point for the angular momenta and to additionally specify the change in linear momentum of the centre of gravity.
  - select the instantaneous centre of revolution or rotation as the reference point for the motion. In the case of guided motion, it is sufficient to specify the body's change in angular momentum about the instantaneous centre of revolution.
- Both the linear and angular momenta are directional quantities. When studying the motion of bodies, the positive direction of motion (coordinates) must be defined. Their mathematical signs and the signs of the acting, resulting, external forces and moments must be observed.
- The principle of linear momentum and of angular momentum result from the fundamental equation of motion (fundamental dynamic law) through integration over time. Thus, all motion events which can be described using the fundamental equation of motion, can also be studied with the aid of the principles of linear momentum and of angular momentum.
- Linear momentum and angular momentum principles are observed in relation to so-called impact phenomena in particular. Impact phenomena cannot be described by the simple fundamental equation of motion, as generally no statement can be made about the magnitude of the acting impact force and the duration of the impact.

Impact laws – central impulse

Definition of impact and impulse

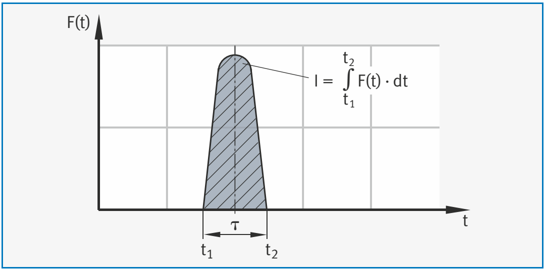
We speak of an impact when a large force F(t) is exerted on a body with the mass m during a very short period of time τ (impact duration) and when this takes place in such a way that the time integral over the acting force assumes a finite value.

The time integral over the force F(t) is referred to as the magnitude of the impulse I:

Equation 22

$I = \int_{t_1}^{t_2} F(t) \cdot dt = \Delta p$	with impact duration	$\tau = t_2 - t_1$

Figure 1
Magnitude of the impulse



Generally, the time curve for the acting force F(t) cannot be determined separately from the impact duration τ. The impulse I can be described by its effect, which **brings about a finite change in the velocity** of the body. In other words, the linear momentum of the body changes in the direction of the acting impact force.

In accordance with the fundamental dynamic law, the following applies:

Equation 23

$$F(t) = \frac{d(m \cdot v)}{dt}$$

This gives the following for the time integral over the acting impact force:

Equation 24

$$I = \int_{t_1}^{t_2} F(t) \cdot dt = m \cdot v_2 - m \cdot v_1 = m \cdot \Delta v$$

where Δv = change in velocity of the body's centre of gravity as a result of the impact.

Figure 2
Impulse

① Before impact (t_1)
② After impact (t_2)

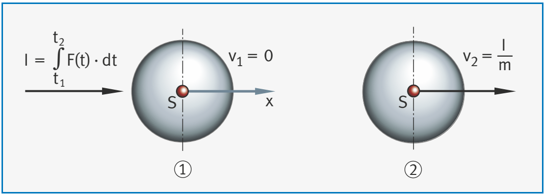
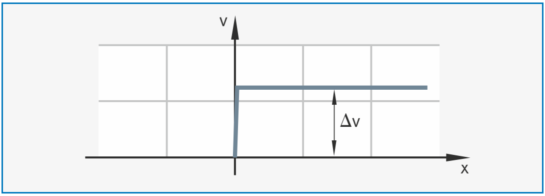


Figure 3
Impulse velocity profile



An impulse causes a “sudden” change in the velocity of the body, which corresponds to a change in linear momentum (see Principle of linear momentum). This change in velocity takes place in such a short space of time that the distance covered by the body during this time is negligible or practically zero.

The influence of other external forces (weights and friction forces etc.) is also negligible during the course of the impact compared with the magnitude of the impact force.

If the impulse is applied “normally” to the body and in the direction of the centre of gravity S and no other external forces are applied (including reaction forces) in the direction of impact, this results in a **sudden change in the translational velocity** in the direction of impact.

Impact laws – angular impulse

Unguided motion

If the impulse is **not** applied in the direction of the centre of gravity S of the body and the line of influence of the impulse is at a distance a from the centre of gravity, the impact not only causes a sudden change in the translational velocity, but also a sudden change in the angular velocity ω.

Angular impulse

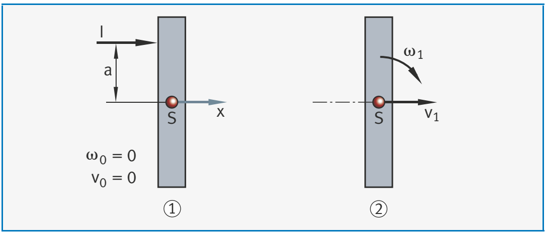
Consequently, there is also a change in the angular momentum (moment of momentum) of the body about the centre of gravity S. This impulse is known as **angular impulse**:

Equation 25

$$H_S = a \cdot I$$

Figure 4
Unguided motions

① Before impact
② After impact



Angular impulse relative to centre of gravity S

If we apply the equation for impulse:

Equation 26

$$I = \int_{t_0}^{t_1} F(t) \cdot dt = m \cdot (v_1 - v_0) = m \cdot v_1$$

this gives the following for the angular impulse relative to the centre of gravity S:

Equation 27

$$H_S = a \cdot \int_{t_0}^{t_1} F(t) \cdot dt = a \cdot I = J_S \cdot (\omega_1 - \omega_0) = J_S \cdot \omega_1$$

Angular impulse relative to instantaneous centre of rotation MP

If the change in angular momentum about the instantaneous centre of rotation MP is described as the angular impulse $H_{MP} = I \cdot b$ (b = distance between the line of action of the impulse I and the instantaneous centre of rotation), this gives:

Equation 28

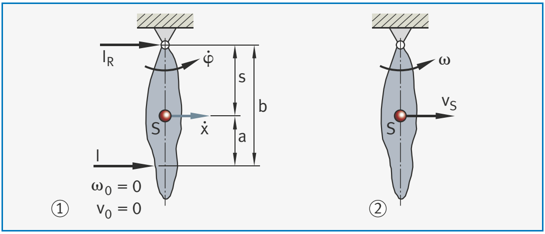
$$H_{MP} = b \cdot \int_{t_0}^{t_1} F(t) \cdot dt = b \cdot I = J_{MP} \cdot (\omega_1 - \omega_0) = J_{MP} \cdot \omega_1$$

Guided motion

In the case of a guided motion, a relationship between the velocity of the centre of gravity and the angular velocity of the body can be described:

Figure 5
Guided motions

① Before impact
② After impact



If several impulses (including reaction impulses) are applied to a body, then the following statements apply:

- The sum of all impulses is equal to the change in the linear momentum.
- The sum of all angular impulses about the centre of gravity (total moment of the impacts) is equal to the change in the angular momentum about the centre of gravity.
- The total moment of the impacts about the instantaneous centre of rotation (of revolution) is equal to the change in the angular momentum (moment of momentum) about the instantaneous centre of rotation.

For the centre of gravity S for example, this gives the following:

Equation 29

$$I + I_R = m \cdot v_S$$

$$I \cdot a - I_R \cdot (b - a) = J_S \cdot \omega \qquad v_S = (b - a) \cdot \omega = s \cdot \omega$$

and for the instantaneous centre of rotation MP:

Equation 30

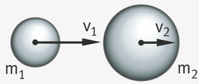
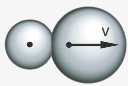
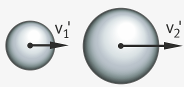
$$I \cdot b = J_{MP} \cdot \omega \qquad v_S = s \cdot \omega$$

Once the positive velocity directions have been defined, the signs of the impulses and angular impulses must be taken into account.

Impact laws for solid bodies

Direct, central impact

In the case of a direct, central impact, the impact normal and the velocity vectors lie on the line connecting the centres of gravity of the two bodies.

Impact period	Principle of linear momentum I =	Energy equation E =
positive direction →		
I. Before impact $v_1 > v_2$ 	$m_1 v_1 + m_2 v_2$	$\frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2$
II. Moment of maximum deformation of the bodies 	$(m_1 + m_2)\, v$	$\frac{(m_1 + m_2)}{2} v^2 + E_{p\,max} + W_{R1,\,2}$
III. After impact $v_1' < v_2'$ 	$m_1 v_1' + m_2 v_2'$	$\frac{m_1}{2} (v_1')^2 + \frac{m_2}{2} (v_2')^2 + W_{R1,\,2}$

Newton's impact hypothesis
Equation 31

In this context, the impact hypothesis is:

$$e\left(v_1 - v_2\right) = v_2' - v_1'$$

Description of the impact periods

Impact period I → II:

- The two bodies are deformed until their centres of gravity are at a minimum distance from one another. At this moment, the velocities of their centres of gravity are equal.

Impact period II → III:

- If the bodies are perfectly elastic ($e = 1$), the energy stored in the bodies in the form of elastic deformation energy is released again and converted into kinetic energy.
- If the bodies are perfectly inelastic ($e = 0$, perfectly plastic impact), the deformation is not reversed.
  It occurs in the bodies as heat and is lost for the remainder of the motion sequence. As a result, impact period II → III does not occur.
- The imperfectly elastic impact ($0 < e < 1$) lies between the two limiting cases.

Calculating the velocities

The velocities of the bodies after impact and the occurring losses can be determined using the principle of conservation of linear momentum[1)], the energy conservation law[1)] and Newton's impact hypothesis[1)].

Velocities of the bodies after impact (III)	Potential elastic energy at the moment of maximum deformation (II)	Energy loss during impact I → II and II → III
Perfectly elastic impact e = 1	$v_2' - v_1' = v_1 - v_2$	
$v_1' = \dfrac{(m_1 - m_2)\, v_1 + 2 m_2 v_2}{m_1 + m_2}$ $v_2' = v_1' = \dfrac{(m_2 - m_1)\, v_2 + 2 m_1 v_1}{m_1 + m_2}$	$E_{p\ max} = \dfrac{1}{2}(v_1 - v_2)^2 \cdot \dfrac{m_1 m_2}{m_1 + m_2}$	$W_{R1,\,2} = 0$
Imperfectly elastic impact 0 < e < 1	$v_2' - v_1' = e \cdot (v_1 - v_2)$	
$v_1' = \dfrac{m_1 v_1 + m_2 v_2 - m_2 (v_1 - v_2)\, e}{m_1 + m_2}$ $v_2' = \dfrac{m_1 v_1 + m_2 v_2 + m_1 (v_1 - v_2)\, e}{m_1 + m_2}$	$E_{p\ max} = \dfrac{1}{2} e^2 (v_1 - v_2)^2 \cdot \dfrac{m_1 m_2}{m_1 + m_2}$	$W_{R1,\,2} = \dfrac{1}{2}\left(1 - e^2\right)(v_1 - v_2)^2 \cdot \dfrac{m_1 m_2}{m_1 + m_2}$
Perfectly plastic impact e = 0	$v_1' = v_2' = v$	
$v_1' = v_2' = v = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$	$E_{p\ max} = 0$	$W_{R1,\,2} = \dfrac{1}{2}(v_1 - v_2)^2 \dfrac{m_1 m_2}{m_1 + m_2}$

1) The signs of v and v' must be observed when evaluating the relationships.

Oblique, central impact

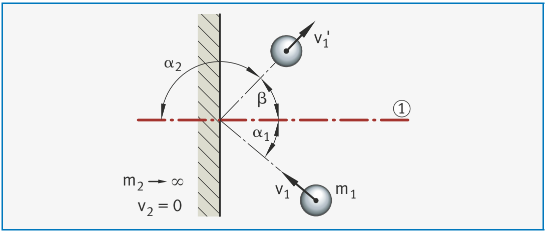
In the case of an oblique, central impact, the impact normal lies on the line connecting the centres of gravity of the two bodies, but the velocity vectors do not.

If no frictional forces occur in the contact area between the two bodies, there will be no change in the bodies' velocity component which is tangential to the contact area. In this instance, the relationships for the direct, central impact can be used by determining the components of the velocities in the direction of the impact normal.

Example system: **oblique central impact of a sphere against a wall**

Figure 6
Oblique, central impact

① Impact normal



The following applies perpendicular to the impact normal:

Equation 32

$$v_1 \cdot \sin\alpha_1 = v_1' \cdot \sin\alpha_2 = v_1' \cdot \sin\beta$$

The following applies in the direction of the impact normal:

Equation 33

$$v_1' \cdot \cos\alpha_2 = -e \cdot v_1 \cdot \cos\alpha_1 = e \cdot v_1 \cdot \cos\beta$$

Therefore:

Equation 34

$$\tan\alpha_2 = -\frac{\tan\alpha_1}{e} = -\tan\beta \qquad v_1' = -e\,\frac{v_1 \cdot \cos\alpha_1}{\cos\alpha_2}$$

Direct, eccentric impact

In the case of a direct, eccentric impact, the velocity vectors lie in the direction of the impact normal, but the impact normal does not lie on the line connecting the two bodies' centres of gravity.

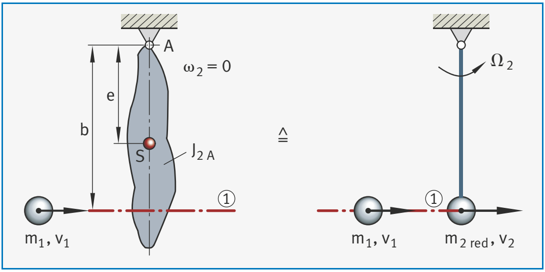
Guided motion

If one of the two bodies (or both) is pivot-mounted, the relationships for a direct, central impact can be used by reducing the rotating mass of the guided bodies to the point of impact.

Example system: **guided motion**

Figure 7
Direct, eccentric impact
Before impact

① Impact normal



Note:

Equation 35

$$m_{2\,red} = \frac{J_{2\,A}}{b^2}$$

With impact factor e, this gives the following:

Equation 36

$$v_1' = \frac{m_1\,v_1 - m_{2\,red} \cdot v_1 \cdot e}{m_1 + m_{2\,red}} \qquad v_2' = \frac{m_1\,v_1 - m_1\,v_1 \cdot e}{m_1 + m_{2\,red}}$$

$$\Omega_2 = \frac{v_2'}{b}$$

Unguided motion

If neither of the two bodies is guided, the equations required to determine the velocities of the bodies (translational velocity and angular velocity) after impact are obtained from the impact laws (for the impulse or angular impulse) and Newton's impact hypothesis for the point of impact.

Example system: **unguided motion**

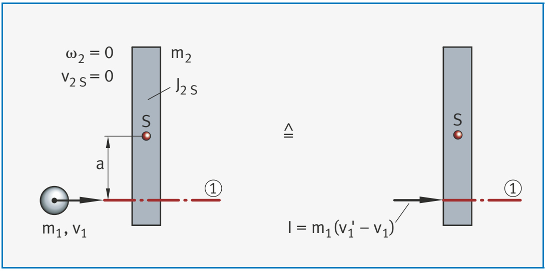


Figure 8
Positive direction before impact

1 Impact normal

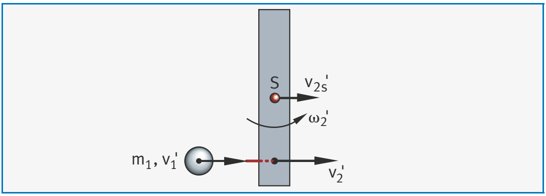


Figure 9
After impact

If we consider the effect of the impulse I occurring between the two bodies, for each body separately, the use of the following equation:

Equation 37

$$I = \int F(t) \cdot dt$$

gives the impulse of body 2 (mass m_2) on body 1 (mass m_1):

Equation 38

$$\int F(t) \cdot dt = m_1 \left(v_1' - v_1\right)$$

For reaction reasons, an impulse of the same magnitude acts on body 2 (mass m_2):

Equation 39

$$\int F(t) \cdot dt = m_2 \left(v_{2\,S}' - v_{2\,S}\right) \qquad v_{2\,S} = 0$$

An angular impulse also acts about the centre of gravity of body 2 (mass m_2):

Equation 40

$$a \cdot \int F(t) \cdot dt = J_{2S} \left(\omega_2' - \omega_2\right) \qquad \omega_2 = 0$$

With Newton's impact hypothesis, this ultimately gives the following for the impact point:

Equation 41

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{\left(v_{2\,S}' + \omega_2' \cdot a\right) - v_1'}{v_1} \qquad v_2 = 0$$

These equations can be used to determine the velocities v_1' and $v_{2\,S}'$ and ω_2' after impact.

Oblique, eccentric impact

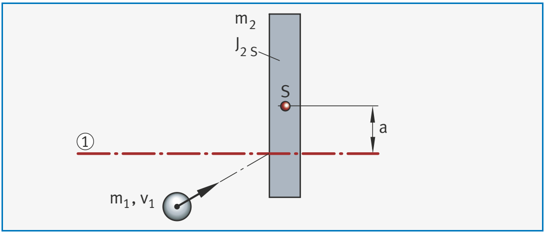
In the case of an oblique, eccentric impact, neither the impact normal nor the line connecting the centres of gravity of the two bodies and the velocity vectors coincide.

If no frictional forces occur in the contact area between the two bodies, there will be no change in the bodies' velocity component which is perpendicular to the impact normal. In this instance, the relationships for the direct, eccentric impact can be used by determining the components of the velocities in the direction of the impact normal.

Example system: **oblique, eccentric impact**

Figure 10
Oblique, eccentric impact

① Impact normal



Mechanical vibrations and acoustics

General definitions

Description of vibration events

In a vibration event, the energy present is transformed from one form of energy to another in defined time periods and periodically transformed, in whole or in part, back into the first form of energy.

Within a mechanical translational or rotational vibration system comprising a mass and a potential energy reservoir, a transformation process occurs between the kinetic and potential energy present.

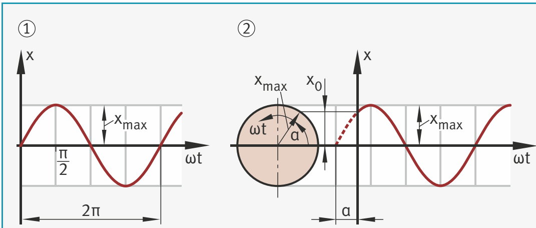
Free, undamped vibration

A free, undamped oscillation is present if energy is neither supplied to nor withdrawn from the oscillating system during the oscillation. As a result, the amount of energy initially introduced remains constant and a periodic energy transformation takes place. In this case, the system would perform stationary natural oscillations, whose frequency depends solely on the characteristics of the system (mass and potential energy storage). Oscillations of this type are also known as **harmonic oscillations**. The vibration pattern x(t) over time can therefore be characterised by a constant **vibration amplitude** (x_{max}) and a harmonic mathematical function (sin, cos), whose argument contains the **natural frequency** of the system.

Figure 1
Free, undamped oscillation

x_{max} = amplitude
2π = period

① Without phase shift
② With phase shift α



$x(t) = x_{max} \cdot \sin(\omega_0 t)$

$x(t) = x_{max} \cdot \sin(\omega_0 t + \alpha)$

Damped vibration

If the oscillating system loses a portion of its dynamic energy in each oscillation period, a damped vibration is present. In the case of linear damping proportional to velocity (e.g. Newtonian friction), the vibration amplitude will decrease according to a geometric progression.

Forced vibration If the oscillating system is excited by an external, periodically acting force F(t) or moment M(t), forced vibrations will occur. Due to the excitation force, energy can be supplied to or withdrawn from the system.
After an initial transient phase, the system no longer oscillates at its natural frequency, but with the frequency of the externally acting excitation force.

Resonance If the frequency of the external excitation acting on the system matches the natural frequency, resonance occurs. In the case of undamped systems, the vibration amplitudes in resonance tend towards "infinity". The vibration amplitudes, when considered as a function of the excitation frequency, are subdivided at the resonance point (natural frequency = excitation frequency) into **subcritical** and **supercritical** frequency ranges.

Coupled vibrations If two oscillating systems are coupled with each other by means of mass or elasticity, a periodic exchange of energy takes place between the systems (multiple mass oscillator).

Mathematical description of vibrations In general, mechanical vibration events can be described, depending on their initial conditions, by means of sine or cosine functions or their superposition. In the analysis of vibration events, Fourier analysis can be useful, as any function that is monotonic and continuous can be represented as a sum of sinusoidal and cosinusoidal basic and harmonic oscillations.

The following relationships exist between the natural circular frequency, the frequency and the period time of a harmonic vibration:

Equation 1

$$\omega_0 = 2 \cdot \pi \cdot f_0 = \frac{2 \cdot \pi}{T}$$

ω_0 = natural circular frequency (vibration in 2 π seconds)

f_0 = frequency (number of vibrations per second)

$$T = \frac{2 \cdot \pi}{\omega_0} = \frac{1}{f_0}$$

T = vibration time for one period

In circumferential motion, the relationship between frequency and speed is as follows:

Equation 2

$$f = \frac{n}{60}$$ in Hz (n in min^{-1})

$$\omega = \frac{2 \cdot \pi \cdot n}{60} = \frac{\pi \cdot n}{30}$$ in s^{-1} (n in min^{-1})

Quantities and units A number of important quantities and units relating to mechanical vibrations are described as follows:

Name	Quantity	Unit	Description
Mass (rigid body)	m	kg	A (rigid) mass undergoing translational acceleration, velocity or displacement through vibration
Mass moment of inertia	J	$kg \cdot m^2$	Inertia of a rigid mass against a change of angular velocity or rotational speed
Displacement	$x(t)$ $\varphi(t)$	m rad[1]	Time dependent local displacement or angular deflection value
Peak displacement	$x_{max},\ \hat{x}$ $\varphi_{max},\ \hat{\varphi}$	m rad	Maximum observed peak value of local displacement and angular deflection (within a time window or angular range)
Displacement velocity	$\dot{x}(t)$ $\dot{\varphi}(t)$	m/s rad/s	Instantaneous value of alternating velocity through mass displacement
Vibration acceleration	$\ddot{x}(t)$	m/s^2	Instantaneous acceleration of the oscillating mass
Force of inertia	$m \cdot \ddot{x}(t)$	N	According to d'Alembert: inertia force or the moment of inertia forces acting against the instantaneous acceleration
Moment of inertia forces	$J \cdot \ddot{\varphi}(t)$	$N \cdot m$	
Spring stiffness, spring constant	c	N/m	For linear springs (springs with proportional characteristic curve): the spring reaction is proportional to the deflection (displacement)
Spring force	$F_F = c \cdot \Delta x$	N	
Torsion spring stiffness, Torsion spring constant	c_T	$N \cdot m/rad$	For torsion springs with linear characteristic curve the spring reaction is proportional to angular deflection
Spring moment (spring torque)	$c_T \cdot \varphi(t)$	$N \cdot m$	
Damping constant	b	$N \cdot s/m$	In the case of Newtonian friction, the damping force is proportional to the velocity and the damping constants (linear damping)
Damping constant for rotary motion	b	$N \cdot m \cdot s/rad$	
Damping factor (damping coefficient)	$\delta = b/(2 \cdot m)$ $\delta = b/(2 \cdot J)$	1/s 1/s	The damping constant relative to twice the mass
Damping factor	$D = \delta/\omega_0$	–	$D < 1$: damped vibration, $D \geqq 1$: aperiodic case

Continuation of table, see Page 151.

[1] The unit rad can be replaced by "1" in calculation.

Continuation of table, Quantities and units, from Page 150.

Name	Quantity	Unit	Description
Damping ratio	$\hat{x}_n / \hat{x}_{n+1}$ $\hat{\varphi}_n / \hat{\varphi}_{n+1}$	– –	The ratio between two amplitudes separated by one period
Logarithmic damping decrement	$\Lambda = \dfrac{2 \cdot \pi \cdot D}{\sqrt{1-D^2}}$	–	$\Lambda = \ln\left(\hat{x}_n / \hat{x}_{n+1}\right)$ $\Lambda = \ln\left(\hat{\varphi}_n / \hat{\varphi}_{n+1}\right)$
Time	t	s	The current time coordinate
Phase angle	α	rad	This characterises the vibration phase, in other words the vibration state in which the system is currently present
Phase displacement angle	$\epsilon = \alpha_1 - \alpha_2$	rad	The difference in phase angle between two vibration events with the same circular frequency; if the value is positive this is a leading angle
Periodic time (cycle duration)	$T = 2 \cdot \pi / \omega_0$	s	The time in which one single vibration occurs
Characteristic frequency of the natural vibration	$f_0 = 1/T$	Hz	The reciprocal value of the period time
Characteristic circular frequency of the natural vibration	$\omega_0 = 2 \cdot \pi \cdot f_0$	1/s	The number of vibrations in $2 \cdot \pi$ seconds
Natural circular frequency (natural frequency)	$\omega_0 = \sqrt{c/m}$ $\omega_0 = \sqrt{c_T/J}$	1/s 1/s	The vibration frequency of the natural vibration of the system (undamped)
Natural circular frequency in the case of damping	$\omega_d = \sqrt{\omega^2{}_0 - \delta^2}$	1/s	In the case of a very small damping factor $D \ll 1$, $\omega_d = \omega_0$
Excitation frequency	Ω	1/s	The circular frequency of the excitation
Circular frequency ratio	$\eta = \Omega/\omega_0$	–	Resonance is present at $\eta = 1$

Free, undamped vibration

Description of the energy approach

Where free, undamped mechanical vibrations take place, a periodic exchange generally occurs between **potential** and **kinetic energy.**

The potential energy is present in this case as:

- the energy of position of the vibrating mass in a gravitational field (the Earth's gravitational field, a centrifugal field etc.)
- the elastic deformation energy (elastic strain energy of a spring, a support or a bar structure etc.)
- or both

The kinetic energy is present in this case as:

- the energy of motion of the vibrating mass

Motion equation, formulation and solution

The simplest mechanical vibration system, to which a range of vibrating bodies can be reduced, is **linear spring vibration**. The oscillator comprises a theoretically massless elastic element (in this case a spring with a linear spring characteristic) and a point mass.

For formulation and solution of the motion equation for this free, undamped vibration, consideration is given to the following forces:

- translational forces
- inertia forces
- reset forces

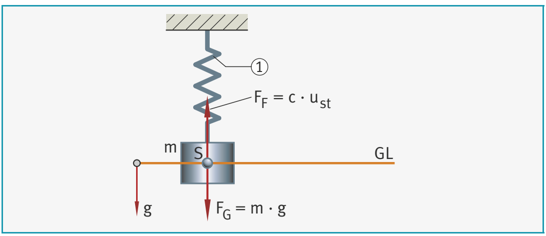
Translational forces

In a resting state, the weight force F_G of the mass m and the spring force F_F are in equilibrium.

Figure 2
Linear spring vibration: translational forces

GL = static equilibrium position
S = centre of gravity

① Massless spring



An equilibrium between the two translational forces can be described as follows:

Equation 3

$$F_G = m \cdot g \qquad F_F = c \cdot u_{st}$$

$$\text{Equilibrium:} \qquad F_G = F_F$$

The spring is deflected by an amount u_{st} compared to its untensioned length:

Equation 4

$$u_{st} = \frac{m \cdot g}{c}$$

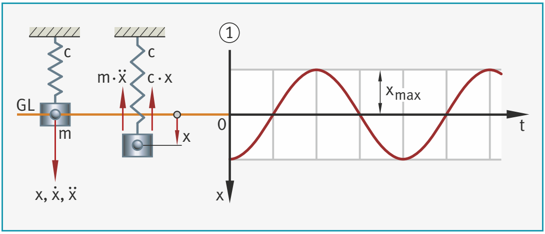
This position is known as the **rest position** or **static equilibrium position.**

If the mass is deflected out of its static equilibrium position in a vertical direction and then released, it performs free, periodic vibrations about the static equilibrium position.

Figure 3
Linear spring vibration: motion pattern

GL = static equilibrium position

① Motion pattern over time



The motion pattern is described by means of the motion equation. This is determined by developing a force approach in accordance with d'Alembert's principle:

- In the direction of motion, plot the coordinates of direction for x, $\dot{x}$ and $\ddot{x}$ **starting from the centre of gravity of the mass in the static equilibrium position.**
- Then plot the forces acting at the centre of gravity of the mass in the direction of motion if these are regarded as being **deflected in the positive coordinate of direction** in the vibration.

Inertia forces The inertia forces are considered using the following approach:

- Force approach according to d'Alembert: plot the kinetic reaction **against the positive direction of acceleration** $\ddot{x}$.
- If the vibration event takes place in an **accelerated reference system** (accelerated elevator, accelerated vehicle or rotating system etc.), there will be an additional system force (kinetic reaction, centrifugal force, Coriolis force) that acts on the mass.
  This **additional system force is plotted against the positively defined direction of system acceleration**. In this case, u, $\dot{u}$, $\ddot{u}$ are the motion coordinates of the accelerated reference system.
  This can result, on the one hand, in a change in the static equilibrium position compared with that in the unaccelerated reference system or, on the other hand, a change in the frequency of vibration.

Reset forces The reset forces are considered using the following approach:

- Plot the **spring force** due to the deflection of the mass out of its static equilibrium position **against the positive direction of deflection.**
- **Weight force** and **static spring force**:
  - If only a periodic exchange occurs in a vibration system between potential elastic and kinetic energy under the influence of a constant gravitational field, the weight force F_G (of the mass m) and the spring force F_F (due to the static deflection) will cancel each other out at each moment of the motion if the motion equation is formulated for vibration about the static equilibrium position. **In this case, the weight force and spring force are not used at all in the approach.**
  - If an exchange takes place between potential energy of position and kinetic energy during the vibration, the **weight force** F_G (of the mass m) must be included in the approach.

Motion equation For an equilibrium of forces in the direction of motion, this gives the following equilibrium relationship in the oscillator presented:

Equation 5

$$\sum F_x = -m \cdot \ddot{x} - c \cdot x = 0$$

The forces occurring are written in the sequence of the derivatives of x, starting with the force having the highest derivative, taking account of their direction.

If this equation is divided by the factor of the highest derivative, this gives the **homogeneous differential equation** as a motion equation for free, undamped vibration of the mass m:

Equation 6

$$\ddot{x} + \frac{c}{m} \cdot x = 0$$

This form of the equation will hereinafter be referred to as the **normal form.** In terms of its mathematical structure, it is typical for all free, undamped and linear vibrations.

This therefore solves the purely mechanical problem of formulating the motion equation. Solving this motion equation is now a mathematical task.

General solution of the motion equation

The **acceleration** of the motion event is a **function of the travel.** As a result, solving this equation by means of double integration over time is not a straightforward matter.

For the existing form of d'Alembert's differential equation with constant coefficients, the general approach to solution is as follows:

Equation 7

$$x = C \cdot e^{s \cdot t}$$

If this approach to solution is applied to the normal form of the differential equation, this gives:

Equation 8

$$C \cdot s^2 \cdot e^{s \cdot t} + \frac{c}{m} \cdot C \cdot e^{s \cdot t} = 0$$

$$s^2 + \frac{c}{m} = 0$$

$$s_{1,2} = \pm \sqrt{-\frac{c}{m}} = \pm i \sqrt{c/m}$$

This gives the general solution of the motion equation:

Equation 9

$$x = C_1 \, e^{+i\sqrt{c/m} \cdot t} + C_2 \, e^{-i\sqrt{c/m} \cdot t}$$

With the aid of Euler's formula $e^{\pm i\varphi} = \cos\varphi \pm i \cdot \sin\varphi$, this can also be expressed as follows:

Equation 10

$$x = C_1 \left(\cos\sqrt{c/m} \cdot t + i \cdot \sin\sqrt{c/m} \cdot t\right) + C_2 \left(\cos\sqrt{c/m} \cdot t - i \cdot \sin\sqrt{c/m} \cdot t\right)$$

$$x = \left(C_1 + C_2\right) \cos\sqrt{c/m} \cdot t + i \cdot \left(C_1 - C_2\right) \sin\sqrt{c/m} \cdot t$$

This relationship only results in a real value as a solution for the motion coordinate x if the constants C_1 and C_2 are conjugated in complex form as follows:

Equation 11

$$C_{1,2} = K_1 \pm i \cdot K_2$$

The general solution for a harmonic vibration is thus expressed as follows:

Equation 12

$$x = 2K_1 \cos \sqrt{c/m} \cdot t - 2K_2 \sin \sqrt{c/m} \cdot t$$

$$\text{where} \quad \omega_0 = \sqrt{c/m} \qquad \omega_0^2 = c/m$$

ω_0 is known as the **natural circular frequency** of the vibration event. The square of the natural circular frequency is always represented in the normal form of the differential equation by the factor of the linear motion coordinate x.

Based on these considerations, the following **general solution of the differential equation** is always to be expected for the **free, undamped vibration**:

Equation 13

$$x = A \sin \omega_0 t + B \cos \omega_0 t \qquad \text{where} \quad \omega_0 = \sqrt{c/m}$$

Consideration of the initial conditions

The two free constants A and B of the general solution are defined by the **initial conditions** of the vibration event, which are normally stipulated.

The different initial conditions will result in the corresponding solutions when the conditions are used in the general solution:

Initial conditions			Solution
$t = 0$	$x = 0$	$\dot{x} = \dot{x}_{max}$	$x = \frac{\dot{x}_{max}}{\omega_0} \cdot \sin \omega_0 t$
$t = 0$	$x = x_{max}$	$\dot{x} = 0$	$x = x_{max} \cdot \cos \omega_0 t$
$t = 0$	$\dot{x} = 0$	$\ddot{x} = \lvert \ddot{x}_{max} \rvert$	$x = \frac{\lvert \ddot{x}_{max} \rvert}{\omega_0^2} \cdot \cos \omega_0 t$
$t = 0$	$x = x_0$	$\dot{x} = \dot{x}_0$	$x = \frac{\dot{x}_0}{\omega_0} \cdot \sin \omega_0 t + x_0 \cdot \cos \omega_0 t$

From a comparison of the maximum vibration deflections x_{max} (amplitudes), two important relationships can be determined between the natural circular frequency of the oscillating body, the vibration amplitude, the maximum velocity and the maximum acceleration:

Equation 14

$$\dot{x}_{max} = \omega_0 \cdot x_{max}$$

$$|\ddot{x}_{max}| = \omega_0^{\,2} \cdot |x_{max}|$$

The general solution of the differential equation can therefore be expressed with the aid of the amplitudes in the following form:

Equation 15

$$x = x_{max} \cdot \sin(\omega_0\, t + \alpha)$$

In this case, α is the phase angle (leading angle) with respect to a vibration $x = x_{max} \cdot \sin(\omega_0\, t)$ and x_{max} is the amplitude of the vibration event. The phase angle and amplitude are determined by the initial conditions.

This gives:

Initial condition	Solution
$t = 0 \quad x = x_0 \quad \dot{x} = \dot{x}_0$	$x_{max} = \sqrt{x_0^{\,2} + \dfrac{\dot{x}_0^2}{\omega_0^2}}$ $\tan\alpha = \dfrac{x_0}{\dot{x}_0 / \omega_0}$

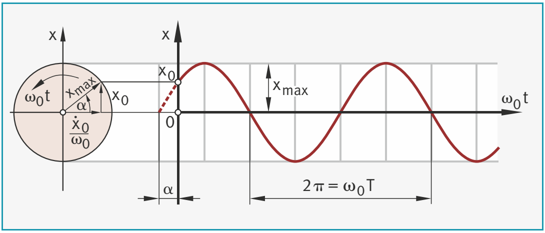


Figure 4
Free, undamped vibration: motion pattern

2π = period

Consideration of overlaid vibrations

The preceding method of representation of the motion equation is particularly useful when considering a superposition of several simultaneous vibrations of identical natural circular frequency ω_0 and differing phase angles α.

Two vibrations with the phase angles α_1 and α_2 have a phase displacement angle ϵ:

Equation 16

$$\epsilon = \alpha_1 - \alpha_2$$

Reduction to linear spring vibration

A series of free, undamped oscillating bodies in which an exchange takes place between elastic and kinetic energy can be reduced to the basic form of the free, undamped oscillator.

The spring constant c of the elasticity at the position and in the direction of vibration of the mass can be determined on the basis of the condition:

Equation 17

$$c = \frac{F}{u}$$

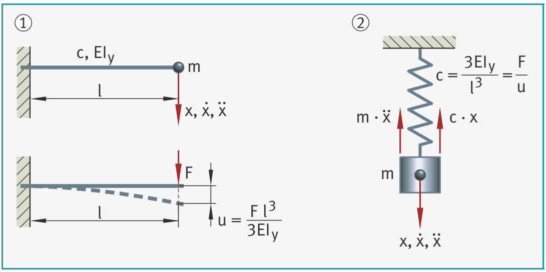
F = force on the elasticity in the direction of vibration of the mass

u = deflection at the force application point in the direction of the force F

Figure 5
Free, undamped vibration and transfer to linear spring vibration

c = spring constant
EI_y = bending stiffness

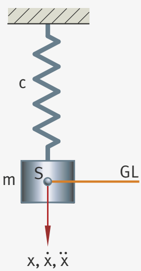
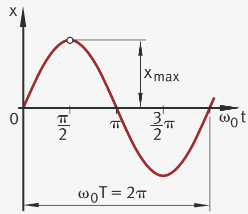
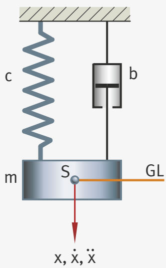
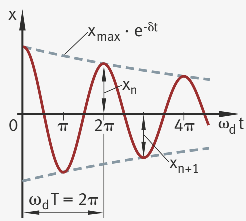
① Oscillator
② Oscillator substitute system



Overview of motion equations and amplification function

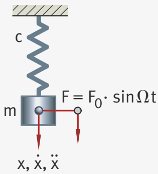
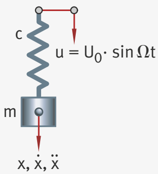
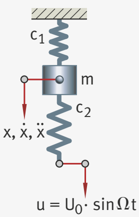
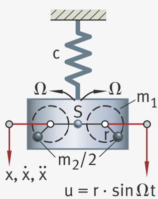
Free, undamped and damped vibration

The following table shows the conditions and relationships for solving the motion equations for free, undamped and damped vibrations:

Oscillator schematic	Differential equation and solution	Time-based vibration pattern
 GL = static equilibrium position	**Approach:** $-m \cdot \ddot{x} - c \cdot x = 0$ **Differential equation:** (normal form of the homogeneous differential equation, 2nd order): $\ddot{x} + \frac{c}{m} \cdot x = 0$ **Initial conditions:** $t = 0; x = 0; \quad \dot{x} = \dot{x}_{max}$ **Solution:** $x = x_{max} \cdot \sin \omega_0 t$ With natural circular frequency: $\omega_0 = \sqrt{\frac{c}{m}}$	 **Period time of one vibration:** $T = 2\,\pi/\omega_0$ **Vibration frequency:** $f = 1/T = \omega_0/2\,\pi$
 GL = static equilibrium position b = damping constant	**Approach:** $-m \cdot \ddot{x} - b \cdot \dot{x} - c \cdot x = 0$ **Differential equation:** (normal form of the homogeneous differential equation, 2nd order): $\ddot{x} + \frac{b}{m} \cdot \dot{x} + \frac{c}{m} \cdot x = 0$ **Initial conditions:** $t = 0; x = x_0; \quad \dot{x} = 0$ **Simplified solution:** $x = e^{-\delta t} \cdot x_{max} \cdot \cos \omega_d t$ With damping factor: $\delta = b/(2\,m)$ With natural circular frequency: $\omega_d = \sqrt{\omega_0^2 - \delta^2}$	**Damped vibration:** $b < 2\sqrt{c \cdot m}$  **Period time of one vibration:** $T = 2\,\pi/\omega_d$ **Amplitude ratio for T/2:** $x_n/x_{n+1} = e^{-\delta\,(T/2)}$

Undamped vibration caused by external excitation

The following table shows the conditions and relationships for solving the motion equations for undamped vibrations caused by external excitation (forced vibrations). The solutions apply to the forced state.

Oscillator schematic	Differential equation	Solution and amplitude (non-homogeneous)	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin(\Omega\, t)$		**Solution function:** $x = x_{max} \cdot \sin(\Omega\, t + \alpha)$	
	**Approach:** $-m \cdot \ddot{x} - c \cdot x + F(t) = 0$ **Differential equation:** $m \cdot \ddot{x} + c \cdot x = F_0 \sin \Omega\, t$	**Solution:** $x = \frac{F_0}{c - m\,\Omega^2} \sin(\Omega\, t + \alpha)$ **Amplitude:** $x_{max} = \frac{F_0}{c} \cdot \left\lvert \frac{1}{1-\eta^2} \right\rvert = \frac{F_0}{c} \cdot V_{03}$	Subcritical range $\eta = \frac{\Omega}{\omega_0} < 1$ $\alpha = -\epsilon$ $\epsilon = 0°$
	**Approach:** $-m \cdot \ddot{x} - c \cdot x + c \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + c \cdot x = c \cdot U_0 \sin \Omega\, t$	**Solution:** $x = \frac{c \cdot U_0}{c - m\,\Omega^2} \sin(\Omega\, t + \alpha)$ **Amplitude:** $x_{max} = U_0 \left\lvert \frac{1}{1-\eta^2} \right\rvert = U_0 \cdot V_{03}$	
	**Approach:** $-m \cdot \ddot{x} - c_1 \cdot x - c_2 \cdot x + c_2 \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + (c_1 + c_2) \cdot x$ $= c_2 \cdot U_0 \cdot \sin \Omega\, t$	**Solution:** $x = \frac{c_2 \cdot U_0}{c_1 + c_2 - m\,\Omega^2} \sin(\Omega\, t + \alpha)$ **Amplitude:** $x_{max} = \frac{c_2 \cdot U_0}{(c_1 + c_2)} \cdot \left\lvert \frac{1}{1-\eta^2} \right\rvert$ $= \frac{c_2 \cdot U_0}{c_1 + c_2} \cdot V_{03}$	Super-critical range $\eta = \frac{\Omega}{\omega_0} > 1$ $\alpha = -\epsilon$ $\epsilon = 180°$
	**Approach:** $-(m_1 + m_2) \cdot \ddot{x} - c \cdot x - m_2 \cdot \ddot{u} = 0$ **Differential equation:** $(m_1 + m_2) \cdot \ddot{x} + c \cdot x$ $= m_2 \cdot r \cdot \Omega^2 \cdot \sin \Omega\, t$	**Solution:** $x = \frac{m_2 \cdot r \cdot \Omega^2}{c - (m_1 + m_2)\,\Omega^2} \sin(\Omega\, t + \alpha)$ **Amplitude:** $x_{max} = \frac{m_2 \cdot r}{m_1 + m_2} \cdot \left\lvert \frac{\eta^2}{1-\eta^2} \right\rvert$ $= \frac{m_2 \cdot r}{m_1 + m_2} \cdot V_{01}$	

Amplification function

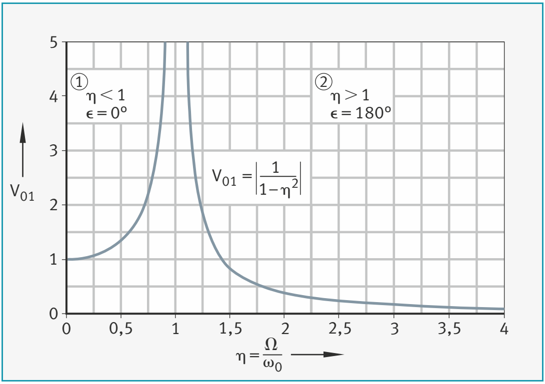
The excitation of undamped vibrations can be achieved in different forms:

- periodic force or spring force excitation

Figure 6
Periodic force or spring force excitation

η = circular frequency ratio
V_{01} = amplification function

① Subcritical range
② Supercritical range

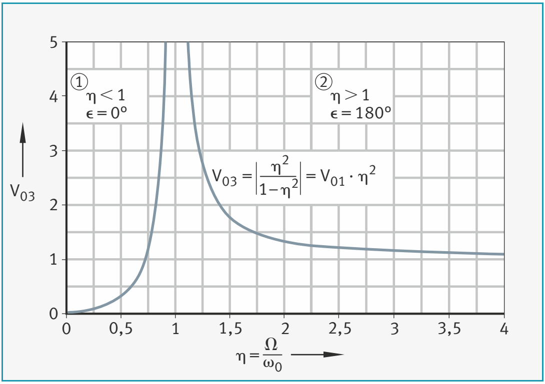


- periodic mass force excitation

Figure 7
Periodic mass force excitation

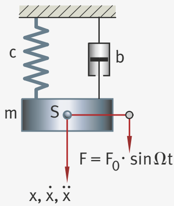
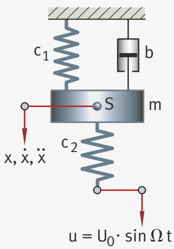
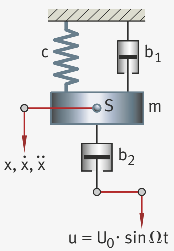
η = circular frequency ratio
V_{03} = amplification function

① Subcritical range
② Supercritical range



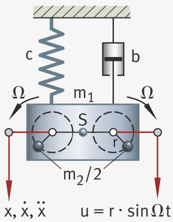
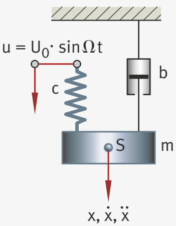
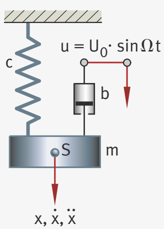
Damped vibration caused by external excitation

The following table shows the conditions and relationships for solving the motion equations for damped vibrations caused by external excitation (forced vibrations). The solutions apply to the turned in state.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin(\Omega t)$	**Solution function:** $x = x_{max} \cdot \sin(\Omega t + \alpha)$	
	**Approach:** $-m \cdot \ddot{x} - b \cdot \dot{x} - c \cdot x + F(t) = 0$ **Differential equation:** $m \cdot \ddot{x} + b \cdot \dot{x} + c \cdot x = F_0 \cdot \sin \Omega t$ **Amplitude:** $x_{max} = \frac{F_0}{c} \cdot \frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4 D^2 \eta^2}}$ $x_{max} = \frac{F_0}{c} \cdot V_3$	$\alpha = -\epsilon_3$ $\tan \epsilon_3 = \frac{2 D \eta}{1-\eta^2}$
	**Approach:** $-m \cdot \ddot{x} - b \cdot \dot{x} - c_1 \cdot x - c_2 \cdot x + c_2 \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + b \cdot \dot{x} + \left(c_1 + c_2\right) \cdot x = c_2 \cdot U_0 \cdot \sin \Omega t$ **Amplitude:** $x_{max} = \frac{c_2 \cdot U_0}{c_1 + c_2} \cdot \frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4 D^2 \eta^2}}$ $x_{max} = \frac{c_2 \cdot U_0}{c_1 + c_2} \cdot V_3$	$\alpha = -\epsilon_3$ $\tan \epsilon_3 = \frac{2 D \eta}{1-\eta^2}$
	**Approach:** $-m \cdot \ddot{x} - b_1 \cdot \dot{x} - b_2 \cdot \dot{x} - c \cdot x + b_2 \cdot \dot{u} = 0$ **Differential equation:** $m \cdot \ddot{x} + \left(b_1 + b_2\right) \cdot \dot{x} + c \cdot x = b_2 \cdot U_0 \cdot \Omega \cdot \cos \Omega t$ **Amplitude:** $x_{max} = \frac{b_2 \cdot U_0}{b_1 + b_2} \cdot \frac{2 D \eta}{\sqrt{\left(1-\eta^2\right)^2 + 4 D^2 \eta^2}}$ $x_{max} = \frac{b_2 \cdot U_0}{b_1 + b_2} \cdot V_2$	$\alpha = \gamma_2 = \frac{\pi}{2} - \epsilon_3$ $\tan \gamma_2 = \frac{1-\eta^2}{2 D \eta}$

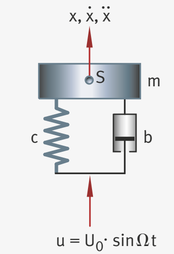
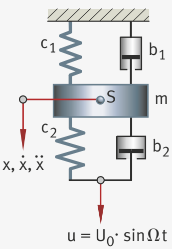
Continuation of table, see Page 163.

Continuation of table, Damped vibration caused by external excitation, from Page 162.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin(\Omega t)$	**Solution function: $x = x_{max} \cdot \sin(\Omega t + \alpha)$**	
	**Approach:** $-(m_1+m_2)\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x-m_2\cdot\ddot{u}=0$ **Differential equation:** $(m_1+m_2)\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = m_2\cdot r\cdot\Omega^2\sin\Omega t$ **Amplitude:** $x_{max} = \frac{m_2\cdot r}{m_1+m_2}\cdot\frac{\eta^2}{\sqrt{(1-\eta^2)^2+4D^2\eta^2}}$ $x_{max} = \frac{m_2\cdot r}{m_1+m_2}\cdot V_1$	$\alpha = -\epsilon_1$ $\tan\epsilon_1 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x+c\cdot u=0$ **Differential equation:** $m\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = c\cdot U_0\cdot\sin\Omega t$ **Amplitude:** $x_{max} = U_0\cdot\frac{1}{\sqrt{(1-\eta^2)^2+4D^2\eta^2}}$ $x_{max} = U_0\cdot V_3$	$\alpha = -\epsilon_3$ $\tan\epsilon_3 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x+b\cdot\dot{u}=0$ **Differential equation:** $m\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = b\cdot U_0\cdot\Omega\cdot\cos\Omega t$ **Amplitude:** $x_{max} = U_0\cdot\frac{2D\eta}{\sqrt{(1-\eta^2)^2+4D^2\eta^2}}$ $x_{max} = U_0\cdot V_2$	$\alpha = \gamma_2 = \frac{\pi}{2}-\epsilon_3$ $\tan\gamma_2 = \frac{1-\eta^2}{2D\eta}$

Continuation of table, see Page 164.

Continuation of table, Damped vibration caused by external excitation, from Page 163.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin(\Omega\, t)$	**Solution function:** $x = x_{max} \cdot \sin(\Omega\, t + \alpha)$	
	**Approach:** $-m \cdot \ddot{x} - b \cdot \dot{x} - c \cdot x + b \cdot \dot{u} + c \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + b \cdot \dot{x} + c \cdot x = b \cdot U_0 \cdot \Omega \cdot \cos \Omega\, t + c \cdot U_0 \cdot \sin \Omega\, t$ **Amplitude:** $x_{max} = U_0 \cdot \sqrt{\dfrac{1 + 4D^2\eta^2}{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}$ $x_{max} = U_0 \cdot V_{2,3}$	$\alpha = -\epsilon_{2,3}$ $\tan \epsilon_{2,3}$ $= \dfrac{2D\eta^3}{1+\eta^2\left(4D^2-1\right)}$
	**Approach:** $-m \cdot \ddot{x} - (b_1 + b_2) \cdot \dot{x} - (c_1 + c_2) \cdot x + b_2 \cdot \dot{u} + c_2 \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + (b_1 + b_2) \cdot \dot{x} + (c_1 + c_2) \cdot x$ $= b_2 \cdot U_0 \cdot \Omega \cdot \cos \Omega\, t + c_2 \cdot U_0 \cdot \sin \Omega\, t$ **Amplitude:** $x_{max} = \dfrac{c_2 \cdot U_0}{c_1 + c_2} \cdot \sqrt{\dfrac{1 + 4D^2\eta^2}{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}$ $x_{max} = \dfrac{c_2 \cdot U_0}{c_1 + c_2} \cdot V_{2,3}$	$\alpha = -\epsilon_{2,3}$ $\tan \epsilon_{2,3}$ $= \dfrac{2D\eta^3}{1+\eta^2\left(4D^2-1\right)}$

Amplification function

The excitation of damped vibrations can be achieved in different forms:

- direct force, indirect spring force and mass force excitation: Amplification function V_3

Figure 8
Amplification function V_3

η = circular frequency ratio for V_3, ϵ_3

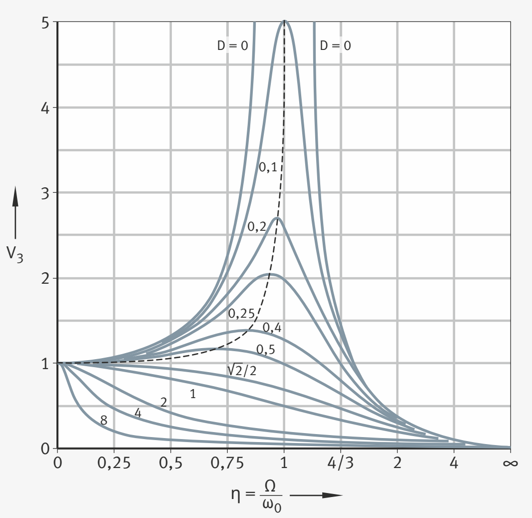
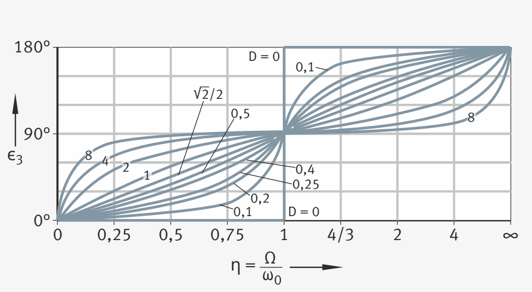


Figure 9
Phase displacement angle ϵ_3

η = circular frequency ratio



- damping force excitation: Amplification function V_2

Figure 10
Amplification function V_2

η = circular frequency ratio

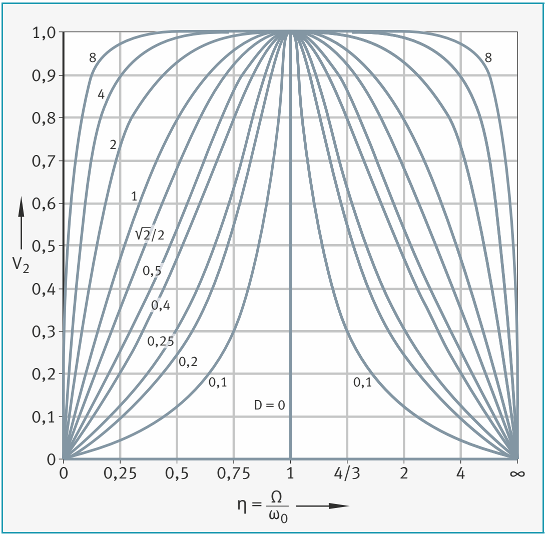
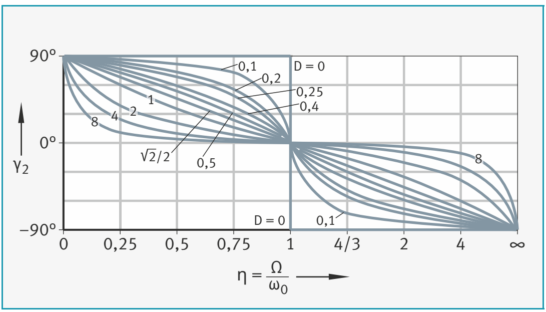


Figure 11
Phase displacement angle γ_2

η = circular frequency ratio



■ spring and damping force excitation: Amplification function $V_{2,\,3}$

Figure 12
Amplification function $V_{2,\,3}$

η = circular frequency ratio

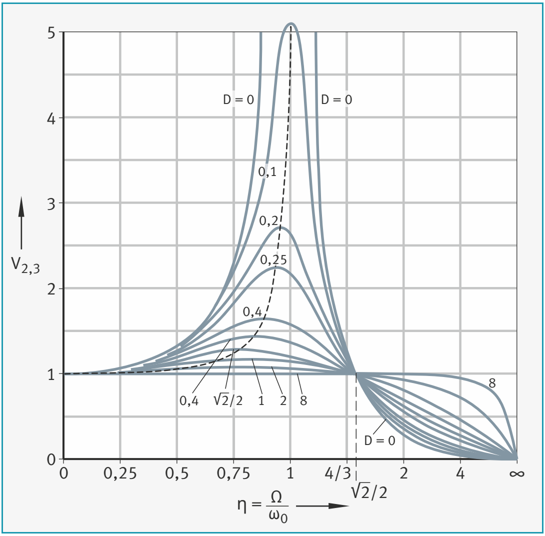
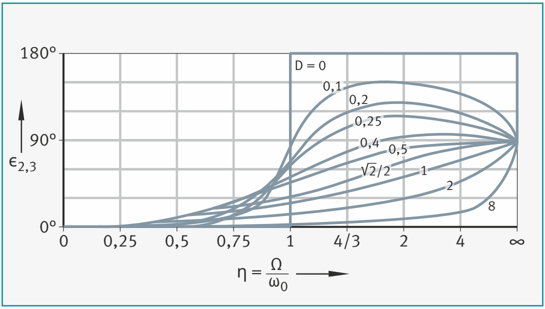


Figure 13
Phase displacement angle $\epsilon_{2,\,3}$

η = circular frequency ratio



Technical acoustics

Sound, sound pressure and sound levels

Sound pressure waves in the medium of air with frequency components within the human hearing range from 20 to 20 000 Hz are described as **audible sound**. Sound at lower or higher frequencies is described as infrasound or ultrasound respectively.

Furthermore, the following terms are used depending on the medium transmitting the sound:

- airborne sound = vibrations in air and gases
- liquid-borne sound = vibrations in liquids
- structure-borne sound = vibrations in solid bodies

In air and other gases as well as in liquids, sound only propagates in the form of compression waves. The alternating pressure p(t) that is superposed on the static air pressure is described as **sound pressure**. The sound pressure is the most important measurement value in these media and is measured by means of microphones or pressure sensors.

For structure-borne sound, the most important measurement value is the vibration velocity v(t) or structure-borne sound speed vertical to the radiating surface of a sound generator.

In general, the acceleration a(t) is measured using piezoelectric quartz sensors and then converted to structure-borne sound speed:

Equation 18

$$a(t) = dv(t)/dt$$

For a frequency band with the centre frequency f, the speed is used as the RMS:

Equation 19

$$\overline{v}(f) = \overline{a}(f)/2 \cdot \pi \cdot f$$

The speed is normally stated in relative terms as a speed level L_v, using the reference value $v_0 = 5 \cdot 10^{-8}$ m/s:

Equation 20

$$L_v = 10 \cdot \lg(\overline{v}/v_0)^2 = 20 \cdot \lg(\overline{v}/v_0) \qquad \text{in dB}$$

The following table shows some values for the acoustic perception of the human ear.

Perception	Volume phon	Sound pressure N/m^2	Sound power W	Sound intensity W/m^2
Auditory threshold[1)]	0 – 10	$2 \cdot 10^{-5}$ [2)]	10^{-12}	10^{-12}
Conversation	50 – 60	0,2	$\approx 10^{-3}$	$\approx 10^{-3}$
Pain threshold	130	20	$\approx 10^{3}$	$\approx 10^{3}$

1) The lowest volume level that is perceptible by the human ear.

2) Reference sound pressure: $p_0 = 2 \cdot 10^{-5}\ N/m^2$, internationally defined reference value for the RMS of sound pressure (DIN ISO 226).

Sound level, noise situation and perception

Some example values for the sound level in certain noise situations and the resulting levels of perception are as follows:

Sound level dB(A)[1)]	Noise situation	Perception
0 0 – 10 10 – 20	Complete silence, start of the auditory range Auditory threshold Rustling of leaves	Calm
30 40 50	Whispered speech Quiet radio music Upper limit for mental work requiring concentration	Faint
50 – 70 75	Office work, people talking Start of a disruptive influence on the nervous system	Moderately loud
80 85	Heavy traffic, limit of hearing recovery Start of danger to hearing	Very loud
90 90 – 100	Truck driving noise Car horn	Extremely loud
110 110 – 120	Pneumatic drill Large forging hammer	Unbearable
130 140	Jet aircraft (100 m), pain threshold Rocket launch	Painful

1) In accordance with IEC 61672:2013.
Sound pressure level L_p with applied A-weighting characteristic.

Quantities and units The following table shows a selection of acoustic quantities.

Name		Quantity	Unit	Description
Speed of sound	Solid materials	$c_L = \sqrt{\frac{2\,G\,(1-\nu)}{\rho\,(1-2\,\nu)}}$	m/s	Longitudinal waves in large bodies
		$c_T = \sqrt{G/\rho}$		Transversal waves in large bodies
		$c_D = \sqrt{E/\rho}$		Longitudinal waves in bars; steel: 5 000 m/s
	Liquids	$c = \sqrt{K/\rho}$		Water: 1 485 m/s
	Gases	$c = \sqrt{\kappa \cdot R \cdot T}$		Air: c_{air} = 331,3 m/s + T(°C) · 0,606 (m/s)/°C 343 m/s (at 1 bar, +20 °C) 331 m/s (at 1 bar, 0 °C) Hydrogen: 1 280 m/s (at 1 bar, 0 °C)
Sound particle velocity		$v = a_0 \cdot \omega$ $v = a_0 \cdot 2 \cdot \pi \cdot f$	m/s	Alternating speed of oscillating particles in the direction of vibration
Sound pressure		p	N/m^2 μbar	Alternating pressure caused by sound vibration
Sound power		$P = A \cdot p \cdot v$	W	Sound energy that passes through a defined surface per unit of time
Sound intensity		$I = P/A$ $= p^2/(c \cdot \rho)$	W/m^2	Sound power per unit of area perpendicular to the direction of propagation
Sound level	Sound pressure level	$L_p = 20 \cdot \lg (p/p_0)$	dB(SPL) dB(A)	$p_0 = 2 \cdot 10^{-5}$ Pa
	Sound intensity level	$L_I = 10 \cdot \lg (I/I_0)$	dB(SIL)	$I_0 = 10^{-12}$ W/m^2
	Sound power level	$L_W = 10 \cdot \lg (P/P_0)$	dB(SWL)	$P_0 = 10^{-12}$ W
Loudness level		$\Lambda = 10 \cdot \lg (I/I_0)$ at 1 000 Hz	phon	Frequency-dependent relation of the perceived loudness of a pure tone to its respective sound pressure level
Sound absorption coefficient		$\alpha = 1 - \left(\frac{I_r}{I_a}\right)$ Index: a = incident r = reflected	1	Intensity measure of the conversion of sound energy into heat as a result of air friction. At 500 Hz: concrete: 0,01 glass: 0,03 slag wool: 0,36 to 0,8 (depending on the layer thickness)
Sound reduction index		$R = 10 \cdot \lg (I_1/I_2)$ Index: 1 = this side of the wall 2 = the other side of the wall	dB	Logarithmic measure of the sound reduction achieved by a wall; sheet steel, 1 mm: R = 29 dB
Acoustic efficiency		$\eta = P_{acou}/P_{mech}$	1	Ratio of acoustic to mechanical power

Continuation of table, see Page 171.

Continuation of table, Quantities and units, from Page 170.

Legend

a_0 Amplitude	m	A Area	m^2
f Frequency	Hz	E Modulus of elasticity	Pa
ρ Density	kg/m^3	G Modulus of rigidity	Pa
κ Isentropic exponent		P Power	W
K Modulus of compression	Pa	R Gas constant	$J/(kg \cdot K)$
ν Poisson's ratio		T Absolute temperature.	K

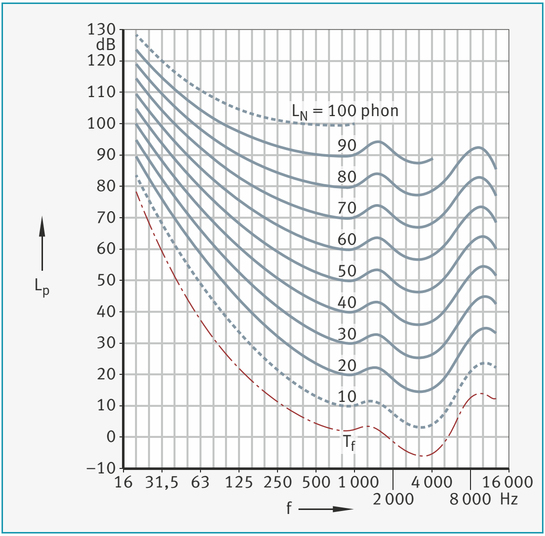
Normal equal-loudness-level contours

The normal equal-loudness-level contours (isophones) are measured for pure tones under free-field conditions in an anechoic chamber.

Figure 14
Isophones according to ISO 226

Source: ISO 226:2023

L_p = sound pressure level
f = frequency
L_N = loudness level of a pure tone
T_f = human absolute threshold of hearing (auditory threshold)



Hydraulics and pneumatics

Hydraulics

Hydraulic transmissions

Hydraulic transmissions contain pumps, motors and control elements (hydraulic valves) interconnected in a circuit in which power is transmitted by means of circulating hydraulic fluid. The circuit can be of an open or closed design. The controller defines the motion and direction of motion, limits the load in the transmission and, where necessary, adjusts the transmission ratio in accordance with the operating conditions.

Hydraulic pumps

Hydraulic pumps are rotary displacement (rotary piston) or stroke displacement (axial piston) machines with a fixed or variable displacement volume.

In practice, displacement principles are allocated to specific application areas. The permissible continuous operating pressure is defined by the type of displacement element and the resulting load on the drive mechanism. A further significant feature is the chamber design, which covers the chamber shape and the size of the stroke volume in comparison with the machine size.
In the case of the cell cross-sections of rotary displacement machines, which are normally of a rectangular cross-section, it is more difficult to maintain the required gap tolerances. Since internal leakage losses occur as a function of pressure, the scope of application is restricted to low and medium-pressure systems.
Cylindrical fits can be easily achieved. Axial piston machines are therefore required for use in the high and very high pressure range.

Rotary displacement machines

Rotary displacement machines feed the hydraulic fluid with uniform rotation into cells whose volume is cyclically varied by the design of the limiting walls or the penetration of a tooth. The rotary displacement machine also ensures that the inlet and pressure chambers are sealed from each other. An adjustable stroke volume is only realised in single-stroke vane-cell pumps.

Stroke displacement machines Stroke displacement machines are characterised by the separation of the drive mechanism and the delivery chamber. The cyclical variation of the cell size is carried out by means of a linear piston. Adjustment of the stroke volume is possible by intervention in the drive mechanism geometry or the controller. Due to the internal flow reversal of the fluid, the machines require a slide or valve control mechanism between the displacement chamber and the flow paths.

Hydraulic motors Hydraulic motors convert the fluid energy made available to them into mechanical work. Depending on their output drive motion, a distinction is drawn between torque motors, swivel motors with a limited rotation angle and thrust motors (cylinders). In contrast to hydraulic pumps, hydraulic motors have a constant stroke volume. Adjustable machines are only used in exceptional cases.

Torque motors All of the design principles described for rotary displacement machines and slide-valve controlled axial piston machines are suitable for use as torque motors.
They convert the hydraulic power $P_h = \dot{V} \cdot \Delta p$ (minus the leakage power loss $P_{v,\,v} = \dot{V}_v \cdot \Delta p$, the hydraulic power loss $P_{v,\,h} = \dot{V} \cdot \Delta p_h$ and the mechanical power loss $P_{v,\,r} = M_r \cdot \omega$) into the mechanical motor power $P_m = M \cdot \omega$.

Swivel motors Swivel motors generate swivel motion either directly by swivelling a vane in the subdivided circular cylinder (vane motor with swivel angle of 300°) or by linear movement of a piston by means of a rack and pinion gear.

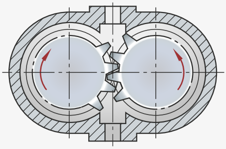
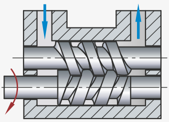
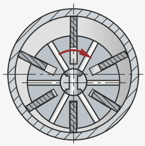
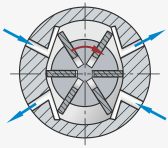
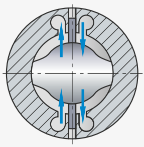
Thrust motors In the case of thrust motors, a distinction is drawn between single action designs (plunger cylinders) and double action designs (differential cylinders). Differential cylinders can be used for thrust and pull operation by means of alternate piston loading.

Hydraulic pumps The following section presents the values, units and relationships for hydraulic pumps, as well as common hydraulic pumps and their normal operating values.

Values, units and relationships The relationships for hydraulic pumps can also be applied analogously to the inverse energy transformation process in hydraulic motors.

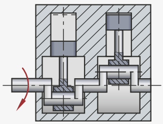
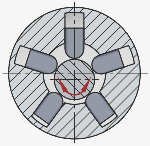
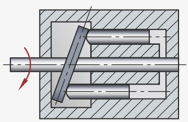
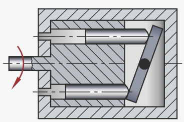
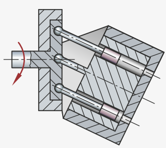
Value	Unit	Name	Relationship, comments
M	Nm	Mechanical drive torque of the pump	Torque provided by the drive unit to the pump shaft
M_r	Nm	Frictional torque within the pump	Friction in the drive mechanism and between the displacement elements
M_{th}	Nm	Theoretical pump moment	$M_{th} = \Delta p \cdot V_H / (2 \cdot \pi)$ $= \Delta p \cdot V_0$
P_m	W	Mechanical drive power of the pump	$P_m = M \cdot \omega$ $P_m = P_{th} + P_{v,r} + P_{v,h}$
P_{th}	W	Displacement power in relation to Δp	$P_{th} = M_{th} \cdot \omega$
P_u	W	Displacement power	$P_u = (M - M_r) \cdot \omega$ The displacement power is transferred to the displacement volume flow and divided up into the displacement power P_{th} in relation to Δp and the hydraulic power loss $P_{v,h}$
$P_{v,h}$	W	Hydraulic power loss	$P_{v,h} = \dot{V}_{th} \cdot \Delta p_h = M_h \cdot \omega$
$P_{v,r}$	W	Frictional power loss of the pump	$P_{v,r} = M_r \cdot \omega$
V_H	m^3	Stroke volume = displacement volume	The displacement volume is determined from the geometrical data for the pump
$\dot{V}$	m^3/s	Actual delivery rate	$\dot{V} = \dot{V}_{th} - \dot{V}_v$ The pressure differential Δp causes a leakage flow $\dot{V}_v$ through the gap that reduces the displacement volume flow
$\dot{V}_{th}$	m^3/s	Theoretical delivery rate (based on the assumption of complete filling of the stroke volume during intake)	$\dot{V}_{th} = n \cdot V_H = \omega \cdot V_0$ n = speed $\omega = 2 \cdot \pi \cdot n$ $V_0 = V_H / (2 \cdot \pi)$ base volume
η	–	Overall efficiency	$\eta = \frac{P_h}{P_m} = 1 - \frac{\sum P_v}{P_m}$ $\eta = \eta_{h,m} \cdot \eta_v$
$\eta_{h,m}$	–	Mechanical-hydraulic efficiency	$\eta_{h,m} = \frac{P_{th}}{P_m} = 1 - \frac{P_{v,r} + P_{v,h}}{P_m}$
η_v	–	Volumetric efficiency	$\eta_v = \frac{P_h}{P_{th}} = 1 - \frac{P_{v,v}}{P_{th}} = 1 - \frac{\dot{V}_v}{\dot{V}_{th}}$

Rotary displacement machines The following table shows an overview of types of common rotary displacement machines and their normal operating values.

Displacement element	Name	Displacement volume cm^3/rev	Pressure range bar	Speed 1/min	Favourable oil viscosity 10^{-6} m^2/s
Gear	Gear pump 	0,4 ... 1 200	... 200	1 500 ... 3 000	40 ... 80
	Screw pump 	2 ... 800	... 200	1 000 ... 5 000	80 ... 200
Vane	Vane pump – Single stroke 	30 ... 800	... 100	500 ... 1 500	30 ... 50
	Vane pump – Multiple stroke 	3 ... 500	... 160 (200)	500 ... 3 000	30 ... 50
	Rotary piston pump 	8 ... 1 000	... 160	500 ... 1 500	30 ... 50

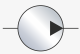
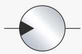
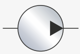
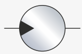
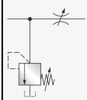
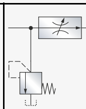
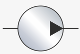
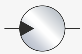
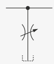
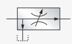
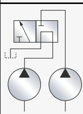
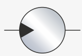
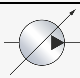
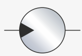
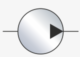
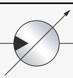
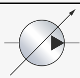
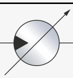
Source: Dubbel (with slight design modifications).

Stroke displacement machines The following table shows an overview of types of common stroke displacement machines and their normal operating values.

Displacement element	Name	Displacement volume cm^3/rev	Pressure range bar	Speed 1/min	Favourable oil viscosity $10^{-6}\ m^2/s$
Piston	In-line piston pump 	... 800	... 400	1000 ... 2000	20 ... 50
	Radial piston pump with internal piston support 	0,4 ... 15000	... 630	1000 ... 2000	20 ... 50
	Axial piston pump or swash plate pump 	1,5 ... 3600	... 400	500 ... 3000	30 ... 50
	Rear swash plate pump 				
	Bent axis piston pump 				

Source: Dubbel (with slight design modifications).

Hydrostatic stationary transmissions Hydrostatic stationary transmissions can be subdivided according to some of their characteristic features as follows:

Transmission type	Displacement machines (fixed or adjustable stroke volume)		Speed transmission ratio i_G (constant or adjustable, dependent on load or independent of load)	Open loop or closed loop control of speed transmission ratio		Torque trans-mission ratio μ_G
	Pump	Motor		Open loop	Closed loop	
I Figure 1, Page 178			Constant, independent of load	Not possible		Constant
II Primary flow throttle transmission Figure 2, Page 178			i_G adjustable, dependent on load in open loop control, independent of load in closed loop control			–
III Secondary flow throttle transmission Figure 3, Page 178			i_G adjustable, dependent on load in open loop control, independent of load in closed loop control			Constant
IV			Adjustable in steps, independent of load	Connection of a machine	–	–
V Figure 4, Page 178			i_G adjustable, speed transmission ratio is independent of the load of the hydraulic motor	Open loop or closed loop control through adjustment of the stroke volume of one or both (VII) displacement machines		–
VI Figure 4, Page 178						–
VII Figure 4, Page 178						–

For corresponding diagrammatic examples, see Page 178.

The following diagrams show examples of the transmission types I to VII for hydrostatic stationary transmissions.

The following image shows an **open circuit**:
the hydraulic pump and hydraulic motor are not adjustable.

Figure 1
Transmission type I

1 Drive unit
2 Operating unit
3 Hydraulic pump
4 Hydraulic motor

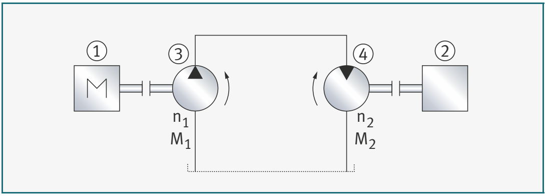


Figure 2
Transmission type II
Primary flow throttle transmission

1 Hydraulic pump
2 Hydraulic motor

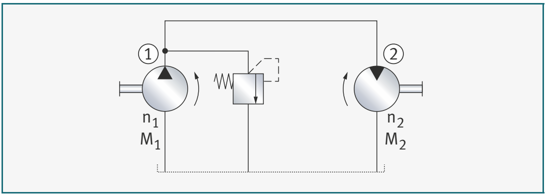
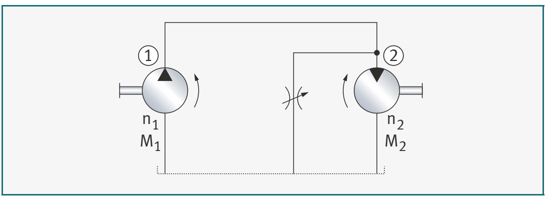


Figure 3
Transmission type III
Secondary flow throttle transmission

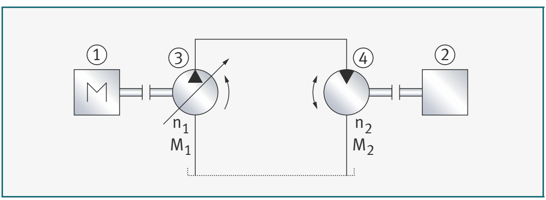
1 Hydraulic pump
2 Hydraulic motor



The following image shows a **closed circuit**:
the hydraulic pump is adjustable and reversible, the hydraulic motor is not adjustable.

Figure 4
Transmission type V to VII

1 Drive unit
2 Operating unit
3 Hydraulic pump
4 Hydraulic motor



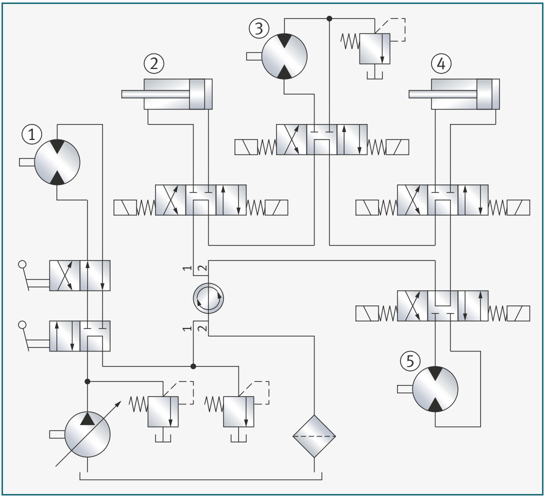
Hydraulic oil systems

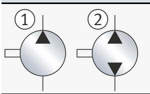
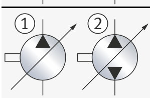
The following tables give an overview of symbols and names in hydraulic oil systems. The hydraulic symbols correspond to DIN ISO 1219-1:2019 “Fluid power systems and components – Graphic symbols and circuit diagrams”.

The associated diagram shows an example of a complete, hydraulic oil system. The idle position of the system is always shown.

Figure 5
Industrial crane

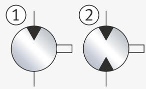
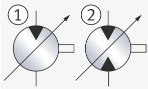
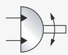
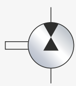
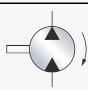
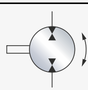
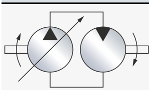
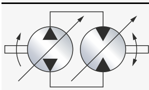
① Travel
② Whipping
③ Lift
④ Push
⑤ Rotate



Symbol	Name and explanation
Hydraulic pump	
	**Pump** With constant displacement volume ① With one flow direction ② With two flow directions
	**Pump** With variable displacement volume ① With one flow direction ② With two flow directions

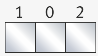
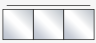
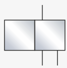
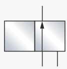
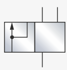
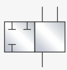
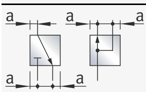
Continuation of table, see Page 180.

Continuation of table, Hydraulic oil systems, from Page 179.

Symbol	Name and explanation
Hydraulic motor	
	**Motor** With constant displacement volume 1 With one flow direction 2 With two flow directions
	**Motor** With variable displacement volume 1 With one flow direction 2 With two flow directions
	**Swivel motor** (with restricted swivel angle)
Hydraulic pump – hydraulic motor	
	**Pump motor** With constant displacement volume As pump in one flow direction As motor in the opposing direction
	**Pump motor** With constant displacement volume As pump or motor in one flow direction
	**Pump motor** With constant displacement volume As pump or motor in two flow directions
Compact hydraulic transmission	
	**Transmission** For one direction of output rotation with adjustment and constant motor for one delivery direction
	**Transmission** For two directions of output rotation with variable pump and variable motor for two delivery directions

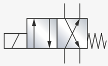
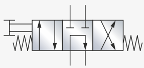
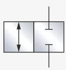
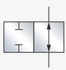
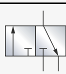
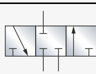
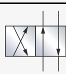
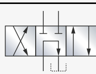
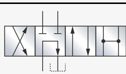
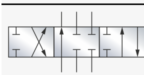
Continuation of table, see Page 181.

Continuation of table, Hydraulic oil systems, from Page 180.

Symbol	Name and explanation
Hydraulic valves (general)	
	The **valve** is represented by a square or rectangle
	Number of fields = number of valve settings, where the neutral position is arranged on the right if there are two boxes
	In the case of valves with a constant functional transition between the switching positions, the boxes are framed by two lines above and below
	The ports, respectively inlet and outlet, are attached to the neutral position box
	Within the boxes, the lines and arrows indicate the direction of flow
	A connection between two paths within a valve is indicated by a dot. Where lines cross but a dot is not present, this indicates that the paths are not connected to each other
	Closed ports are indicated by perpendicular bars
	The respective positions of the paths and arrows (angled or straight) within the boxes correspond to the positions of the ports
	If a position is changed and the inlet or outlet remains connected to a port, the arrow is displaced relative to the ports

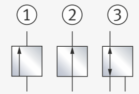
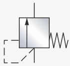
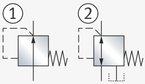
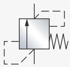
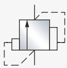
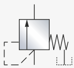
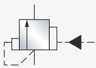
Continuation of table, see Page 182.

Continuation of table, Hydraulic oil systems, from Page 181.

Symbol	Name and explanation
Hydraulic valve actuation	
The symbols for the actuation modes and auxiliary elements are arranged perpendicular to the ports and outside of the rectangle (for further actuation modes, see Actuation and drive modes, Page 187). The valves are drawn in the current-free starting position.	
	**$^{4}/_{2}$-way valve** (valve with 4 ports and 2 switching positions) With magnetic coil actuation and spring return
	**$^{4}/_{3}$-way valve** (valve with 4 ports and 3 switching positions) With manual actuation by pressing or pulling and spring centring at neutral position
Hydraulic directional control valves	
The description as a way valve is preceded by the number of ports and the number of switching positions; e.g. way valve with three controlled ports and two switching positions: 3/2-way valve (vocalised: three stroke two way valve).	
	**$^{2}/_{2}$-way valve** Locked in neutral position
	**$^{2}/_{2}$-way valve** With free flow in neutral position
	**$^{3}/_{2}$-way valve** Flow shut off in neutral position
	**$^{3}/_{3}$-way valve** With shut-off neutral position, forward and reverse settings
	**$^{4}/_{2}$-way valve** With forward and reverse settings
	**$^{4}/_{3}$-way valve** With recirculating neutral position, forward and reverse settings
	**$^{4}/_{4}$-way valve** As 4/3, but with floating position after forward setting
	**$^{6}/_{3}$-way valve** In neutral position, 1 inlet free, 2 inlets locked

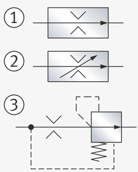
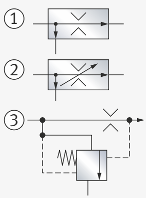
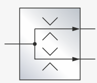
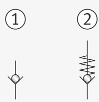
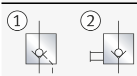
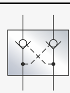
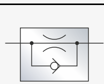
Continuation of table, see Page 183.

Continuation of table, Hydraulic oil systems, from Page 182.

Symbol	Name and explanation
Hydraulic pressure valves	
	**Pressure valve (general)** ① Single edge valve with closed neutral position ② Single edge valve with open neutral position ③ Double edge valve, three controlled ports
	**Pressure relief valve** Pressure limited at inlet by opening the outlet against a return force
	**Pressure control valve** Maintenance of constant outlet pressure ① Without outlet port = pressure regulating valve ② With outlet port = pressure reducing valve
	**Pressure drop valve** Reduction of outlet pressure by a fixed amount compared to the inlet pressure
	**Proportional pressure valve** Reduction of outlet pressure in a fixed ratio to the inlet pressure
	**Sequence valve** Opening of path to further devices upon reaching the inlet pressure defined by the spring force
	**Proportional pressure relief valve** Restriction of inlet pressure to a value proportional to the pilot pressure
Hydraulic flow control valves	
	**Choke** Valve with integral, constant constriction; flow rate and pressure drop are dependent on viscosity
	**Orifice** Sharp-edged constriction, substantially independent of viscosity and effective in both directions
	**Choke valve** Adjustable constriction, effective in both directions

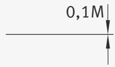
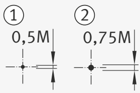
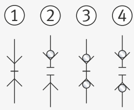
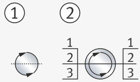
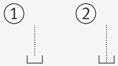
Continuation of table, see Page 184.

Continuation of table, Hydraulic oil systems, from Page 183.

Symbol	Name and explanation
Hydraulic flow control valves	
	**2-way flow control valve** ① 2-way flow limiting valve ② 2-way flow adjustment valve ③ Flow control valve maintains constant outlet flow by automatic closure
	**3-way flow control valve** ① 3-way flow limiting valve ② 3-way flow adjustment valve ③ Flow control valve maintains constant outlet flow by automatic opening of outlet (bypass valve)
	**Flow divider** Valves for dividing or combining several outlet or inlet flows. Substantially independent of pressure
Hydraulic shut-off valves	
	**Shut-off valves** Shut-off of flow in one direction and release of flow in the opposing direction ① Check valve: shut-off if the outlet pressure is greater than the inlet pressure, ② Shut-off if the outlet pressure is greater than or equal to the inlet pressure (with spring)
	**Check valve** ① Shut-off can be deactivated ② Flow can be shut off
	**Delockable double check valve** With 2 check valves for 2 separate flows, automatic locking of which is alternately deactivated by the inlet pressure
	**Choke check valve** With flow in one direction and choke in the other direction

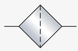
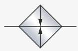
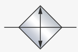
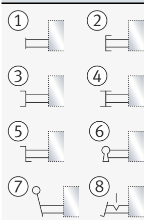
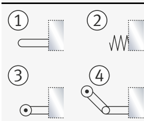
Continuation of table, see Page 185.

Continuation of table, Hydraulic oil systems, from Page 184.

Symbol	Name and explanation
Hydraulic pipelines and accessories	
	**Working pipeline** Pipeline and energy transfer
	Control pipeline, oil leakage pipeline, venting pipeline, flushing pipeline For transfer of control energy, for setting and control, for discharge of leaking fluids
	**Flexible pipeline** Pipeline being flexible in operation, rubber hose, corrugated tube etc.
	**Pipeline connection** Rigid connection, for example welded, soldered or connected by screws (including fittings) 1 Within a symbol 2 Outside a symbol
	**Pipeline crossover** Crossover of pipelines that are not connected with each other
	**Quick release coupling** 1 Connected without mechanically opened check valve 2 Disconnected, with one check valve 3 Connected, with two check valves 4 Disconnected, with two check valves
	**Rotating connection** Pipeline connection that can be rotated during operation, for example a pivot 1 With one path 2 With three paths
	**Vent** Without connection option
	**Reservoir** Open, connected to the atmosphere 1 With pipe end above the fluid level 2 With pipe end below the fluid level
	**Hydraulic reservoir** For storage of hydraulic energy

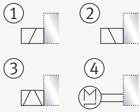
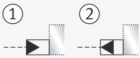
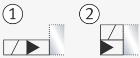
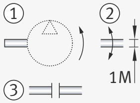
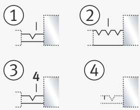
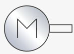
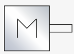
Continuation of table, see Page 186.

Continuation of table, Hydraulic oil systems, from Page 185.

Symbol	Name and explanation
Hydraulic pipelines and accessories	
	**Filter or mesh** For separation of contaminant particles
	**Heating system** Arrows indicate the supply of heat
	**Cooling system** Arrows indicate the dissipation of heat
	**Manometer**
	**Thermometer**
	**Pressure switch** Electromechanical, adjustable
	**Shut-off valve**
Actuation and drive modes	
	**Manual actuation modes** ① General ② By pressing ③ By pulling ④ By pressing and pulling ⑤ By rotating ⑥ By key ⑦ By lever ⑧ By pedal with detent
	**Mechanical actuation modes** ① By plunger ② By spring ③ By sensing roller ④ By roller lever

Continuation of table, see Page 187.

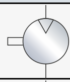
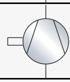
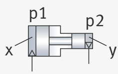
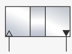
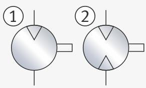
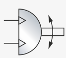
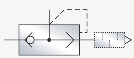
Continuation of table, Hydraulic oil systems, from Page 186.

Symbol	Name and explanation
Actuation and drive modes	
	**Electric actuation** ① Magnetic coil, one winding, effective direction towards the adjustment element ② Magnetic coil, one winding, effective direction away from the adjustment element ③ Two mutually acting windings ④ Stepper motor
	**Pressure actuation (direct)** ① By application of pressure ② By withdrawal of pressure
	**Pressure actuation (indirect)** ① By application of pressure on control line ② By withdrawal of pressure on control line
	**Combined actuation** ① Magnetic coil and pilot directional valve ② Magnetic coil or pilot directional valve
	**Shaft** ① Pump drive shaft in one direction of rotation (right hand rotation from viewpoint of shaft end) ② In two directions of rotation ③ With shaft coupling
	**Detent** ① 1 lockable engaged position ② 3 lockable engaged positions ③ 4 lockable engaged positions ④ No lockable engaged position
	**Pressure source** Hydraulic energy
	**Pressure source** Pneumatic energy
	**Electric motor** with almost constant speed
	**Heat engine**

Pneumatics

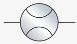
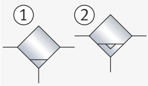
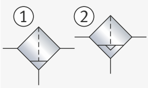
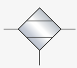
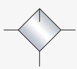
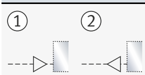
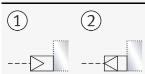
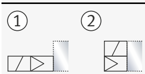
Pneumatic systems

The following table gives an overview of symbols and names in pneumatic systems.

Symbol	Name and explanation
Pneumatics – compressor	
	**Compressor** With constant displacement volume, only one flow direction
	**Vacuum pump** For removing gaseous media from a low pressure area
	**Pressure converter** Comprising two different pressure chambers x and y
	**Pressure medium converter** Changeover from one pressure medium to another while maintaining the same pressure
Pneumatics – motor	
	**Pneumatic motor** With constant displacement volume 1 With one flow direction 2 With two flow directions
	**Swivel motor**
Pneumatic valve	
	**Quick ventilation valve** Shut-off valve in which the outlet line is vented to the outside when the inlet line is in the vented stage (with noise damper in the outlet pipeline)
The remaining symbols are identical to those used for hydraulic oil systems	

Continuation of table, see Page 189.

Continuation of table, Pneumatic systems, from Page 188.

Symbol	Name and explanation
Pneumatic pipelines and accessories	
	**Noise damper** For reduction of resulting noise
	**Compressed air reservoir**
	**Flow measurement device** (also for use with hydraulic systems) Volume flow measurement device
Maintenance unit	
	**Water separator** Separation and drainage of condensation water from the system ① Manually actuated ② Automatic drainage
	**Filter** With water separator ① Manually actuated ② Automatic drainage
	**Vent** Without connection option
	**Dryer** For drying of air by means of chemicals
	**Oiler** For adding a small quantity of oil to the air flow
Actuations	
	**Pressure actuation (direct)** ① By application of pressure ② By withdrawal of pressure
	**Pressure actuation (indirect)** ① By application of pressure ② By withdrawal of pressure
	**Combined actuation** ① Electromagnet and directional valve ② Electromagnet or directional valve

Mechatronics

Terms and definitions

Mechatronics

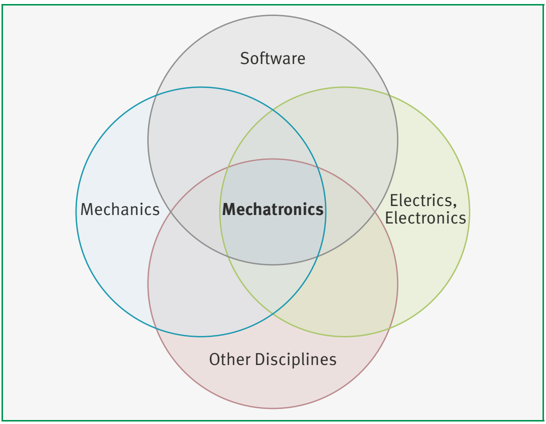
The development and production of mechatronic products involves an integrated partnership between the disciplines of mechanics, hydraulics, pneumatics, electrical engineering and electronics, control technology and information technology.

While mechatronics was defined in its earliest days (approx. 1970) as simply the integration of mechanics and electronics, the term now has a wider scope:
"Mechatronics is a branch of engineering which seeks to enhance the functionality of technical systems by close integration of mechanical, electronic and data processing components."
(Bosch, Automotive Handbook 1995).

A similar definition was issued by the IEEE/ASME in 1996 and was also used as a basis for VDI Guideline 2206 "Development of mechatronic and cyber-physical systems":
"Mechatronics is the synergetic integration of mechanical engineering with electronic and intelligent computer control in the design and manufacturing of industrial products and processes."

Figure 1
Structure of mechatronic systems

Source: VDI/VDE Guideline 2206:2021



Mechatronics can therefore achieve synergies through the integration of mechanical engineering, electrical engineering and information technology, as well as other disciplines if necessary, leading to improvements in the functionality of technical systems.

The spectrum of mechatronic products extends from simple implements through household appliances ("white goods, brown goods"), automotive engineering and medical technology to aviation and aerospace engineering.

Technical system

A technical system consists of subsystems or elements that are interconnected by relationships and segregated from the environment by a system boundary (enveloping surface), giving rise to input and output variables in the form of energy, material and information. For comparisons, refer to Figure 2, Page 192.

Technical process

A technical process converts energy, material and information in a technical system, whereby the temporal sequence is characterised by states.

Materials, energy and information

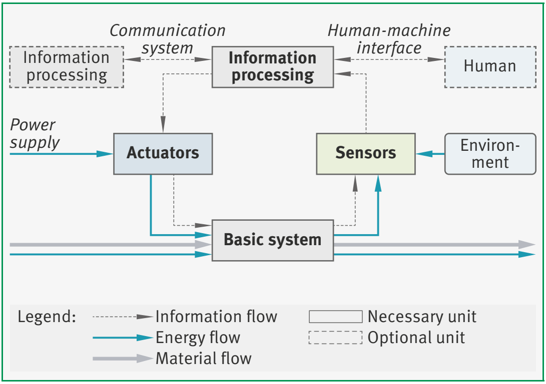
Raw products, substances, components, gases and liquids are examples of materials. Energy can be made available in a number of forms, including mechanical, thermal, electrical and chemical. Information can take the form of a signal, a measurement value, various data or a control pulse.

Mechatronic system

Mechatronic systems are technical systems which have the special feature, as illustrated in Figure 1, Page 190, of combining mechanical engineering, electrical engineering and information technology as well as other disciplines where applicable.
If the mechatronic system is regarded as a "black box", it can be represented as follows as a reference architecture:

Figure 2
Reference architecture of a mechatronic system

Source: VDI/VDE Guideline 2206:2021



The input and output values are material, energy and information flows. The technical component, the basic system (or even the technical process), represents the supporting basis system in practical terms and essentially comprises elements taken from mechanics, electrical engineering, hydraulics and pneumatics. It is controlled on the open loop or closed loop principle by means of the information processing component. The sensors convert the signals from the technical component into information elements for the information processing component and the actuators intervene by modifying the parameters of the technical component.

Depending on the complexity of the system, the following tasks can be undertaken in mechatronic systems:

- open loop control, closed loop control
- monitoring and error diagnosis
- coordination
- management

They thus cover the entire spectrum ranging from reactive to cognitive activities.

Cyber-physical system VDI Guideline 2206 defines the term cyber-physical system (CPS) as a “system based on a mechatronic core that is characterized by its networking with the Internet of Things and Services, which enables, for example, changes in behaviour or properties during operation”. The basis is, therefore, a mechatronic system that can communicate beyond the system boundary.
This intensified networking of technical systems and the connection to the Internet of Things opens up further potential in relation to the digitalisation of products, processes and associated services, but also comes with increased responsibility in terms of cyber security.

Sensors

Main task In the context of mechatronics, a sensor is defined as a device that records and provides information about the current properties of the basic system and the environment. A sensor thus records all characteristic, physical measurement values and usually converts these into proportional electric signals, in line with requirements, which are fed to the information-processing component.

A non-electrical quantity is usually recorded at the sensor input, converted into intermediate quantities by transformers where appropriate (also repeatedly) and converted into the primary analog electrical quantity by a converter. Depending on the integration level of the sensor, the primary analog electrical quantity directly represents the output signal, or is subject to further processing by evaluation units and, where necessary, converted into a digital output signal by an analog-to-digital converter.

The sensors must therefore be selected on the basis of:

- the physical parameter to be recorded
- the measuring range to be mapped
- the required accuracy
- any pre-processing required
- the required output signal

Working principles Established sensors are commonly based on three basic working principles established in metrological practice. Resistive sensors detect changes in electrical resistance, while the basis of inductive sensors is on measuring changes in magnetic resistance. By contrast, the function of capacitive sensors is based on measuring changes in electrical capacitance.

There are also other sensors which use optical, piezoelectric or pyroelectric effects, to cite just a few examples.

Examples The following table, which does not claim to be exhaustive, provides an overview of established sensors categorised according to their measurement value (input) and sensor output signal (output).

Measurement value (input)	**Sensor output signal (output)**					
	Electric resistance			**Voltage U**	**Current I**	**Charge Q**
	Resistive R	**Inductive L**	**Capacitive C**			
Elongation $\epsilon = \Delta l / l_0$	Strain gauge	–	–	–	Fibre-optic sensor	–
Position: Length l Displacement s Angle φ	Potentiometer, magneto-resistive sensor, Gaussian field plate	Differential transformer, plunger-type displacement sensor	Capacitive displacement sensor	Hall sensor, optoelectronic light barrier sensor	Eddy current sensor	–
Velocity $v = ds/dt$	Magneto-resistive rotational angle sensor	Inductive rotational angle sensor	–	Magnetic pole speed sensor	Opto-electronic speed sensor	Gyrometer/ piezo sensor
Acceleration $a = dv/dt$	Seismic sensor: mass-damper-spring system, retransmission to displacement measurement: resistive, inductive, capacitive, optoelectronic, piezoresistive; Hall/Gaussian sensor technology or strain measurement (strain gauge)					
Force F Moment $F \cdot l$	Piezoresistive sensor, strain gauge sensor	Magneto-elastic sensor	Force compensation sensor	Spring elements or strain gauges, retransmission to: displacement(s) measurement with $F = f(s)$, or elongation(ϵ) measurement with $F = f(\epsilon)$		Piezo electric sensor
Pressure $p = \text{force}/\text{area}$	Piezoresistive sensor, strain gauge sensor	Magneto-elastic sensor	Capacitive pressure sensor	Spring elements or strain gauges, retransmission to: displacement(s) measurement with $F = f(s)$, or elongation(ϵ) measurement with $F = f(\epsilon)$		Piezo electric sensor
Temperature T	NTC and PTC resistance	–	–	Thermocouple	Opto-electronic pyrometer	–
Humidity $f_{abs} \cdot f_{rel}$	Resistive hygrometer	–	Capacitor, hygrometer	–	–	–

Source: Horst Czichos, Mechatronik, Grundlagen und Anwendungen technischer Systeme, 4. überarbeitete und erweiterte Auflage.
Stephan Rinderknecht, Rainer Nordmann, Herbert Birkhofer, Einführung in die Mechatronik für den Maschinenbau, 2. überarbeitete Auflage, Skripte im Shaker Verlag, Shaker Verlag, Aachen, 2018.

Information processing

Main task

Information processing generally represents the link between sensor technology and actuator technology. The sensor signal is processed within a process computer and a control signal is output for the actuator. The process computer is a programmable, electronic hardware component that provides the entire system with a certain level of artificial intelligence. A distinction is made here between open loop control and closed loop control of the system. A human-machine interface is required in order to allow users to influence the mechatronic system during operation. In the cyber-physical system, an interface is also possible without human interaction.

Open loop control and closed loop control

Open loop control
In open loop control, the control signal output to the actuator is directly influenced by the reference variable without feedback via sensors. Therefore, in open loop control, the output variable of the basic system is not checked against its desired reference variable. Inaccuracies in the mapping of system behaviour or the occurrence of external disturbances render open loop control imprecise.

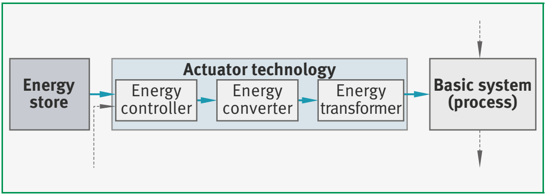
Closed loop control
In closed loop control, the output variable of the basic system is returned via the sensors in a process known as feedback, creating a closed action flow. A nominal/actual comparison with the reference variable enables a corresponding continuous adjustment of the control variable. This allows a closed loop control system to compensate for unknown disturbances, parameter fluctuations and model uncertainties and ensure stable operation.

Actuators

Main task In the context of mechatronics, an actuator is defined as a device that specifically influences state variables in mechatronic systems. The actuator represents the link between information processing and the basic system to be influenced. The often digital, low-power control signal from information processing may first need to be converted into an analog signal.

Depending on the application, actuator technology can include several functional elements, but as a minimum the energy controller, see Figure 3:

Figure 3
Structure of an actuator



Energy controller The actuator is activated by the low-power control variable via the energy controller. The higher energy required is provided by powerful energy stores, as determined by the control variable at the energy controller output. Examples of energy controllers include transistors, thyristors and valves. The energy provided by the energy store can be in hydraulic, pneumatic, electrical, mechanical or other form.

Energy converter Depending on requirements, the energy provided by the energy controller will need to be converted into another form of energy by the energy converter. For example, electric motors convert electrical energy into mechanical rotational energy and hydraulic cylinders convert hydraulic energy into mechanical translational energy.

Energy transformer The energy obtained from the energy converter can be further adapted to process-related requirements using energy transformers of the same type, thus allowing special requirements such as short travel ranges for large loads (so-called force actuators) and large travel ranges for small loads (so-called travel actuators) to be realised. Examples of mechanical converters include gearboxes, spindles and levers.

Examples The following table, which does not claim to be exhaustive, provides an overview of the established functional elements of actuators.

Working principle		Functional element
Electromechanical	Electromagnetic	■ Electromagnet ■ Reluctance motor
	Electrodynamic	■ Plunger coil ■ Direct current motor ■ EC motor ■ Synchronous and asynchronous machine
Hydromechanical	Pneumatic	■ Pneumatic cylinder ■ Radial and axial piston motor ■ Multi-disc motor
	Hydrostatic and hydrodynamic	■ Hydrostatic motor ■ Hydraulic cylinder ■ Hydrodynamic motor, fluid motor ■ Bent-axis motor ■ In-line motor ■ Vane motor ■ Radial piston motor
Material-mechanical	Thermomechanical	■ Shape memory alloy ■ Bimetal ■ Expansion element
	Electromechanical	■ Piezo crystal ■ Electroactive polymer ■ Electrorheological fluid
	Magnetomechanical	■ Ferromagnetic crystal ■ Magnetic shape memory alloy ■ Magnetorheological fluid
	Chemomechanical	■ Fuel cell ■ Oxygen pump ■ Polymer gel ■ Cross-linked polymers

Source: Horst Czichos, Mechatronik, Grundlagen und Anwendungen technischer Systeme, 4. überarbeitete und erweiterte Auflage.
Stephan Rinderknecht, Rainer Nordmann, Herbert Birkhofer, Einführung in die Mechatronik für den Maschinenbau, 2. überarbeitete Auflage, Skripte im Shaker Verlag, Shaker Verlag, Aachen, 2018.

Development process

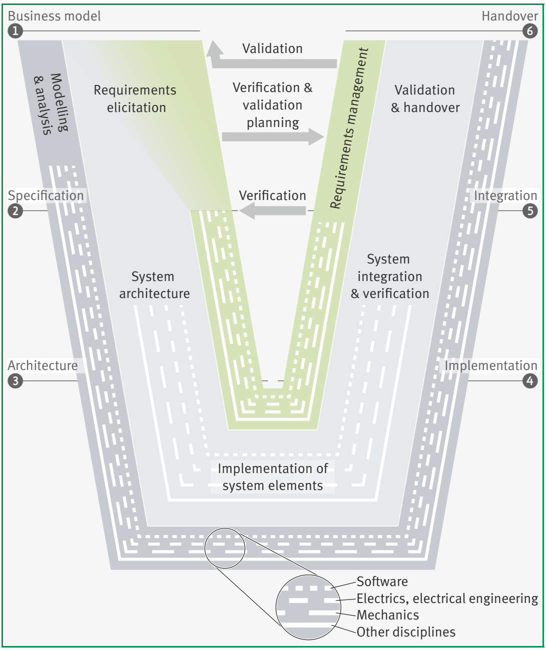
The development of mechatronic and cyber-physical systems requires ways of thinking and working that are holistic and interdisciplinary.

Since the development methods and processes in the individual disciplines will demonstrate some similarities but will also show differences in decisive respects, it is vital to achieve good communication in the interdisciplinary team in order to ensure an effective method of working across the disciplines. A decisive role is played here by the issues of modelling and orientation to the functional concept.

In VDI Guideline 2206, a process model in a V shape is presented for the development of mechatronic and cyber-physical systems. It should be viewed as a framework that can be adapted to the respective corporate structure and describes the linking of interdisciplinary product development tasks in iterative terms during the design stage for the purpose of ensuring product properties.

Figure 4
V model for development of mechatronic and cyber-physical systems

Source: VDI/VDE Guideline 2206:2021



In relation to the quality of the resulting products, it has become clear that there are three definitive aspects: the integration of the technical disciplines, a holistic approach and correct modelling (as a basis for simulation of the complete system).

It is important to prepare a process landscape in which mechatronic engineers can apply the most suitable method. Employees in such a team must be able to “see the big picture” and see themselves as system architects who can, as necessary, draw on the competences of the technical specialists.

Structure of the V model

The V model is divided into three strands, which must be processed simultaneously and are closely linked.

Middle strand of the V model

The middle strand represents the system development stage complete with the iterative and partially overlapping core tasks:

- Elicit requirements.
- Develop system architecture.
- Implement system elements.
- Consolidate and verify subsystems to form the overall system.
- Validate and hand over the system.

Inner strand of the V model

The inner strand describes the requirements development stage comprising requirements elicitation and requirements management. The purpose of requirements management is to analyse, structure, assign and integrate changes in requirements across the entire development project.

Outer strand of the V model

The outer strand is concerned with modelling and analysis of the system and various subsystems. The objective of modelling is to create mathematical and descriptive substitution models of the system, subsystems or system elements, which describe the structure and behaviour with sufficient accuracy.

It is important that powerful software tools are provided that can be used, by means of simulations, to secure the necessary characteristics in the various stages of development.

Modelling and simulation

Main task

The objective of modelling is to analyse and/or predict the actual system in question and/or the behaviour of the system in question, and thereby obtain important information as early as the design phase regarding the optimal design of the system or replacement of components.

Modelling involves converting an actual (existing or intended) system into an abstract image (model). As an abstraction, a model is always a simplified representation of an actual system. The principle “as simple as possible, as complex as necessary” should be applied here. In other words, whilst it is necessary to map all relevant physical effects in order to adequately predict actual system behaviour, the numerical effort needed to generate the simulation must remain manageable.

Theoretical modelling

In theoretical modelling, the system is described using mathematically formulated physical laws – usually in the form of ordinary and partial differential equations.

The most frequently required laws include:

- Newton’s axioms
- Leverage laws
- Fundamental laws of thermal dynamics
- Ohm’s law
- Kirchhoff’s laws

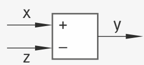
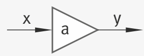
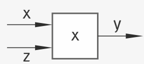
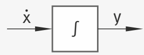
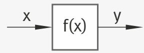
The theoretical modelling process is divided into three stages. First, the actual system must be converted into a limited, simplified substitute system. The static and dynamic behaviour of the system is then described using the differential equations. Once the differential equations have been concurrently solved, the actual behaviour of the system can be simulated.

Block diagrams

The causal relationships between a system and its environment or between subsystems are best described using functional diagrams. In practice, this interlinking of differential equations is commonly represented in symbolic form as a block diagram, in which the characteristic transfer function between the input variables (cause) and output variables (effect) is represented as a simple block for each of the elements in question. The individual blocks are linked to each other via their input and output variables. Block diagrams are a widely used form of representation in the field of engineering and encountered in many numerical solution tools such as MATLAB®/SIMULINK® and LabVIEW™.

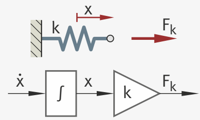
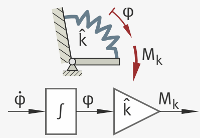
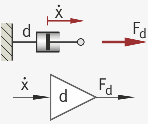
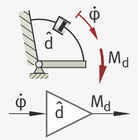
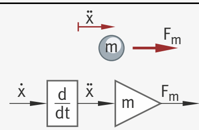
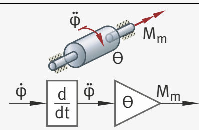
Symbols for standard operators

The following table shows the symbolic representation for standard operators, which can be used to build differential equations and thus system elements in the block diagram.

Operation	Mathematical formulation	Symbolic representation
Addition and subtraction (also with more than two outputs)	$y = x - z$	
Multiplication (multiplier/gain) with constant expression a	$y = x \cdot a$	
Multiplication of two variables	$y = x \cdot z$	
Integration over time	$y = x$	
Any functions (with constant expressions or multiple inputs)	$y = f(x)$	

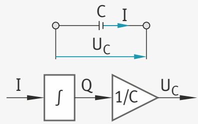
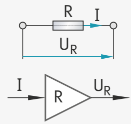
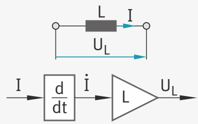
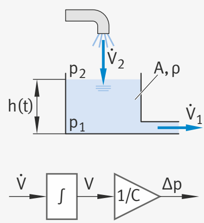
Equivalent circuit diagrams for system components

The following tables show common equivalent circuit diagrams for the various system components from the fields of mechanics, electrical engineering, hydraulics and pneumatics:

Component	Physical/mathematical value	Mathematical formulation	Symbolic representation
Mechanical system components			
Spring	Elasticity (store)	$F_k = k \cdot x = k \cdot \int \dot{x} \cdot dt$	
Torsion spring		$M_k = \hat{k} \cdot \varphi = \hat{k} \cdot \int \dot{\varphi} \cdot dt$	
Damper	Resistance (drain)	$F_d = d \cdot \dot{x}$	
Rotational damper		$M_d = \hat{d} \cdot \dot{\varphi}$	
Mass	Inertia (store)	$F_m = m \cdot \ddot{x} = m \cdot \dfrac{d\dot{x}}{dt}$	
Rotating mass		$M_m = \theta \cdot \ddot{\varphi} = \theta \cdot \dfrac{d\dot{\varphi}}{dt} \int \dot{\varphi} \cdot dt$	

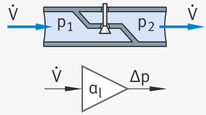
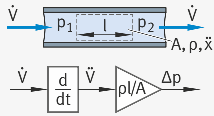
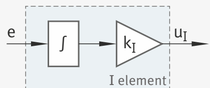
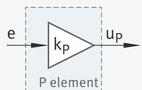
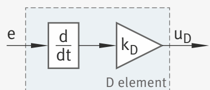
Continuation of table, see Page 203.

Continuation of table, Equivalent circuit diagrams for system components, from Page 202.

Component	Physical/mathematical value	Mathematical formulation	Symbolic representation
Electrical system components			
Condenser	Capacitance (store)	$U_C = \frac{1}{C} \cdot \int I \cdot dt = \frac{1}{C} \cdot Q$	
Ohmic resistance	Resistance (drain)	$U_R = R \cdot I = R \cdot \dot{Q}$	
Coil	Inductance (store)	$U_L = L \cdot \dot{I} = L \cdot \ddot{Q}$	
Hydraulic and pneumatic system components			
Reservoir	Capacitance (store)	Weight accumulator: $(p_1 - p_2) \cdot A = \rho \cdot g \cdot A \cdot h$ $\dot{V} = (\dot{V}_2 - \dot{V}_1) = A \cdot \dot{h}$ $\Delta\dot{p} = \dot{p}_1 - \dot{p}_2$ $= \frac{\rho \cdot g}{A} \cdot \dot{V} = \frac{1}{C_{hyd}} \cdot \dot{V}$ General pressure accumulator: $\Delta p = p_1 - p_2$ $= \frac{1}{C_{hyd}} \cdot \int \dot{V} \cdot dt$ $C_{hyd} = \frac{dV}{d\Delta p}$	

Continuation of table, see Page 204.

Continuation of table, Equivalent circuit diagrams for system components, from Page 203.

Component	Physical/mathematical value	Mathematical formulation	Symbolic representation
Hydraulic and pneumatic system components			
Choke, orifice	Resistance (drain)	Linear case: $\Delta p = p_1 - p_2 = \alpha_l \cdot \dot{V}$ Nonlinear case: $\Delta p = p_1 - p_2 = \alpha_{nl} \cdot \dot{V}^n$	
Fluid mass	Inertia (store)	$(p_1 - p_2) \cdot A = \rho \cdot l \cdot A \cdot \ddot{x}$ $= \rho \cdot l \cdot \ddot{V}$ $\Delta p = p_1 - p_2$ $= \frac{\rho \cdot l}{A} \cdot \ddot{V} = L_{hyd} \cdot \ddot{V}$	
Controller			
I element of the controller	Integral coefficient k_I	$u_I = k_I \cdot \int e \cdot dt$	
P element of the controller	Amplification factor k_P	$u_P = k_P \cdot e$	
D element of the controller	Differential coefficient k_D	$u_D = k_D \cdot \dot{e}$	

In addition to the function-oriented schematic representation, the preceding tables illustrate the translation of the mathematical description into block diagrams using basic operations for important system components, see table Symbols for standard operators, Page 201. All other elements, such as sources (for example batteries) and converters (for example transmissions) etc. can also be represented accordingly.

Motivation and indicators for use

The motives and reasons for the implementation of mechatronic systems can be as follows:

- functional transfer, in other words the distribution at optimum cost of the main function to various disciplines
- the implementation of new functions
- increasing the precision of motion under real time conditions
- robustness against mechanical malfunctions
- capability of adaptation to changes in environmental conditions
- autodiagnosis, autocorrection
- improvement of operational security
- increasing autonomy
- automatic learning
- compensation of mechanical deficiencies

In spite of the advantages stated, the blind use of components from different technical disciplines can lead to a more complicated problem and a more expensive solution. Mechatronic solutions should be applied in such a way that they give an increased benefit.

Examples of mechatronic systems

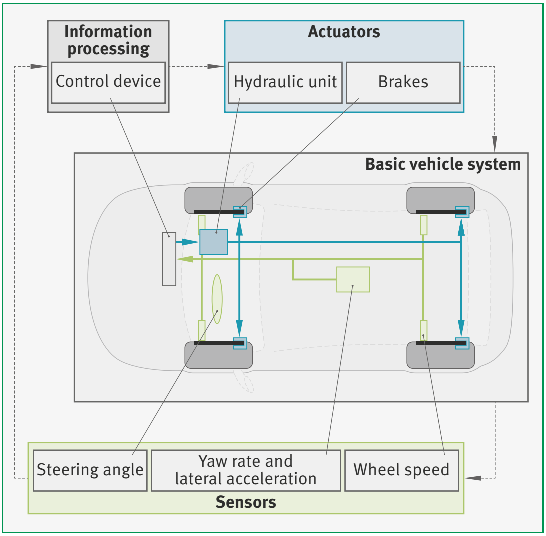
ABS and ESP

The anti-lock braking system (ABS) system ensures, through the interaction of mechanics, electronics and software, that a vehicle retains steerability and driveability under braking. Locking of the wheels is prevented.

Like anti-lock braking systems, electronic stability programs (ESP) are practically standard equipment in modern cars. They ensure stable and thus safe driving around bends, since braking is applied in a targeted manner to the individual wheels independently of the driver order to counteract swerving of the vehicle.

The vehicle is the complete system and also acts as the supporting technical component (basic system), see Figure 5.
The current state of the vehicle is detected by means of sensors measuring the wheel speed, yaw rate, lateral acceleration, steering wheel angle and by the pre-pressure sensor. An electronic control device (here, information technology component) processes these data and, if necessary, sends control signals to the hydraulic unit (actuator).

Figure 5
ABS and ESP as an example of a mechatronic system in a passenger car



Magnet bearings

These bearings are suitable for extremely high speeds (in applications such as machine tool spindles) where conventional bearing arrangements are susceptible to failure. The rotor is held without contact and without friction in a magnetic field. The bearing forces are held in equilibrium by means of magnetic forces and these are controlled so that the rotor floats in a stable manner.

Mechatronics in rolling bearings

Functional expansion of rolling bearings

Rolling bearings are essentially designed to facilitate rotations about an axis or displacements along an axis with little friction, but prevent certain other movements through the support of forces and moments. In the course of development, bearing components have been designed such that they can take on additional mechanical functions such as guidance, support or mounting, such as in the case of flanged bearings.

The integration of sensors and actuators offers the possibility of expanding even further the functional scope of bearings.

Sensors detect operating parameters and bearing conditions and forward this information, normally in the form of electrical signals.

The values detected include:

- rotational angle (rotary bearings)
- position (linear bearings)
- velocity and rotational speed
- axial and radial force
- torque and tilting moment
- temperature
- lubricant condition
- vibrations, running noise
- wear, damage

In combination with a rolling bearing, actuators can fulfil the following functions:

- driving
- braking, locking
- setting of operating clearance or rigidity
- damping of vibrations
- relubrication

Inversely, generators can also convert mechanical energy into electrical energy. In many cases, the motion of the bearing can act as a source here.

In order that the mechatronic bearing can fulfil its task, the sensor and actuator functions must be matched to each other.
The operating parameters detected by the sensor system are electronically processed in a closed loop control system according to their programming and the actuator is initiated. Nowadays, closed loop control is mostly achieved by means of microcontrollers; some of these are so small that they can even be integrated in the bearing.

If the microprocessor is controlling a motor in a rolling bearing, this can induce acceleration or deceleration in the direction of motion of the bearing and influence the system state in the required direction via the closed control loop (such as speed stabilisation).

The objective is to create a mechatronic unit in which the components are optimally matched to each other, in order to make best possible use of the advantages of mechanical and electronic components.

In contrast to solutions comprising individual components that have to be installed retrospectively, mechatronic rolling bearings offer the advantages of integrated solutions, such as reduced design space, easier assembly and a smaller number of components.

Sensors in and on the bearing

Rolling bearings are generally fitted with sensors in order to fulfil one of the following three objectives:

- Detection of actual operating conditions such as loads, shocks and temperature.
  The detected data are used in order to achieve well-grounded bearing design for identical or similar applications. Such measurement initiatives often prevent unexpected bearing damage or failures. Once the load conditions have been clarified, the design can be adapted. Further measurements are normally no longer required in regular operation.
- Continuous condition monitoring of the rolling bearing and other adjacent machine elements such as gears.
  The sensors generally record several measurement values relevant to bearings. Automatic signal assessment is often carried out by means of defined algorithms. If necessary, an optical or acoustic alarm is triggered for the operating personnel.
- Online measurement of operating data for the open or closed loop control of the system in which the bearing is fitted.
  Applications range from electric motors (commutation) through machine tools (load determination), vehicles such as cars (ABS, ESP, steer-by-wire), e-bikes (pedal torque measurement) or railways (braking control) to stationary production plant (overload protection), robots (collaboration, safety functions, force-/torque-controlled processes) and household appliances.

Matching of measurement values and measurement purpose

Depending on the purpose of the measurement, different physical measurement values are recorded for the bearing. The sensors required for this measurement are selected accordingly. The table shows the relationship between the measurement values and purpose.

Measurement value	Purpose		
	Design	Condition monitoring	Closed loop control of complete system
Position, velocity	–	■	■
Axial and radial force	■	–	■
Torque and tilting moment	■	–	■
Temperature	■	■	■
Lubricant condition	–	■	–
Vibration, noise	–	■	–
Wear, damage	–	■	–

Examples of measurement methods for some selected values that play a role in rolling bearing technology are given below.

Measurement of position and velocity

Recording of the rotational speed is, due to its use in robotics and in anti-lock braking systems in cars, the most widely used form of integrated sensor technology for rolling bearings. In general, it is used to determine the position and velocity of a moving bearing component relative to a stationary component.

A fundamental distinction is made between:

- Absolute positional measurement, where the relative position between fixed reference points is present as a datum at any point in time, in particular immediately after switching on.
- Incremental measurement, where primary information is generated on the basis of changes in position and direction.

If an index is added, an incremental measurement system can be developed further in the direction of an absolute measurement system. This requires appropriate signal processing, especially recognition of the direction of motion. An orientation movement is always required after switching on, however, in order to arrive at the index point.

The typical design of a rotary bearing comprises an encoder, the scale, applied to the rotating bearing ring. Its position is measured by one or more sensors connected to the stationary ring. The signal created is transmitted by cable. Such bearings are often greater in width than the original types but match these in terms of the other mounting dimensions and performance data, see Figure 6.

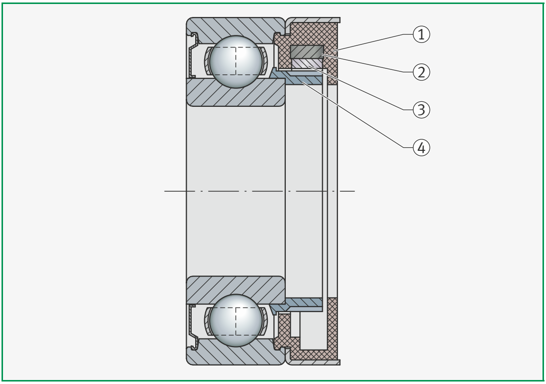


Figure 6
Rotary rolling bearing with measurement system

1 Sensor housing
2 Magnet
3 Hall IC (sensor)
4 Pulse emitter ring (encoder)

In the case of linear bearings, the encoder is normally applied to the profiled guideway as a linear scale. The sensor is mounted on the carriage. Encoders are used to measure the position and velocity of the carriage on the guideway, see Figure 7.

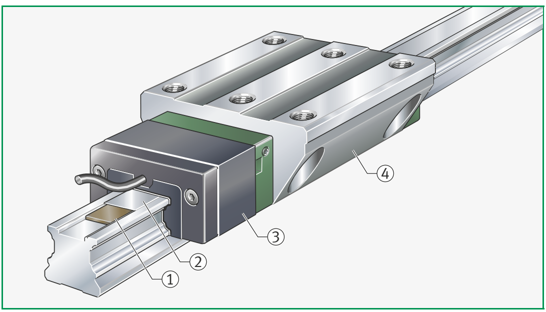


Figure 7
Linear rolling element guidance system with measurement system

1 Guideway with integrated linear scale
2 Covering strip
3 Adaptive measuring head
4 Carriage

Working principles

Sensors may use different working principles. For rolling bearings, the sensors used so far are optical, inductive, capacitive and magnetic types. By far the largest number of applications so far have been based on magnetic methods.

Measurement of force and torque

For determining rotational speed, some other components such as the shaft or gear are equally as suitable as the bearing. For measuring force, however, the bearing is often the only machine element that can be used.

Directly measurable forces

Where forces act and are presented with resistance by a rolling bearing, several effects occur that can in principle be used for measurement:

- An increase in the pressures at internal and external contact points.
- Stresses and thereby elongations at the location of particular volume elements and surface contours.
- Changes in distance between certain reference points. The bearing undergoes deflection.

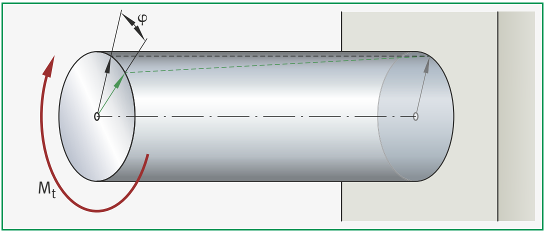
Torque determination

None of these effects occur, however, if a torque is generated along the rotational axis provided by a rotary bearing. Direct torque measurement within a bearing is therefore difficult.

In arrangements with two bearings at both ends of a shaft subjected to torsion by a torque, the difference in the bearing rotational angle can be used as a measurement value, see Figure 8. Gears ultimately convert the torque into reaction forces with the result that, in transmissions, the torque can be determined indirectly from the bearing forces.

Figure 8
Torsion of a shaft

φ = torsion angle
M_t = torsional moment



The method of measuring torque using the torsion angle, as described above, cannot be implemented in practice for shafts with higher torsional stiffness. Instead, various magnetic measuring methods can be used to measure the torque, which directly or indirectly record the shear stresses in a twisted component. The use of strain gauges or sensor coatings on, for example, a shaft is also conceivable.

Selection of the measurement method

For selection of the measurement method, one of the decisive questions is whether it must be possible to measure the forces while the bearing is stationary. If this is not necessary, signals can be used that vary in a periodic manner with the passage of rolling elements and whose amplitude can be used as an indicator of the force. Long term drift and temperature influence have a much less disruptive effect than in the other case.

Measurement of pressures by means of sensor coatings

At the points where forces are transmitted, namely shaft/inner ring, inner ring/rolling element, rolling element/outer ring and outer ring/housing, there is normally no space for sensors unless these are very thin. Piezoelectric and piezoresistive coating systems fulfil this requirement. The former generate electric charges if there is a change in the active force and therefore their thickness; the latter develop a different electric resistance as a function of the specific load. Both methods are suitable for dynamic measurements while piezoresistive systems are preferable for static measurements.

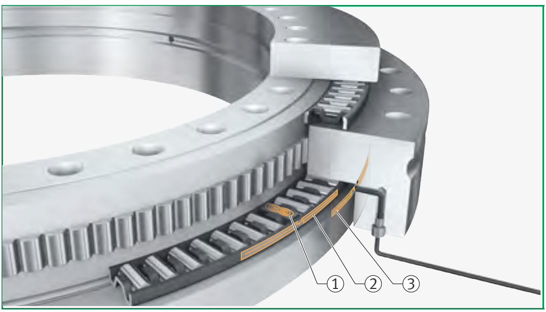
Condition monitoring

In addition to classical condition monitoring, bearings can also be subjected to structural health monitoring. Sensors for non-destructive inspection are integrated in the bearing.

One of these inspection methods is eddy current testing of the raceways, see Figure 9. The probes are located in one or more cage pockets and can check for cracks or spalling during rotation of the inner ring and outer ring. Supply of energy and transmission of signals is carried out wirelessly.

Figure 9
Inductive damage monitoring of rolling bearings

① Inductive sensor
② Antenna on cage
③ Antenna on outer ring



Actuator bearings

There is a wide range of operating principles that can be integrated in the rolling bearing or at least mounted on the bearing in a space-saving form.

Those with a purely mechanical function include:

- one way clutch for locking of one direction of rotation
- brake disc on the rotating ring and brake calliper on the static ring
- passive damping elements for reducing vibration

The use of active elements with a thermal or electrical activation function offers further possibilities for equipping bearings with additional functions. Some have already been realised, while others are still at the development stage.

The following table shows the relationship between the actuator principle and the possible applications.

Actuator principle	Application				
	Drive	Braking Locking	Positioning	Damping	Relubri-cation
Electric motor	■	■	–	–	■
Magnetic coil	–	■	■	–	■
Piezo element	■	■	■	■	■
Magnetostriction	■	■	■	■	■
Rheological liquid	–	■	–	■	–
Shape memory alloy	–	■	■	■	■
Dielectric polymer	■	■	■	–	■
Thermal expansion	–	■	–	–	–

Driving and braking

Where an electric motor is required to generate a particular torque, the rotor and stator must be of an appropriate size. If the bearing required for guidance of the shaft is significantly smaller than the electric motor, integration is not advisable.

Integration of electric motors

For the integration of electric motors, bearings with a relatively large diameter are more suitable if this is determined not as a result of high loads but of other design requirements. Such bearings are, despite their size, not particularly large in section or heavy.

They are used, for example, in conveying equipment and computer tomographs, see Figure 10. If the generation of torque is distributed to the circumference by means of a ring motor, solutions can be realised that give space savings and run quietly. Segmented motors, in which the winding is not required to span the entire circumference of the bearing, are also possible.

Figure 10
Bearing for computer tomographs with integrated ring motor

Positioning and damping

These two functions are similar in that only small actuation distances of between 1 μm and 100 μm are necessary, but in some cases considerable forces are involved. The setting of bearing parameters such as rigidity, preload or internal clearance does not require actuators that operate particularly quickly. In many cases, it is only necessary to compensate for temperature influences. In active vibration reduction, the actuator frequency response must however be matched to the excitation spectrum present.

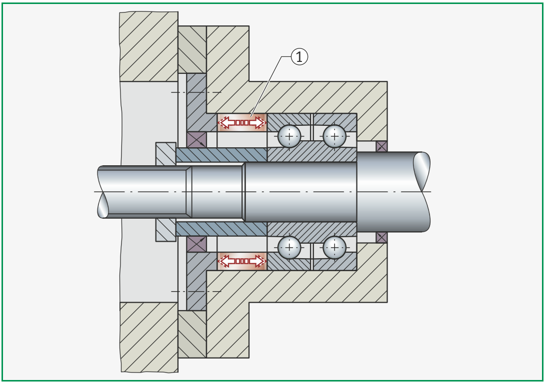
Change in preload

Spindle bearings are characterised by particularly precise shaft guidance. In order to ensure zero backlash, they are preloaded. If the thermal influence is excessively high, there is a danger that the preload will become excessively large, which will thus reduce the bearing operating life.

With an axially active piezo element, the preload can be held constant independently of the temperature, see Figure 11.

Figure 11
Spindle bearing arrangement, preload by means of piezo tensioning elements

① Piezo tensioning element



Energy supply and data transmission

Sensors, like actuators, normally require electrical energy in order to function. The rolling bearing can be used here as a source of mechanical energy.

Energy must be supplied, depending on the application, to the following:

- sensors
- signal processing and evaluation units
- cableless data transmitters
- electronic controllers
- actuators

Data transmission is more difficult to achieve than energy supply. Cables can be used for both but cause problems in mounting in many cases and are easily damaged. Plugs on the bearing are expensive if they are required to fulfil the normal requirements for mechanical stability and leak tightness. There are arguments in favour of wireless data transmission.

Development potential of mechatronic rolling bearings

Although rolling bearings have a history stretching back 120 years, they are still a long way off reaching the end point in their evolution. One branch of this development, the integration of electronic components, has been sketched out in this chapter.

The solutions presented here reflect the current status of development in mechatronic bearings. There is still room for optimisation and possibly also completely new ideas. The rolling bearing will continue to evolve and move from being a component to (also hopefully) a reliable and robust mechatronic system.

Condition monitoring of machines with rolling bearings by means of vibration analysis

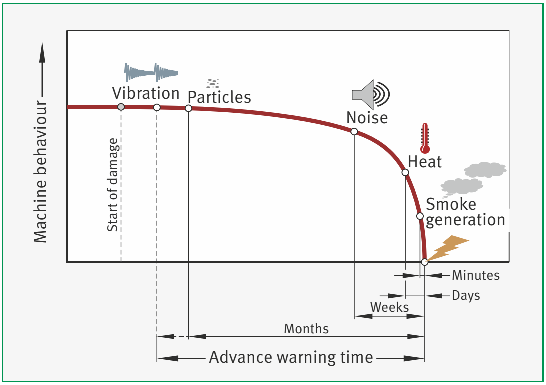
Vibration analysis

Vibration condition monitoring began with the observations of machine operators, who used the “human sensory system”, i.e. hearing, touch and sight, to detect abnormalities and assess these to a certain extent.
With the advancement of measurement technology came the ability to not only identify conspicuous machine vibrations subjectively, but also more comprehensively and objectively, leading in turn to continuous improvements in the quality of the knowledge gained.

Today, vibration-based machine monitoring (vibration measurement and analysis) is an established and reliable tool for detecting and identifying the root causes of machine problems at an early stage, and thus supports the timely planning of necessary maintenance measures.

Vibration measurement and vibration analysis can be used to detect fault conditions such as unbalance and misalignment as well as rolling bearing damage and gear tooth defects. Depending on the application, advance warning times of several months can be achieved, see Figure 12.

Figure 12
Damage curve and detectability as a function of time



Vibrations are induced by occurring forces. Every machine in operation will have a certain basic vibration that reflects its mechanical condition. If the forces acting in the machine change, for example as a result of unbalance, damaged machine components or electrical issues, the vibration behaviour of the machine will also change. If the vibration level increases while the operating parameters remain unchanged, this can indicate a deterioration in the condition of the machine.

With vibration analysis, various machine faults and damage types can be identified on the basis of characteristic vibration patterns in the measurement signals. Early detection of the changed condition may indeed render further operation of the machine permissible in the first instance. This method of condition monitoring offers considerable cost-saving potential provided that the service life of the plant and machinery can be almost fully utilised and their availability can be increased.

In most cases, attention is focussed on the measurement and analysis of structure-borne sound. This involves recording vibrations at the surface of a solid body, for example a machine housing, which are transmitted within the body from their point of origin to the measurement location. Alternatively, measurement methods which involve observing the vibration motion of an entire body, such as a shaft, can also be used.

Vibrations are measured using vibration sensors, which usually measure accelerations, and are broken down into piezoelectric sensors, which detect vibrations in a frequency range of up to 20 kHz, and semiconductor sensors (also known as MEMS sensors), which have a measuring range below 10 kHz but are considerably cheaper.

In the recording of measurement data, a distinction is made between online and offline measuring systems. With online measuring systems, the sensors, which are already intelligent in some cases, are permanently attached to the machine and data recording takes place continuously. With offline measuring systems (usually in the form of hand-held measuring devices) that are used for temporary measurements, the sensors are usually installed on the machines on a temporary basis using magnets.

Time domain and frequency domain

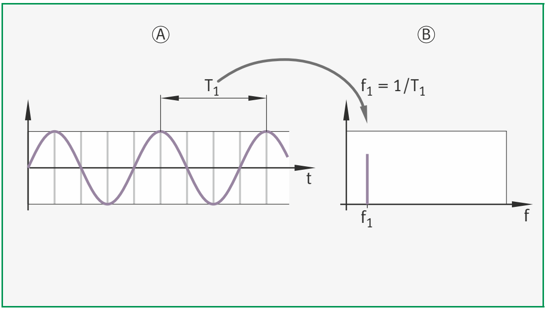
The objective of vibration-based machine monitoring is to identify the cause of a machinery problem.
In order to do so, the emphasis must always be on gaining as much information as possible from the measurement data. Vibration signals can be analysed using the time domain or frequency domain. Based on the same measurement, they are two different display formats that provide the information.

When a vibration signal from the time domain is transferred to the frequency domain, the result is a frequency spectrum, or spectrum for short. For the vibration signal in the time domain, the measurement quantity is plotted against the time axis, whereas for the spectrum, it is plotted against the frequency axis, see Figure 13.

Figure 13
Time and frequency representation of a sine wave signal

Ⓐ Time domain
Ⓑ Frequency domain

t = time
T_1 = period
f = frequency
f_1 = frequency of the sine wave vibration



Depending on the manufacturer and measuring system, analysis software not only provides different forms of representation but also various functionalities that assist the user in the evaluation.
These comprise simple analysis tools such as various cursor types and zoom functions, extending through to mathematical functions.
Through the use of these media, the causes or sources of vibration patterns can be precisely identified.

Amplitude modulation and demodulation (envelope signal)

Amplitude-modulated signals are a frequent occurrence in the vibration monitoring of machines, particularly in the monitoring of gearboxes and rolling bearings. These signals are characterised by the fact that their amplitude values regularly increase and decrease.

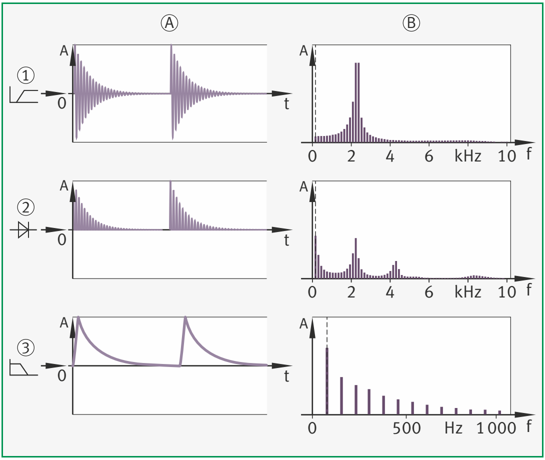
In the monitoring of machinery, amplitude modulation is very important where shock-type excitations are present, for example in the case of rolling bearing damage, impacting or gearbox damage. In these cases, the carrier frequency is a system natural frequency. It is modulated by the frequency of the impacts occurring. In general, the carrier frequency is not of interest, which is why demodulation is used to separate the carrier signal from the useful signal (impact frequencies).

The variants of demodulation used for this purpose by the various measuring device manufacturers all use the shock-type excitation in the case of damage and extract this signal component from the overall signal. The resulting signal is also known as the envelope signal. In contrast, velocity or acceleration signals that are not processed in this form are also called raw signals. In order to obtain the envelope signal, the filters are set such that natural frequencies in the mechanical structure that are excited by impacts are enveloped and filtered out, see Figure 14. In the spectrum, this makes it possible to see only the repeat frequency of the impacts with multiples. This repeat frequency of the impacts is evaluated in envelope signal analysis.

Figure 14
Envelope signal in detail

Ⓐ Time domain
Ⓑ Frequency domain
① High pass filter
② Rectifier
③ Low pass filter

A = measurement quantity
t = time
f = frequency



Bearing damage patterns and calculation of frequencies

The envelope signal analysis is usually used for bearing damage analysis which shows typical forms of bearing damage very clearly.
Although advanced bearing damage may also be visible in the acceleration spectrum, this form of analysis is often very difficult.

The following figures show a theoretical envelope signal spectrum and a measured envelope signal spectrum, respectively, for an instance of bearing damage.

If the raceway of the outer ring is damaged, the envelope signal spectrum will show the ball pass frequency of the outer race (BPFO) with its harmonics.

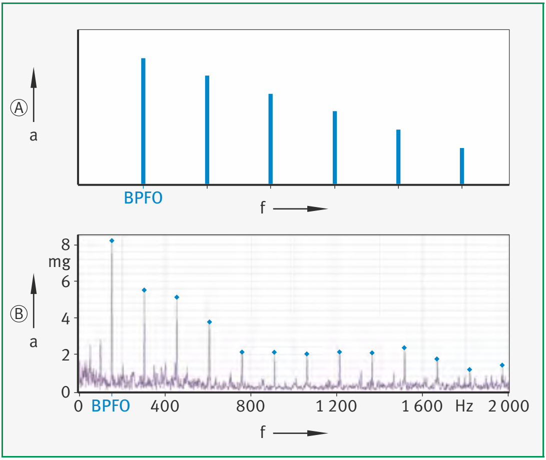


Figure 15
Outer ring damage (envelope spectrum)

Ⓐ Theoretical vibration pattern
Ⓑ Measured signal

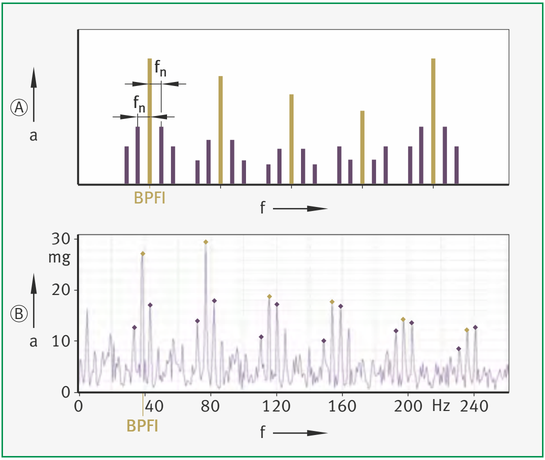
a = acceleration
f = frequency
BPFO = ball pass frequency outer race

In the event of damage to the inner ring, the envelope signal spectrum will show the ball pass frequency of the inner race (BPFI) with its harmonics and with sidebands spaced at the rotational frequency of the rotating bearing ring.

Figure 16
Inner ring damage (envelope spectrum)

Ⓐ Theoretical vibration pattern
Ⓑ Measured signal

a = acceleration
f = frequency
f_n = rotational frequency
BPFI = ball pass frequency inner race

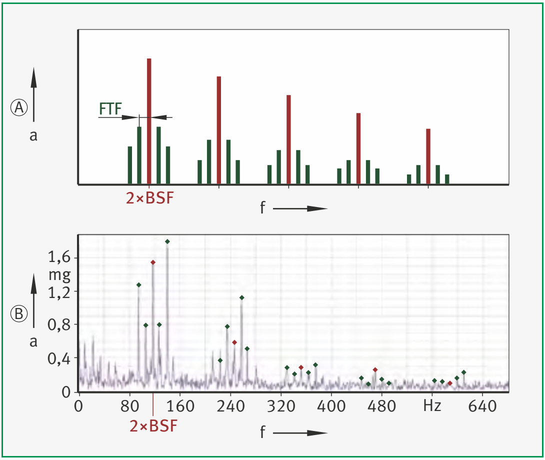


In a case of rolling bearing damage, the envelope signal spectrum will show the double ball spin frequency (2×BSF) with its harmonics and with sidebands spaced at the fundamental train frequency.

Figure 17
Rolling element damage (envelope spectrum)

Ⓐ Theoretical vibration pattern
Ⓑ Measured signal

a = acceleration
f = frequency
BSF = ball spin frequency
FTF = fundamental train frequency



To enable the allocation of the patterns described above to the installed bearing types, the overrolling frequencies can be calculated using the following equations. Alternatively, the frequencies can be found on the web pages operated by bearing manufacturers, such as on the Schaeffler **medias** platform: *https://medias.schaeffler.de*.

Typical frequencies

The frequency components that form the typical damage patterns in the vibration signals by repeated overrolling of a defective location correspond to the kinematic frequencies of the particular bearing that are the product of its geometry.

For the individual damage frequencies, it is common practice to use designations and abbreviations such as:

- ball pass frequency outer race:
  f_o or BPFO
- ball pass frequency inner race:
  f_i or BPFI
- ball spin frequency:
  f_w or BSF
- fundamental train frequency:
  f_c or FTF

The kinematic frequencies can be calculated using the following equations.

The following applies to the ball pass frequency outer race:

Equation 1

$$BPFO = f_o = \frac{1}{2} \cdot f_n \cdot z \cdot \left[1 - \frac{D_w}{D_{pw}} \cdot \cos\alpha \right]$$

The following applies to the ball pass frequency inner race:

Equation 2

$$BPFI = f_i = \frac{1}{2} \cdot f_n \cdot z \cdot \left[1 + \frac{D_w}{D_{pw}} \cdot \cos\alpha \right]$$

The following applies to the ball spin frequency:

Equation 3

$$BSF = f_w = \frac{1}{2} \cdot f_n \cdot \frac{D_{pw}}{D_w} \cdot \left[1 - \left(\frac{D_w}{D_{pw}} \cdot \cos\alpha \right)^2 \right]$$

Further information relating to the monitoring of gearboxes, for example, can be found in the Schaeffler Condition Monitoring Handbook. Support is also available from the Schaeffler Condition Monitoring Service department.

Strength of materials

Terms, values and definitions

Strength theory

The strength theory provides us with the basis for calculating the mechanical stress and dimensioning technical designs.

The following two questions are answered here:

- distribution of the inner forces on the cross-sectional surface of a stressed machine part
- the changes in shape that are caused by these cross-sectional values

The answer to the first question gives the mechanical stress that is acting on the design. The answer to the second question provides information about the elastic deformations of a machine part which are associated with the load, such as changes in length or deflections for example.

Stress in area elements

The external forces and moments acting on a body are balanced by corresponding reaction forces inside the body. If a homogeneous mass distribution is assumed, the occurring inner reaction forces are distributed over a large area. The force density (quotient of internal force and effective area) prevailing in every area element is the **stress**. It mostly changes its size and direction from one point to another.

To describe the stress state in a cross section, the stresses are split:

- **normal stress** σ = components perpendicular to the cross-sectional plane
- **shear stresses** τ = two components in the cross-sectional plane

If a cross-sectional plane is placed in such a way that both shear stresses become zero, then the normal stress reaches an extreme value, which is referred to as the **principal stress**.

Deformation

Stresses are always linked with deformations. A distinction is made between two kinds of deformation:

- elastic deformations
- plastic deformations

Elastic deformation

Elastic deformations disappear again once the imposed external load has been removed. They often follow Hooke's Law.

Changes in the length of a line element per unit of length are described as elongations. Length changes are caused by **normal stresses**.

According to Hooke's Law, elongations are proportional to the stresses accompanying them:

Equation 1

$$\sigma = E \cdot \varepsilon$$

ε = elongations

Here, the proportionality constant is the material parameter modulus of elasticity E.

Shear stresses result in angle changes. The change in an angle that was originally a right angle is, relative to this, described as torsional shear strain or shear γ.

Torsional shear strain (dislocation) is proportional to the shear stress:

Equation 2

$$\tau = G \cdot \gamma$$

γ = torsional shear strains (dislocation)

Here, the proportionality constant is the material parameter shear modulus of rigidity G.

The following relationship exists between the two material parameters:

Equation 3

$$G = \frac{E}{2(1+\nu)}$$

ν = Poisson's ratio

Plastic deformation

If the external forces on a component and thus the inner stresses exceed a certain limit that is intrinsic to the material, either plastic deformations occur, which remain after removal of the external load or the component breaks. The theory of yield, strain hardening and fracture applies.

As a rule, only the elastic range of the material is utilised in the design of components. For this reason, we will only be considering the elastic behaviour of materials here and will be ignoring plastic deformation.

Values and units The following table shows a selection of values and units associated with the strength of materials.

Value	Unit	Designation	Comments
x, y, z	mm	Cartesian coordinates	Right-handed coordinate system
u, v, w	mm	Deformation in x, y, z direction	
a	mm	Distance, lever arm, major elliptical semi-axis	In contrast to the SI system, the basic unit metre (m) is not used in mechanical engineering, but the derived unit millimetre (mm) instead.
b	mm	Width, minor elliptical semi-axis	
d, D	mm	Diameter	
r, R	mm	Radius	
f	mm	Deflection, sagging	
h	mm	Height	
l	mm	Length	$1\ mm = 10^{-3}\ m$
A	mm^2	Area, cross-sectional area	$1\ mm^2 = 10^{-6}\ m^2$
E	N/mm^2	Modulus of elasticity	$E = \sigma/\epsilon$
F	N	Force	$1\ N = 1\ kg \cdot m/s^2$
F_G	N	Weight	$F_G = m \cdot g$
g	mm/s^2	Gravitational acceleration	$g = 9{,}806\,65\ m/s^2$
G	N/mm^2	Shear modulus	$G = \tau/\gamma$
H	mm^3	First moment of area	$H_y = \int z\, d A$
I_a	mm^4	Axial second moment of area	$I_y = \int z^2\, d A$
I_p	mm^4	Polar second moment of area	$I_p = \int r^2\, d A$
I_t	mm^4	Torsional area moment	–
m	kg	Mass	SI base unit
M_b	$N \cdot mm$	Bending moment	Cross-sectional value
M_t	$N \cdot mm$	Torsional moment	Cross-sectional value
F_N, N	N	Normal force	Cross-sectional value
p	N/mm^2	Pressure, Hertzian pressure	–
Q	N	Transverse force	Cross-sectional value

Continuation of table, see Page 227.

Continuation of table, Values and units, from Page 226.

Value	Unit	Designation	Comments
R_e	N/mm^2	Yield point	see material tables
R_m	N/mm^2	Tensile strength, breaking strength	
$R_{p0,2}$	N/mm^2	0,2 proof stress	
T	K	Temperature	SI base unit
W_a	mm^3	Axial section modulus	W_x, W_y, W_z
W_p	mm^3	Polar section modulus	$W_p = I_p/R$ (circle)
W_t	mm^3	Torsional section modulus	–
W_i	$N \cdot mm$	Internal deformation work	of the internal stresses
$W_ä$	$N \cdot mm$	External deformation work	of the forces, moments
α	1/K	Coefficient of linear thermal expansion	$\Delta l = \alpha \cdot l \cdot \Delta T$
α_k	1	Stress concentration factor, notch concentration factor	–
β	1/K	Coefficient of cubical thermal expansion	$\beta = 3\,\alpha$
β_k	1	Fatigue notch factor	–
γ	1	(Torsional) shear strain	$\gamma = \tau/G$
ϵ	1	Elongation	$\epsilon = \Delta l/l$
ϵ_q	1	Transverse elongation	$\epsilon_q = \Delta d/d = -\nu\,\epsilon$
ϵ_m	1	Elongation at fracture	–
Θ	rad/mm	Twist	$\Theta = \varphi/l$
ν	1	Poisson's ratio	$\nu = 0{,}3$ (for most metallic materials) Further designations for ν: $1/m$, ν_E
ρ	kg/mm^3	Density, mass density	–
σ	N/mm^2	Normal stress (tensile stress, compressive stress)	$\sigma = F_N/A$
σ_W	N/mm^2	Fatigue strength under reversed stresses	see Smith diagram
σ_{Sch}	N/mm^2	Fatigue strength under pulsating stresses	
σ_A	N/mm^2	Amplitude strength	
σ_D	N/mm^2	Fatigue limit (general)	
τ	N/mm^2	Shear stress, torsional shear stress	–
φ	rad	Angle, torsional angle	–

Material characteristics

The following table lists a number of important material characteristics. For additional or detailed values, refer to the Engineering materials chapter of this Pocket Guide.

Material	Modulus of elasticity[1)] E kN/mm^2 = GPa	Poisson's ratio ν	Coefficient of linear expansion α 10^{-6}/K	Density ρ kg/dm^3	Tensile strength[2)] R_m N/mm^2 = MPa
Metals					
Aluminium	72,2	0,34	23,9	2,7	40 ... 160
Aluminium alloys	59 ... 78	0,33 ... 0,34	18,5 ... 24,0	2,6 ... 2,9	300 ... 700
Brass	78 ... 123	0,35	17,5 ... 19,1	8,3 ... 8,7	140 ... 780
Brass (60% Cu)	100	0,36	18	8,5	200 ... 740
Bronze	108	0,35	16,8 ... 18,8	7,2 ... 8,9	300 ... 320
Cast iron	64 ... 81	0,24 ... 0,29	9 ... 12	7,1 ... 7,4	140 ... 490
Copper	125	0,35	16,86	8,93	200 ... 230
Gold	79	0,42	14,2	19,3	130 ... 300
Iron	206	0,28	11,7	7,86	300
Lead	16	0,44	29,1	11,34	10 ... 20
Magnesium	44	0,33	26,0	1,74	150 ... 200
Nickel	167	0,31	13,3	8,86	370 ... 800
Nickel alloys	158 ... 213	0,31	11 ... 14	7,8 ... 9,2	540 ... 1 275
Platinum	170	0,22	9,0	21,5	220 ... 380
Silver	80	0,38	19,7	10,5	180 ... 350
Steel, alloyed	186 ... 216	0,2 ... 0,3	9 ... 19	7,8 ... 7,86	500 ... 1 500
Steel, unalloyed	210	0,3	12	7,85	300 ... 700
X5CrNi18-10	190	0,27	16	7,9	500 ... 700
100Cr6, hardened	208	0,30	12	7,85	2 000 ... 2 400
Tin	55	0,33	21,4	7,29	15 ... 30
Titanium	105	0,33	8,35	4,5	300 ... 740
Zinc	94	0,25	29	7,14	100 ... 150

Continuation of table, see Page 229.

1) The following relationship applies between the modulus of elasticity E and the shear modulus G of the materials:

$$G = \frac{E}{2(1+\nu)}$$

2) Detailed values for the tensile strength R_m and the yield point R_e of materials can be found in the relevant DIN standards and in the Engineering materials chapter of this Pocket Guide.

Continuation of table, Material characteristics, from Page 228.

Material	Modulus of elasticity[1)] E kN/mm^2 = GPa	Poisson's ratio ν	Coefficient of linear expansion α $10^{-6}/K$	Density ρ kg/dm^3	Tensile strength[2)] R_m N/mm^2 = MPa
Non-metallic materials (inorganic)					
Brick	10 ... 40	0,20 ... 0,35	8 ... 10	1,7 ... 1,9	–
Concrete	22 ... 39	0,15 ... 0,22	5,4 ... 14,2	2,0 ... 2,8	10 ... 40
Construction glass	62 ... 86	0,25	9	2,4 ... 2,7	30 ... 90
Glass (general)	39 ... 98	0,10 ... 0,28	3,5 ... 5,5	2,2 ... 6,3	30 ... 90
Granite	50 ... 60	0,13 ... 0,26	3 ... 8	2,6 ... 2,8	10 ... 20
Marble	60 ... 90	0,25 ... 0,30	5 ... 16	1,8 ... 2,7	–
Porcelain	60 ... 90	–	3 ... 6,5	2,2 ... 2,5	15 ... 40
Quartz glass	62 ... 76	0,17 ... 0,25	0,5 ... 0,6	2,21	30 ... 90
Non-metallic materials (organic)					
Araldite	3,2	0,33	50 ... 70	–	–
Plexiglas® (PMMA)	2,6 ... 3,2	0,35	70 ... 100	1,18	40 ... 70
Polyamide (Nylon®)	1,3 ... 1,7	–	70 ... 100	1,01 ... 1,14	40 ... 80
Polyethylene (HDPE)	0,15 ... 1,6	–	150 ... 200	0,91 ... 0,97	25 ... 30
Polyvinyl chloride	1 ... 3	–	70 ... 100	1,2 ... 1,7	45 ... 60

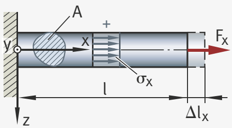
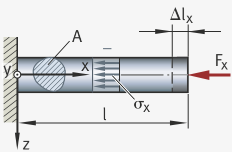
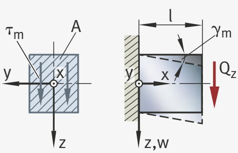
1) The following relationship applies between the modulus of elasticity E and the shear modulus G of the materials:

$$G = \frac{E}{2(1+\nu)}$$

2) Detailed values for the tensile strength R_m and the yield point R_e of materials can be found in the relevant DIN standards and in the Engineering materials chapter of this Pocket Guide.

Load types

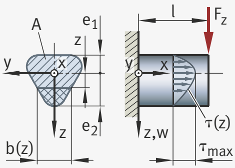
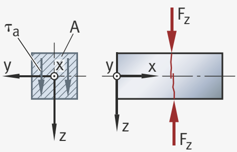
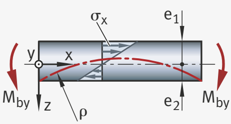
Load types The most important load types, complete with occurring stresses and associated deformations, are presented below.

Load type	Stress	Deformation
Tensile loading 	**Normal stress** $\sigma_x = \frac{F_x}{A}$ **Tensile loading** F_x is positive σ_x is positive **Compressive loading** F_x is negative σ_x is negative	**Elongation (compression[1])** $\varepsilon_x = \frac{\Delta l_x}{l} = \frac{\sigma_x}{E} = \frac{F_x}{E \cdot A}$ **Increase (decrease[1]) in length** $\Delta l_x = \frac{F_x \cdot l}{E \cdot A}$ **Transverse contraction (transverse compression)** $\epsilon_y = -\nu \cdot \epsilon_x$ $\epsilon_z = -\nu \cdot \epsilon_x$
Compressive loading 		
Shear loading (average) 	**Shear stress** (average) $\tau_m = \frac{Q_z}{A}$	**Torsional shear strain (dislocation)** (average) $\gamma_m = \frac{\tau_m}{G} = \frac{Q_z}{G \cdot A}$ $w(l) = \gamma_m \cdot l$

Continuation of table, see Page 231.

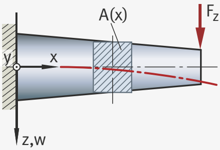
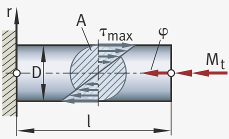
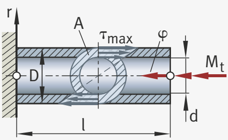
1) Depending on sign of F_x.

Continuation of table, Load types, from Page 230.

Load type	Stress	Deformation
Shear loading (actual)  $Q_z = F_z$	**Shear stress distribution** $\tau(z) = \frac{Q_z H_y(z)}{I_y b(z)}$ with static moment $H_y(z) = \int_z^{e_2} z\, b(z)\, dz$ $\tau_{max} = \frac{Q_z H_y(z = 0)}{I_y(z = 0)}$	**Deflection of the beam** (only as the result of shear stress) $w(x) = k \frac{Q_z}{G \cdot A} \cdot x < \frac{\tau_{max}}{G} \cdot x$ $w(l) = k \frac{Q_z}{G \cdot A} \cdot l$ k = cross-sectional factor
Direct shear loading 	**Average shear stress** $\tau_a = \frac{F_z}{A}$	Shearing occurs when the material's shear strength is exceeded
Bending without transverse force  M_{by} = const. I_y = const. $e_1 = e_{max}$ I_y = axial second moment of area about the y axis	**Bending stress** Distribution $\sigma(z) = -\frac{M_{by}}{I_y} z$ Maximum value $\sigma_{max} = \frac{M_{by}}{I_y} e_{max} = \frac{M_{by}}{W_y}$	**Curvature** $k = \frac{1}{\rho} = \frac{M_{by}}{E I_y}$ ρ = curvature radius **Differential equation of the elastic curve** $w''(x) = -\frac{M_{by}}{E I_y}$

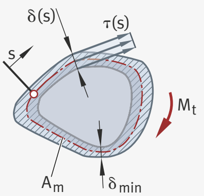
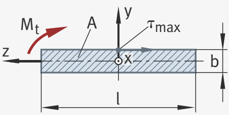
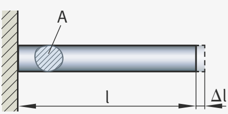
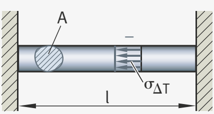
Continuation of table, see Page 232.

Continuation of table, Load types, from Page 231.

Load type	Stress	Deformation
Bending (general)  $I_y(x) \neq$ const. $M_{by}(x) \neq$ const. (influence of transverse force is ignored, justified for long, slender beams)	**Bending stress** Distribution $\sigma_b(x, z) = -\dfrac{M_{by}(x)}{I_y(x)} z$ Maximum value $\sigma_{b_{max}}(x) = \dfrac{M_{by}(x)}{W_y(x)}$	**Differential equation of the elastic curve** $w''(x) = \dfrac{M_{by}(x)}{E I_y(x)}$
Torsion of circular solid cross sections  I_p = polar second moment of area	**Torsional stress** Distribution $\tau(r) = \dfrac{M_t}{I_p} r$ Maximum value $\tau_{max} = \dfrac{M_t}{I_p} \cdot \dfrac{D}{2} = \dfrac{M_t}{W_p}$	**Twist** $\vartheta = \dfrac{\varphi}{l} = \dfrac{M_t}{G I_p}$ **Angle of twist** $\varphi = \dfrac{M_t\, l}{G I_p}$
Torsion of circular hollow cross sections (tubes)  	**Torsional stress** Maximum value $\tau_{max} = \dfrac{M_t}{W_p}$ $W_p = \dfrac{I_p(D) - I_p(d)}{D/2}$	**Angle of twist** $\varphi = \dfrac{M_t\, l}{G I_p}$ $I_p = I_p(D) - I_p(d)$

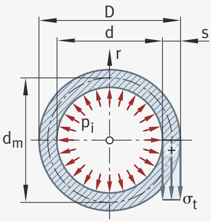
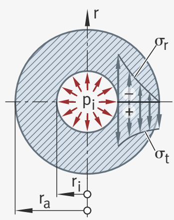
Continuation of table, see Page 233.

Continuation of table, Load types, from Page 232.

Load type	Stress	Deformation
Torsion of thin-walled, closed hollow cross sections (Bredt's formulas) 	**Shear stress** Distribution over circumference $\tau(s) = \frac{M_t}{2\,A_m\,\delta(s)}$ Maximum value $\tau_{max} = \frac{M_t}{W_t} = \frac{M_t}{2A_m \cdot \delta_{min}}$	**Twist** $\vartheta = \frac{M_t}{G \cdot I_t} = \frac{M_t}{G}\,\frac{\oint \frac{ds}{\delta(s)}}{4\,A_m^2}$ $I_t = \frac{4\,A_m^2}{\oint \frac{ds}{\delta(s)}}$
Torsion of narrow rectangular cross sections 	**Shear stress** Distribution $\tau = \frac{2\,M_t}{I_t}\,y$ Maximum value $\tau_{max} = \frac{M_t}{W_t} = \frac{2\,M_t}{I_t}\,\frac{b}{2} = \frac{3\,M_t}{b^2\,h}$	**Twist** $\vartheta = \frac{M_t}{G \cdot I_t} = \frac{M_t}{G}\,\frac{3}{b^3\,h}$ $I_t = \frac{b^3\,h}{3}$
Heating of a bar with free elongation 	**No stresses** Elongation occurs without stresses.	**Length change** $\Delta l = l \cdot \alpha \cdot \Delta T$ α = coefficient of linear thermal expansion ΔT = temperature change
Heating of a bar, clamped at both ends 	**Thermal stress** $\sigma_{\Delta T} = -E \cdot \alpha \cdot \Delta T$ α = coefficient of linear thermal expansion ΔT = temperature change	**No deformation** An increase in length is not possible on account of the clamping operation and must be absorbed by compression in the bar.

Continuation of table, see Page 234.

Continuation of table, Load types, from Page 233.

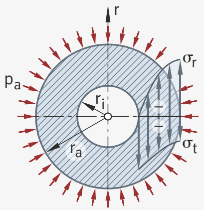
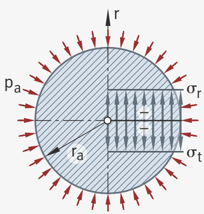
Load type	Stress	Deformation
Thin-walled tube under internal pressure p_i  $D/d < 1{,}2$	**Tangential stress** (Barlow's formula) $\sigma_t = \frac{p_i\, d_m}{2\, s}$ **Axial stress** (Barlow's formula) $\sigma_a = \frac{p_i\, d_m}{4 \cdot s}$ where $d_m = \frac{D+d}{2}$ $s = \frac{D-d}{2}$	**Diameter change** $\Delta d_m = \frac{d_m\, \sigma_t}{E}$ **Length change** $\Delta l = \frac{l\, \sigma_a}{E}$
Thick-walled tube under internal pressure p_i[1)]  Ratio: $Q = \frac{r_i}{r_a} = \frac{d_i}{d_a}$	**Tangential stress** $\sigma_t = p_i \frac{(r_a/r)^2 + 1}{(r_a/r_i)^2 - 1}$ **Radial stress** $\sigma_r = -p_i \frac{(r_a/r)^2 - 1}{(r_a/r_i)^2 - 1}$ **Axial stress**[2)] $\sigma_a = p_i \frac{1}{(r_a/r_i)^2 - 1}$	**Radial displacement** $u(r) = \frac{p_i}{E} \frac{\left[(1-\nu)\, Q^2\, r + (1+\nu)\, r_i^2/r\right]}{1-Q^2}$ **Diameter change** $\Delta d_a = \frac{p_i\, d_a}{E} \frac{2\, Q^2}{1-Q^2}$ $\Delta d_i = \frac{p_i\, d_i}{E} \left[\frac{1+Q^2}{1-Q^2} + \nu\right]$

Continuation of table, see Page 235.

1) In the presence of p_i and p_a, the relationships for the stresses and deformations can be superimposed. The results of FEM calculation may deviate by up to 10% from the results determined using these equations.

2) System with axially impeded expansion.

Continuation of table, Load types, from Page 234.

Load type	Stress	Deformation
Thick-walled tube under external pressure p_a[1)]  $r_i \leq r \leq r_a$	**Tangential stress** $\sigma_t = -p_a \dfrac{(r_a/r_i)^2 + (r_a/r)^2}{(r_a/r_i)^2 - 1}$ **Radial stress** $\sigma_r = -p_a \dfrac{(r_a/r_i)^2 - (r_a/r)^2}{(r_a/r_i)^2 - 1}$ **Axial stress**[2)] $\sigma_a = -p_a \dfrac{(r_a/r_i)^2}{(r_a/r_i)^2 - 1}$	**Radial displacement** $u(r) = -\dfrac{p_a}{E} \dfrac{\left[(1-\nu)\,r + (1+\nu)\,r_i^2/r\right]}{(1-Q^2)}$ **Diameter change** $\Delta d_a = -\dfrac{p_a\,d_a}{E}\left[\dfrac{1+Q^2}{1-Q^2} - \nu\right]$ $\Delta d_i = -\dfrac{p_a\,d_i}{E}\,\dfrac{2}{1-Q^2}$
Solid shaft under external pressure p_a  $0 \leq r \leq r_a$	**Tangential stress** $\sigma_t = -p_a = \text{const.}$ **Radial stress** $\sigma_t = -p_a = \text{const.}$ **Axial stress**[2)] $\sigma_a = -p_a = \text{const.}$	**Radial displacement** $u(r) = -\dfrac{p_a\,r}{E}(1-\nu)$ **Diameter change** $\Delta d_a = -\dfrac{p_a\,d_a}{E}(1-\nu)$ ν = Poisson's ratio

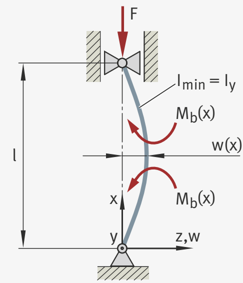
1) In the presence of p_i and p_a, the relationships for the stresses and deformations can be superimposed. The results of FEM calculation may deviate by up to 10% from the results determined using these equations.

2) System with axially impeded expansion.

Buckling of slender bars

A buckling load represents a special case of compressive loading, as occurs, for example, with long spindles, pin supports of slip-on gears, frame members and in other similar cases. Slender bars, when subjected to a compressive load, move out of the unbent (unstable) equilibrium state into an adjacent bent (stable) state when a critical compressive stress is reached.

Figure 1
Buckling of a slender bar



With the aid of compressive stress:

Equation 4

$$\sigma_d = \frac{F}{A}$$

this gives the following for the buckling stress (in the Euler range for Euler buckling case 2, see Figure 1):

Equation 5

$$\sigma_K = \frac{F_K}{A} = \frac{\pi^2 \cdot E \cdot I_y}{A \cdot l^2} \qquad I_y = I_{min}$$

Buckling in the elastic (Euler) range

If we consider the deformed equilibrium state of the bar shown (see Figure 1), the differential equation for buckling around the major cross-sectional axis y (with I_y as the smallest second moment of area), in the case of small deflections w(x), is:

Equation 6

$$E \cdot I_y \cdot w''(x) = -M_b(x) = -F \cdot w(x)$$

$$w''(x) + \alpha^2 \cdot w(x) = 0 \qquad \text{where } \alpha = \sqrt{\frac{F}{E \cdot I_y}}$$

The solution to this differential equation is:

Equation 7

$$w(x) = c_1 \cdot \sin(\alpha \cdot x) + c_2 \cdot \cos(\alpha \cdot x)$$

From the marginal conditions for the bar shown:

Equation 8

$$w(x = 0) = 0 \qquad \text{and } w(x = l) = 0$$

it follows that $c_2 = 0$ and $\sin(\alpha \cdot l) = 0$ (eigenvalue equation) with the eigenvalues:

Equation 9

$$\alpha_K = \frac{n \cdot \pi}{l} \qquad n = 1, 2, 3...$$

This gives the buckling load:

Equation 10

$$F_K = \alpha_K{}^2 \cdot E \cdot I_y = n^2 \cdot \pi^2 \cdot E \cdot I_y / l^2$$

and the smallest buckling load:

Equation 11

$$F_K = \frac{\pi^2 \cdot E \cdot I_y}{l^2} \qquad n = 1$$

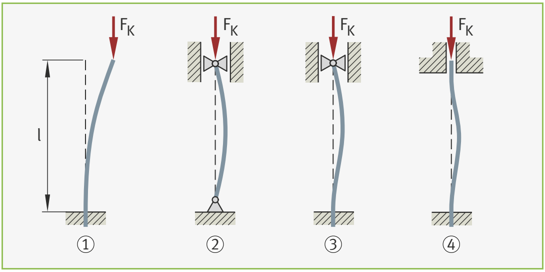
Appropriate eigenvalues are produced for other bearing arrangement cases which, however, can all be ascribed to Euler's buckling load with the reduced buckling length l_K.

Euler's four buckling cases

The following diagram shows the four buckling cases for slender bars in accordance with Euler:

Figure 2
Euler's buckling cases

① Case 1: $l_K = 2\,l$
② Case 2: $l_K = l$
③ Case 3: $l_K = 0{,}7\,l$
④ Case 4: $l_K = 0{,}5\,l$



With the radius of gyration:

Equation 12

$$i_y = \sqrt{I_y / A}$$

and the slenderness ratio:

Equation 13

$$\lambda = l_K / i_y = l_K \Big/ \sqrt{(I_y / A)}$$

the following is given for the buckling stress:

Equation 14

$$\sigma_K = F_K / A = \pi^2 \cdot E \Big/ \lambda^2$$

These relationships for F_K and σ_K apply only in the linear, elastic material range, i.e. as long as the following applies:

Equation 15

$$\sigma_K = \pi^2 \cdot E \Big/ \lambda^2 < R_p \qquad \text{or} \qquad \lambda > \sqrt{\pi^2 \cdot E / R_p}$$

The transition from the elastic to the inelastic (plastic) range lies at the limit slenderness:

Equation 16

$$\lambda_0 = \sqrt{\pi^2 \cdot E / R_p}$$

In this case, R_p is the proportional limit of the material.

Buckling in the inelastic (plastic) range

For smaller slenderness ratios than the limit slenderness, Euler's hyperbola is replaced by Tetmajer's line (see Figure 3, Page 239), which has the following form:

Equation 17

$$\sigma_K = a - b \cdot \lambda$$

The values for limit slenderness λ_0 and for a and b are presented in the following table for a number of materials.

Material	Old designation	E N/mm^2	λ_0	a	b
S235JR	St 37	$2{,}1 \cdot 10^5$	104	310	1,14
E295, E335	St 50, St 60	$2{,}1 \cdot 10^5$	89	335	0,62
5% Ni steel		$2{,}1 \cdot 10^5$	86	470	2,30
Flake graphite cast iron		$1{,}0 \cdot 10^5$	80	$\sigma_K = 776 - 12 \cdot \lambda + 0{,}053 \cdot \lambda^2$ [1)]	
Softwood		$1{,}0 \cdot 10^4$	100	29,3	0,194

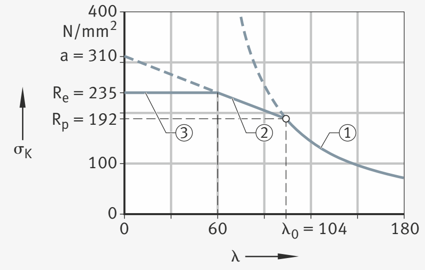
1) No longer a line, rather Engesser's hyperbola.

Tetmajer's line runs from the intersection with Euler's hyperbola to the intersection with the yield point R_e of the material used.

Consequently, three areas exist as a function of the slenderness ratio:

Figure 3
Buckling stress diagram for S235

① Euler's hyperbola
② Tetmajer's line
③ Yield point



In the example "Buckling stress diagram for material S235", we arrive at the following for the yield point R_e:

Equation 18

$$R_e = 235\ \text{N/mm}^2$$

and for the proportional limit R_p:

Equation 19

$$R_p = 0{,}8 \cdot R_e = 188\ \text{N/mm}^2$$

Uniaxial and multi-axial stress states

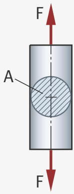
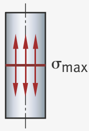
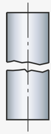
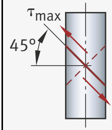
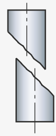
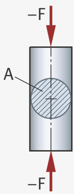
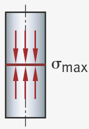
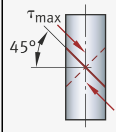
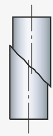
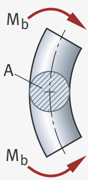
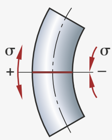
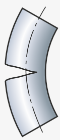
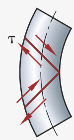
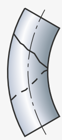
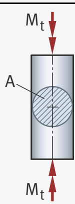
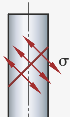
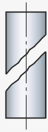
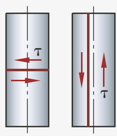
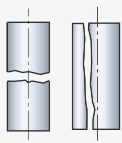
Fracture forms

The mechanism of fracture forms for the uniaxial stress state is presented below.

The most important types of failures under mechanical loading are:

Type of failure	Crucial strength characteristic
Start of yield	Yield point, 0,2 proof stress
Cleavage fracture	Breaking strength
Fatigue fracture	Fatigue strength for present dynamic load case

The following fracture forms can be described for the uniaxial stress state:

External load	Brittle materials[1]		Tough materials	
	Maximum normal stress	Cleavage fracture	Maximum shear stress	Shear or torsional shear deformation
Tension 	 $\sigma_{max} = \frac{F}{A}$		 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Compression 	 $\sigma_{max} = -\frac{F}{A}$	Cleavage fracture not possible	 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Bending 	 $\sigma_{max} = \frac{M_b}{W_a}$		 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Torsion 	 $\sigma_{max} = 2\,\tau_{max}$		 $\tau_{max} = \frac{M_t}{W_p}$	
Applicable fracture hypothesis	Normal stress hypothesis		Shear stress hypothesis, distortion energy hypothesis	

[1] In accordance with the FKM guideline, materials with an elongation at fracture of < 8–12% are regarded as brittle.

Strength hypotheses The following important strength hypotheses can be formulated for the multi-axial stress state:

Failure by		Cleavage fracture	Deformation, ductile fracture	
Strength hypothesis		Normal stress hypothesis (NSH)	Shear stress hypothesis (SSH)	Distortion energy hypothesis (DEH)
Stress state		Equivalent stress σ_v		
$\sigma_1, \sigma_2, \sigma_3$ $\sigma_1 \geqq \sigma_2 \geqq \sigma_3$	3-axial	σ_1	$\sigma_1 - \sigma_3 = 2\,\tau_{max}$	$\frac{1}{\sqrt{2}} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$
$\sigma_1, \sigma_2,$ $\sigma_3 = 0$	2-axial	σ_1	$\sigma_1 = 2\,\tau_{max}$	$\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\,\sigma_2}$
$\sigma_1, \sigma_3,$ $\sigma_2 = 0$	2-axial	σ_1	$\sigma_1 - \sigma_3 = 2\,\tau_{max}$	$\sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\,\sigma_3}$
σ_x, σ_y, τ	2-axial	$\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2} \cdot \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$	$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2}$ [1] $\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2} \cdot \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$ [2]	$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\,\sigma_y + 3\,\tau^2}$
$\sigma_x, \tau,$ $\sigma_y = 0$ [3]	2-axial	$\frac{1}{2}\sigma_x + \frac{1}{2}\sqrt{\sigma_x^2 + 4\,\tau^2}$	$\sqrt{\sigma_x^2 + 4\,\tau^2}$	$\sqrt{\sigma_x^2 + 3\,\tau^2}$

1) The following applies:

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2} > \sigma_x + \sigma_y$$

2) The following applies:

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2} < \sigma_x + \sigma_y$$

3) Bending and torsion:
Application of the distortion energy hypothesis (DEH) has proven itself in cases of superimposed bending and torsion. For dynamic loads (for example, alternating bending load with superimposed torsion load), the correction factor α_0 must be taken into account:

$$\sigma_v = \sqrt{\sigma_x^2 + 3 \cdot (\alpha_0 \cdot \tau)^2}$$

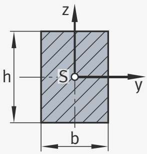
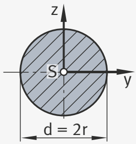
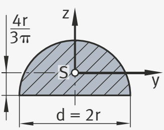
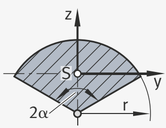
In the case of alternating bending and static torsion: $\alpha_0 = 0{,}7$ [4]
In the case of alternating bending and alternating torsion: $\alpha_0 = 1{,}0$ [4]
In the case of static bending and alternating torsion: $\alpha_0 = 1{,}5$ [4]

4) Approximation values only valid for unnotched components made from general structural steel.

Area moments and section moduli

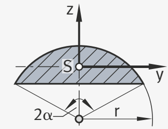
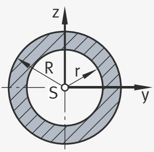
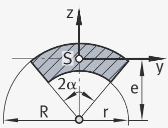
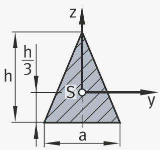
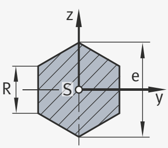
Axial area moments and section moduli

Axial 2nd moments of area and section moduli are calculated using:

Rectangle 	$I_y = \frac{b\,h^3}{12} = A\,\frac{h^2}{12}$ $I_z = \frac{h\,b^3}{12} = A\,\frac{b^2}{12}$	$W_y = \frac{b\,h^2}{6} = A\,\frac{h}{6}$ $W_z = \frac{h\,b^2}{6} = A\,\frac{b}{6}$
Circle 	$I_y = I_z = \frac{\pi\,d^4}{64} = \frac{\pi\,r^4}{4} = \frac{A\,r^2}{4}$	$W_y = W_z = \frac{\pi\,d^3}{32} = \frac{\pi\,r^3}{4} = \frac{A\,r}{4}$
Semicircle 	$I_y = \left(\frac{\pi}{8} - \frac{8}{9\,\pi}\right) r^4 = \frac{I_y}{e} = 0{,}1907\cdot r^3$ $I_z = \frac{\pi\,r^4}{8} = \frac{A\,r^2}{4}$	$W_y = 0{,}0198\;r^4$ $e = r - \frac{4\,r}{3\,\pi}$ $W_z = \frac{\pi\,r^3}{8} = \frac{A\,r}{4}$
Sector of a circle 	$I_y = r^4\left[\frac{2\,\alpha + \sin 2\,\alpha}{8} - \frac{2\left(1-\cos 2\,\alpha\right)}{9\,\alpha}\right]$ $I_z = \frac{r^4}{8}\left(2\,\alpha - \sin 2\,\alpha\right)$	$W_y = \frac{I_y}{\lvert z_{max}\rvert}$ $z = \frac{2\,r\sin\alpha}{3\,\alpha}$ α in radians

Continuation of table, see Page 243.

Continuation of table, Axial area moments and section moduli, from Page 242.

Segment of a circle  	$I_y = r^4\left[\frac{4\alpha - \sin 4\alpha}{16} - \frac{8}{9}\cdot\frac{\sin^6\alpha}{2\alpha - \sin 2\alpha}\right]$ $I_z = \frac{r^4}{48}\left[12\alpha - 8\sin 2\alpha + \sin 4\alpha\right]$	$W_y = \frac{I_y}{\lvert z_{max}\rvert}$ $z = \frac{4r\sin^3\alpha}{3\left(2\alpha - \sin 2\alpha\right)}$ α in radians
Annulus  	$I_y = I_z = \frac{\pi}{4}\left(R^4 - r^4\right)$	$W_y = W_z = \frac{\pi}{4}\,\frac{\left(R^4 - r^4\right)}{R}$
Annular sector  	$I_y = \frac{R^4 - r^4}{8}\cdot\left(2\alpha + \sin 2\alpha\right) - e^2\alpha\left(R^2 - r^2\right)$ $I_z = \frac{R^4 - r^4}{8}\cdot\left(2\alpha - \sin 2\alpha\right)$	$W_y = \frac{I_y}{R - e}$ $e = \frac{2}{3}\cdot\frac{\left(R^3 - r^3\right)\sin\alpha}{\left(R^2 - r^2\right)\alpha}$ α in radians
Triangle  	$I_y = \frac{a h^3}{36} = \frac{A h^2}{18}$ $I_z = \frac{h a^3}{48} = \frac{A a^2}{24}$	$W_y = \frac{a h^2}{24} = \frac{A h}{12}$ $W_z = \frac{h a^2}{24} = \frac{A a}{12}$
Hexagon  	$I_y = I_z = \frac{5\sqrt{3}}{16}R^4 = \frac{5\sqrt{3}}{256}e^4$	$W_y = \frac{5\sqrt{3}}{16}R^3 = \frac{5\sqrt{3}}{128}e^3$ $W_z = \frac{5}{8}R^3 = \frac{5}{64}e^3$

Continuation of table, see Page 244.

Continuation of table, Axial area moments and section moduli, from Page 243.

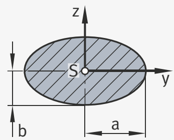
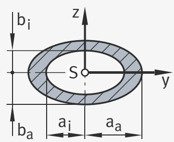
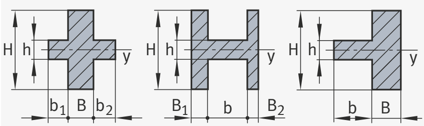
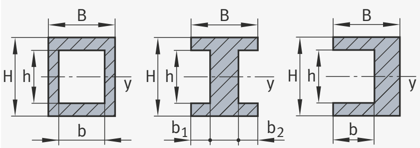
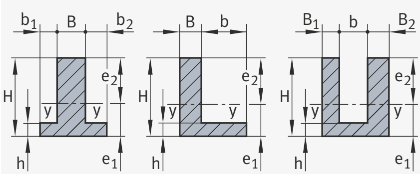
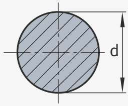
Ellipse 	$I_y = \frac{\pi a b^3}{4} = \frac{A b^2}{4}$ $I_z = \frac{\pi a^3 b}{4} = \frac{A a^2}{4}$	$W_y = \frac{\pi a b^2}{4} = \frac{A b}{4}$ $W_z = \frac{\pi a^2 b}{4} = \frac{A a}{4}$
Hollow ellipse 	$I_y = \frac{\pi\left(a_a b_a^3 - a_i b_i^3\right)}{4}$ $I_z = \frac{\pi\left(a_a^3 b_a - a_i^3 b_i\right)}{4}$	$W_y = \frac{\pi\left(a_a b_a^3 - a_i b_i^3\right)}{4 b_a}$ $W_z = \frac{\pi\left(a_a^3 b_a - a_i^3 b_i\right)}{4 a_a}$
Cross, H profile, T profile 		$I_y = \frac{B H^3 + b h^3}{12}$ $W_y = \frac{B H^3 + b h^3}{6 H}$ where $B = B_1 + B_2$ and $b = b_1 + b_2$
Rectangular hollow profile, I profile, C profile 		$I_y = \frac{B H^3 - b h^3}{12}$ $W_y = \frac{B H^3 - b h^3}{6 H}$ where $b = b_1 + b_2$
T profile, L profile, U profile 		$I_y = \frac{B H^3 + b h^3}{3} - \left(B H + b h\right) e_1^2$ where $B = B_1 + B_2$ and $b = b_1 + b_2$ $W_{y1,\,2} = \frac{I_y}{e_{1,\,2}}$ $e_1 = \frac{1}{2}\,\frac{B H^2 + b h^2}{B H + b h}$ $e_2 = H - e_1$

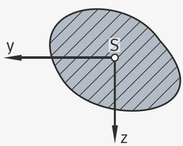
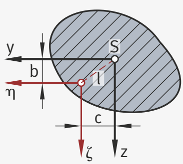
Table: Values for circular sections

For circular sections with diameter d, we arrive at the following 2nd moments of area and section moduli:

	Area moment		Section modulus	
	axial	polar	axial	polar
	$I_a = \dfrac{\pi\, d^4}{64}$	$I_p = 2 \cdot I_a$	$W_a = \dfrac{\pi\, d^3}{32}$	$W_p = 2 \cdot W_a$

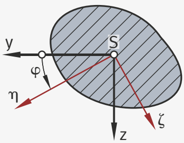
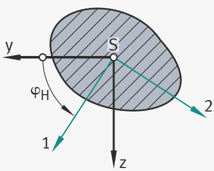
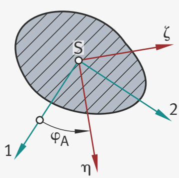
Area moments for various reference axes

The following table lists a number of 2nd moments of area for various reference axes.

Second moment of area for	Axial second moment of area	Centrifugal moment	Polar second moment of area
Any vertical centre-of-gravity axes yz perpendicular to each other			
	$I_y = \int_A z^2 \cdot dA$ $I_z = \int_A y^2 \cdot dA$	$I_{yz} = \int_A y \cdot z \cdot dA$	$I_{ps} = \int_A r^2 \cdot dA$ $I_{ps} = \int_A \left(y^2 + z^2\right) \cdot dA$ $= I_y + I_z$
Axes that are offset in parallel with the yz axes			
	$I_\eta = I_y + b^2 \cdot A$ $I_\zeta = I_z + c^2 \cdot A$	$I_{\eta\zeta} = I_{yz} + b \cdot c \cdot A$	$I_p = I_{ps} + l^2 \cdot A$ $= I_{ps} + (b^2 + c^2) \cdot A$ $= I_\eta + I_\zeta$

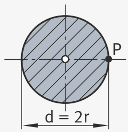
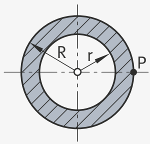
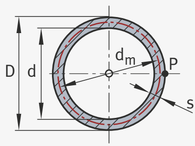
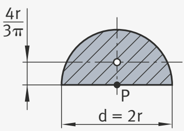
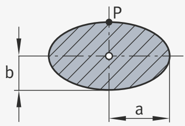
Continuation of table, see Page 246.

Continuation of table, Area moments for various reference axes, from Page 245.

Second moment of area for	Axial second moment of area	Centrifugal moment	Polar second moment of area
Axes which are rotated in the positive sense about the angle φ in relation to the yz axes			
	$I_\eta = \frac{I_y+I_z}{2} + \frac{I_y-I_z}{2}\cdot\cos 2\varphi - I_{yz}\cdot\sin 2\varphi$ $I_\zeta = \frac{I_y+I_z}{2} - \frac{I_y-I_z}{2}\cdot\cos 2\varphi - I_{yz}\cdot\sin 2\varphi$	$I_{\eta\zeta} = \frac{I_y-I_z}{2}\cdot\sin 2\varphi - I_{yz}\cdot\cos 2\varphi$	$I_{ps} = I_\eta + I_\zeta$ $= I_1 + I_2$
Principal inertia axes which are rotated in the positive sense about the angle φ_H in relation to the yz axes			
	$\sin 2\varphi_H = \frac{I_{yz}}{\sqrt{\left(\frac{I_y-I_z}{2}\right)^2 + I_{yz}^2}}$ $\cos 2\varphi_H = \frac{\frac{I_y+I_z}{2}}{\sqrt{\left(\frac{I_y-I_z}{2}\right)^2 + I_{yz}^2}}$ $I_{1,2} = \frac{I_y+I_z}{2} \pm \sqrt{\left(\frac{I_y-I_z}{2}\right)^2 + I_{yz}^2} = 0$		$I_{ps} = I_1 + I_2$ $= I_h + I_z$
Axes which are rotated in the positive sense about the angle φ_A in relation to the principal inertia axes			
	$I_\eta = \frac{I_1+I_2}{2} + \frac{I_1-I_2}{2}\cdot\cos 2\varphi_A$ $I_\zeta = \frac{I_1+I_2}{2} - \frac{I_1-I_2}{2}\cdot\cos 2\varphi_A$	$I_{\eta\zeta} = \frac{I_1-I_2}{2}\cdot\sin 2\varphi_A$	$I_{ps} = I_\eta + I_\zeta$ $= I_1 + I_2$

Torsional area moments and torsional section moduli

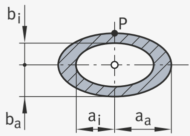
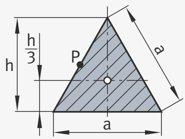
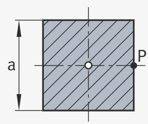
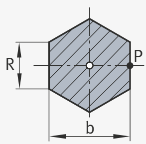
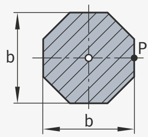
Torsional area moments and torsional section moduli can be calculated as follows (with "P" as the locations for τ_{max}):

Circle[1] 	$I_p = \frac{\pi\, d^4}{32} = \frac{\pi\, r^4}{2}$ $W_p = \frac{\pi\, d^3}{16} = \frac{\pi\, r^3}{2}$
Annulus[1] 	$I_p = \frac{\pi}{2}\left(R^4 - r^4\right)$ $W_p = \frac{\pi}{2}\,\frac{\left(R^4 - r^4\right)}{R}$
Thin-walled annulus[1] 	$\frac{D}{d} < 1{,}2$ $\qquad d_m = \frac{D+d}{2}$ $I_p = \frac{\pi}{4}\, d_m{}^3\, s$ $W_p = \frac{\pi}{2}\, d_m{}^2\, s$
Semicircle 	$I_t = 0{,}296\, r^4$ $W_t = 0{,}348\, r^3$
Ellipse[2] 	$I_t = \pi\, \frac{a^3\, b^3}{a^2 + b^2}$ $W_t = \frac{\pi}{2}\, a\, b^2$

Continuation of table, see Page 248.

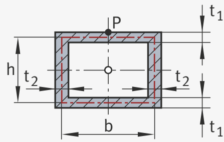
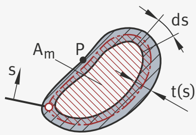
[1] τ_{max} prevails over entire circumference.
[2] P only applies if $a/b \geqq 1$.

Continuation of table, Torsional area moments and torsional section moduli, from Page 247.

Hollow ellipse 	$a_a : b_a = a_i : b_i = c$ $I_t = \pi \dfrac{c^3 \left(b_a{}^4 - b_i{}^4\right)}{1 + c^2}$ $W_t = \dfrac{\pi\, c \left(b_a{}^4 - b_i{}^4\right)}{2\, b_a}$
Equilateral triangle 	$I_t = \dfrac{a^4}{46{,}19} \approx \dfrac{h^4}{26}$ $W_t = \dfrac{a^3}{20} \approx \dfrac{h^3}{13}$
Square 	$I_t = 0{,}141\ a^4$ $W_t = 0{,}208\ a^3$
Hexagon 	$I_t = 0{,}115\ b^4 = 1{,}037\ R^4$ $W_t = 0{,}188\ b^3 = 0{,}977\ R^3$
Octagon 	$I_t = 0{,}108\ b^4$ $W_t = 0{,}185\ b^3$

Continuation of table, see Page 249.

Continuation of table, Torsional area moments and torsional section moduli, from Page 248.

Thin-walled box section 	$t_1 < t_2 \ll b, h$ $I_t = \dfrac{2\left(b\,h\right)^2}{\dfrac{b}{t_1} + \dfrac{h}{t_2}}$ $W_t = 2\,b\,h\,t_{min}$
Thin-walled hollow section 	$I_t = \dfrac{4\,A_m{}^2}{\oint \dfrac{ds}{t(s)}}$ $W_t = 2\,A_m t_{min}$ A_m = content of the area enclosed by the centre line

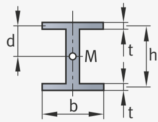
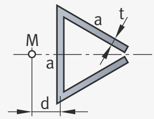
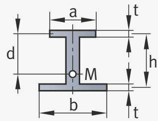
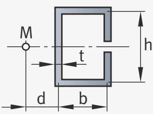
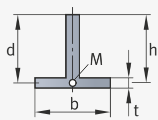
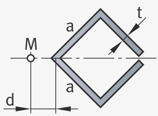
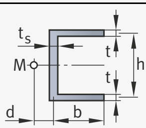
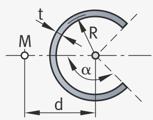
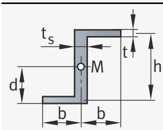
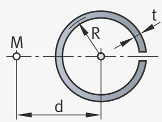
Bending due to transverse force

Shear centres of thin-walled profiles

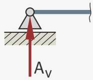
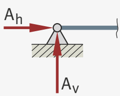
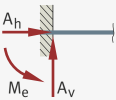
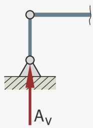
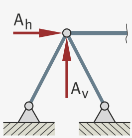
When symmetrical profiles are bent by a transverse force, the cross-section is devoid of torsion. The shear centre (transverse force centre) lies on the symmetry plane. If the profile cross-section has two axes of symmetry, the shear centre falls within the symmetry point, i.e. in the centre of gravity.

In general, this is no longer the case if the load plane does not coincide with a plane of symmetry belonging to the profile. Torsional loading of the profile can be avoided by shifting the load plane appropriately. This is purely dependent on the type of profile, but not on the size of the load (profile constant).

The positions d of the shear centres M are given below for a number of profiles.

Profile	Position d	Profile	Position d
	$d = \frac{h}{2}$		$d = \frac{a\sqrt{3}}{6}$
	$d = \frac{h b^3}{a^3 + b^3}$		$d = \frac{b}{2} \cdot \frac{3b + 2h}{3b + h}$
	$d = h$		$d = \frac{a\sqrt{2}}{4}$
	$d = \frac{3 t b^2}{h t_s + 6 b t}$		$d = 2R \frac{\sin \alpha - \alpha \cos \alpha}{\alpha - \sin \alpha \cos \alpha}$ α in radians
	$d = \frac{h}{2}$		$d = 2R$

Flat support types The following possible bearing reactions exist for flat support types:

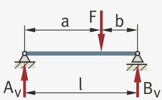
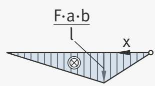
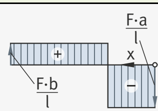
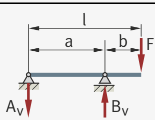
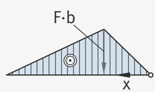
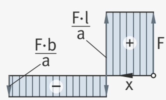
Support type	Degrees of freedom	Bearing reactions	Explanation
Movable pivoting bearing 	2	1	The beam coupled to the movable pivoting bearing can be moved in a horizontal direction and can be rotated about the pivot point. Consequently, it has two degrees of freedom. The bearing can only apply one reaction force acting perpendicular to the sliding direction.
Fixed pivoting bearing 	1	2	In the case of a fixed pivoting bearing, the beam cannot be moved in any direction and can only be rotated about the joint. The effect that the fixed pivoting bearing has on the beam can generally be represented by a force in any direction that can be split into two independent components.
Fixed restraint 	0	3	A securely clamped beam can neither be moved nor rotated. It has no degrees of freedom. The bearing arrangement can be loaded by forces and moments in any direction. The effect of the fixed restraint on the beam can therefore be represented by two forces and one restraining moment.
Pin support 	2	1	The effect of a pin support on the beam coupled to it is equivalent to that of a movable pivoting bearing. Perpendicular to the pin support, the beam can be moved and rotated about the joint. A reaction force can only be transmitted to the beam in the direction of the support.
Three-hinged support 	1	2	The effect of the three-hinged support corresponds to that of the fixed pivoting bearing. It prevents every translational movement in the plane clamped by the supports. Only one degree of freedom remains for rotation about the pivot point. The effect on the beam is covered by two independent forces.

Intermediate elements The following possible intermediate conditions exist for intermediate elements:

Intermediate element	Intermediate conditions	Intermediate reactions	Explanation
Joint 	$M_b = 0$	$Q \neq 0$ $N \neq 0$	A joint supplies the intermediate condition that the bending moment on the joint must disappear if freedom from friction of the beam connection is presumed. Therefore, in the event of a section through the joint, only one transverse force and one normal force occur as cross-sectional values or intermediate reactions.
Sliding sleeve 	$N = 0$	$Q \neq 0$ $M_b \neq 0$	A sliding sleeve cannot transmit a normal force. The disappearance of the normal force on it can be evaluated as an intermediate condition. Transverse force and bending moment can be transmitted as intermediate reactions. Once again, it is presumed that the connection is devoid of friction.

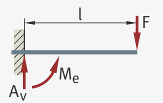
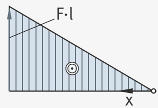
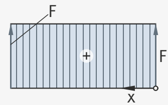
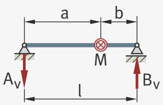
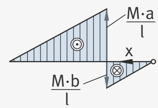
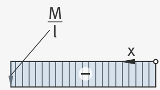
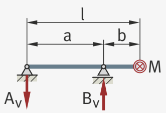
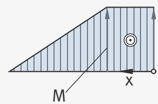
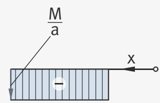
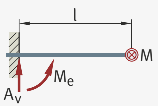
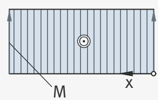
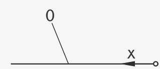
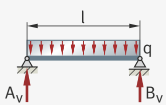
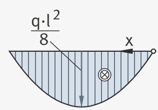
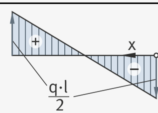
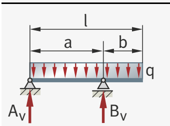
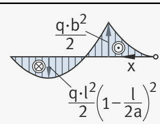
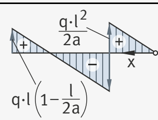
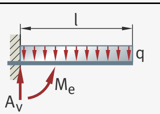
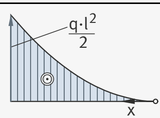
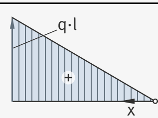
Bending due to transverse force for simple loaded beams It is possible to calculate bearing reactions, moment and transverse force distributions for simple loaded beams and formulate equations for the elastic curve.

Bearing reactions, moment and transverse force distributions The bearing reactions, moment and transverse force distributions are as follows for simple loaded beams:

System	Bearing reactions	Bending moment distribution	Transverse force distribution
	$A_v = F \cdot \frac{b}{l}$ $B_v = F \cdot \frac{a}{l}$		
	$A_v = F \cdot \frac{b}{a}$ $B_v = F \cdot \frac{l}{a}$		

Continuation of table, see Page 253.

Continuation of table, Bearing reactions, moment and transverse force distributions, from Page 252.

System	Bearing reactions	Bending moment distribution	Transverse force distribution
	$A_v = F$ $M_e = F \cdot l$		
	$A_v = \frac{M}{l}$ $B_v = \frac{M}{l}$		
	$A_v = \frac{M}{a}$ $B_v = \frac{M}{a}$		
	$A_v = 0$ $M_e = M$		
	$A_v = \frac{q \cdot l}{2}$ $B_v = \frac{q \cdot l}{2}$		
	$A_v = q \cdot l \left(1 - \frac{l}{2a}\right)$ $B_v = \frac{q \cdot l^2}{2a}$		
	$A_v = q \cdot l$ $M_e = \frac{q \cdot l^2}{2}$		

Elastic curve equation

If the elastic curve equations w(x) are formulated for simple loaded beams, the following applies:

Equation 20

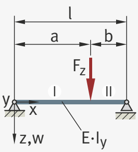
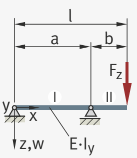
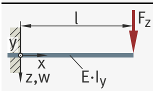
$$w''(x) = -\frac{M_{by}(x)}{E \cdot I_y(x)}$$

and:

Equation 21

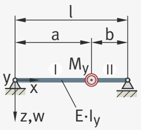
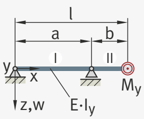
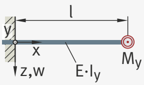
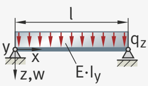
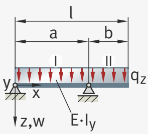
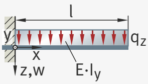
$$I_y(x) = \text{const.}$$

This gives the following:

System	Elastic curve equation w(x)	w_{max}
	$w_I = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(2\frac{a \cdot x}{l^2} - 3\frac{a^2 \cdot x}{l^3} + \frac{a^3 \cdot x}{l^4} + \frac{a \cdot x^3}{l^4} - \frac{x^3}{l^3}\right)$ $w_{II} = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(-\frac{a^3}{l^3} + 2\frac{a \cdot x}{l^2} + \frac{a^3 \cdot x}{l^4} - 3\frac{a \cdot x^2}{l^3} + \frac{a \cdot x^3}{l^4}\right)$	$w(a) = \frac{F_z \cdot a^2 \cdot b^2}{3\,E \cdot I_y \cdot l}$ $a > b$ $w_{I,\,max}$ where $x = l\sqrt{\frac{2\,a}{3\,l} - \frac{a^2}{3\,l^2}}$ $a < b$ $w_{II,\,max}$ where $x = l - l\sqrt{\frac{1}{3} - \frac{a^2}{3\,l^2}}$
	$w_I = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(\frac{a \cdot x}{l^2} - \frac{a^2 \cdot x}{l^3} - \frac{x^3}{a \cdot l^2} + \frac{x^3}{l^3}\right)$ $w_{II} = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(\frac{a^2}{l^2} - 4\frac{a \cdot x}{l^2} + \frac{a^2 \cdot x}{l^3} + 3\frac{x^2}{l^2} - \frac{x^3}{l^3}\right)$	$w_{I,\,max} = \frac{\sqrt{3} \cdot F_z \cdot a^2 \cdot b}{27\,E \cdot I_y}$ $w_{II,\,max} = \frac{F_z\, l \cdot b^2}{3\,E \cdot I_y}$
	$w = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(3\frac{x^2}{l^2} - \frac{x^3}{l^3}\right)$	$w_{max} = \frac{F_z \cdot l^3}{3\,E \cdot I_y}$

Continuation of table, see Page 255.

Continuation of table, Elastic curve equation, from Page 254.

System	Elastic curve equation w(x)	w_{max}
	$w_{\mathrm{I}} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(6\,\frac{a \cdot x}{l^2} - 2\,\frac{x}{l} - 3\,\frac{a^2 \cdot x}{l^3} - \frac{x^3}{l^3}\right)$ $w_{\mathrm{II}} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(-3\,\frac{a^2}{l^2} + 2\,\frac{x}{l} + 3\,\frac{a^2 \cdot x}{l^3} - 3\,\frac{x^2}{l^2} + \frac{x^3}{l^3}\right)$	$w_{\mathrm{I},\,max} = \frac{M_y \cdot l^2}{3\,E \cdot I_y} \cdot \left(-\frac{2}{3} + \frac{2\,a}{l} - \frac{a^2}{l^2}\right)^{\frac{3}{2}}$ $w_{\mathrm{II},\,max} = \frac{M_y \cdot l^2}{3\,E \cdot I_y}\left(\frac{1}{3} - \frac{a^2}{l^2}\right)^{\frac{3}{2}}$
	$w_{\mathrm{I}} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(\frac{a \cdot x}{l^2} - \frac{x^3}{a \cdot l^2}\right)$ $w_{\mathrm{II}} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(\frac{a^2}{l^2} - 4\,\frac{a \cdot x}{l^2} + 3\,\frac{x^2}{l^2}\right)$	$w_{\mathrm{I},\,max} = \frac{\sqrt{3} \cdot M_y \cdot a^2}{27\,E \cdot I_y}$ $w_{\mathrm{II},\,max} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(3 - \frac{4\,a}{l} + \frac{a^2}{l^2}\right)$
	$w = -\frac{M_y}{2\,E \cdot I_y}\,x^2$	$w_{max} = \frac{M_y \cdot l^2}{2\,E \cdot I_y}$
	$w = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(\frac{x}{l} - 2\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{max} = \frac{5\,q_z \cdot l^4}{384\,E \cdot I_y}$
	$w_{\mathrm{I}} = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(-2\,\frac{a \cdot x}{l^2} + 4\,\frac{a^2 \cdot x}{l^3} - \frac{a^3 \cdot x}{l^4} + 2\,\frac{x^3}{a \cdot l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$ $w_{\mathrm{II}} = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(2\,\frac{a^2}{l^2} - 8\,\frac{a \cdot x}{l^2} + 4\,\frac{a^2 \cdot x}{l^3} - \frac{a^3 \cdot x}{l^4} + 6\,\frac{x^2}{l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{\mathrm{II},\,max} = \frac{q_z \cdot l^4}{24\,E \cdot I_y} \cdot \left(3 - 8\,\frac{a}{l} + 6\,\frac{a^2}{l^2} - \frac{a^3}{l^3}\right)$
	$w = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(6\,\frac{x^2}{l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{max} = \frac{q_z \cdot l^4}{8\,E \cdot I_y}$

Principle of passive deformation work

Area of application

The principle of passive deformation work is used to:

- calculate deformation (deflection, torsion) in systems with a statically determinate bearing arrangement
- calculate bearing reactions in externally statically indeterminate systems
- calculate cross-sectional values in systems with a statically indeterminate internal structure

Checking

In every case, it is first necessary to check whether the system has an external statically determinate bearing arrangement and a statically determinate internal structure.

Example 1: Statically determinate system

If the system is both externally and internally statically determinate, the following guidelines apply:

- Apply a unit force "1" (for deflection) and a unit moment "M" (for torsion) to the system, in the direction of the sought-after deformation, as an external load, at the point in the system where the sought-after deformation occurs (deflection or torsion). During the loading sequence, the system is only loaded using the unit value to start with and only then are the effective external forces applied (point 1).
- Determine normal force, transverse force (mostly negligible) and moment distributions (bending moment, torsion moment) separately for each external load, including the unit values. The same running coordinate x must be retained in order to determine all cross-sectional values. The cross-sectional values resulting from the unit values are identified by a dash (point 2).
- Specify the entire passive internal deformation work occurring in the system. Thus:
  external passive deformation work = internal passive deformation work (point 3):

Equation 22

$$\begin{matrix} ① & \text{"1"} \cdot w = \\ ② & \text{"M"} \cdot \varphi = \end{matrix} \int \frac{M_b \cdot \bar{M}_b}{E \cdot I_a} \cdot dx + \int \frac{M_t \cdot \bar{M}_t}{G \cdot I_p} \cdot dx + \int \frac{N \cdot \bar{N}}{E \cdot A} \cdot dx + ③$$

① Deflection
② Torsion
③ Passive spring work (transverse forces are ignored)

- Evaluate the integrals with the aid of the integral tables or by mathematical calculation (observe the signs of the cross-sectional values) (point 4)
- Divide the resulting relationship by the unit value and calculate the desired value (point 5)

Example 2: Statically indeterminate system

If the system is statically indeterminate, it may:

- have a statically indeterminate bearing arrangement
- have a statically indeterminate internal structure

System with statically indeterminate bearing arrangement

If the system has a statically indeterminate bearing arrangement, the following guidelines apply:

- Make the system statically determinate by detaching surplus support joints and by applying the bearing reactions to the system as external impressed forces at these points. At the same time, a marginal condition must also be specified for this, which identifies the initial state (point 1).
- At the point where the marginal condition exists, apply a unit force “1” (for deflection) or a unit moment “M” (for torsion) (point 2).
- See points 2 to 4 for statically determinate systems (point 3).
- Apply the marginal condition, divide the relationship by the unit value and calculate the unknown bearing reaction (point 4).

System with a statically indeterminate internal structure

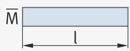
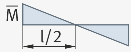
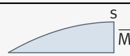
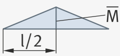
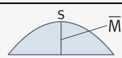
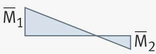
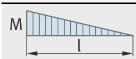
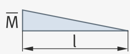
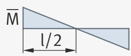
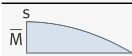
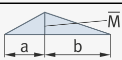
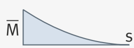
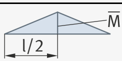
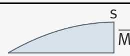
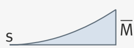
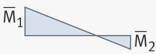
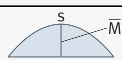
If the system has a statically indeterminate internal structure, the following guidelines apply:

- Make the system statically determinate by installing joints or movable sleeves, or by guiding sections etc. Apply cross-sectional values as external impressed forces and define the marginal conditions.
- See points 1 to 4 for systems with statically indeterminate bearing arrangements.

In current practice, systems with statically indeterminate bearing arrangements are calculated numerically.

Tables of integrals The following table shows a selection of sample calculations for the integral: $\int M \cdot \bar{M} \cdot dx$

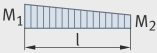
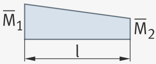
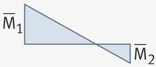
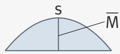
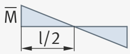
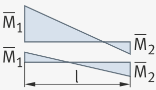
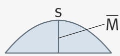
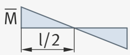
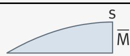
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$M \cdot \bar{M} \cdot l$		0
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		
			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{1}{6} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{5}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(1 + \frac{b}{l}\right)$		$\frac{1}{4} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot \bar{M} \cdot l$		$\frac{1}{4} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot (2\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot (2\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$

Continuation of table, see Page 259.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 258.

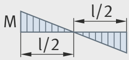
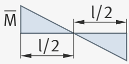
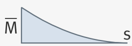
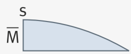
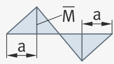
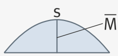
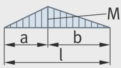
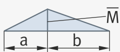
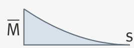
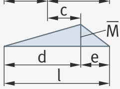
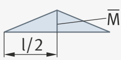
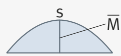
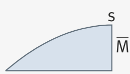
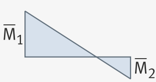
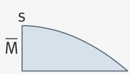
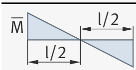
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{6}\left[M_1 \cdot \left(2\bar{M}_1 + \bar{M}_2\right) + M_2 \cdot \left(2\bar{M}_2 + \bar{M}_1\right)\right] \cdot l$		$\frac{1}{12}\bar{M} \cdot \left(M_1 + 3M_2\right) \cdot l$
	$\frac{1}{6}\left[2\left(M_1 \cdot \bar{M}_1 + M_2 \cdot \bar{M}_2\right) + M_2 \cdot \bar{M}_1 + M_1 \cdot \bar{M}_2\right] \cdot l$		$\frac{1}{3}\bar{M} \cdot \left(M_1 + M_2\right) \cdot l$
	$\frac{1}{6}\bar{M} \cdot \left(M_1 + M_2\right) \cdot l$		$\frac{1}{12}\bar{M} \cdot \left(3M_1 + 5M_2\right) \cdot l$
	$\frac{1}{12}\bar{M} \cdot \left(3M_1 + M_2\right) \cdot l$		$\frac{1}{12}\bar{M} \cdot \left(5M_1 + 3M_2\right) \cdot l$
			
	$\frac{1}{6}\left[2\left(M_1 \cdot \bar{M}_1 + M_2 \cdot \bar{M}_2\right) + M_2 \cdot \bar{M}_1 + M_1 \cdot \bar{M}_2\right] \cdot l$		$\frac{1}{3}\bar{M} \cdot \left(M_1 + M_2\right) \cdot l$
	$\frac{1}{6}\bar{M} \cdot \left(M_1 - M_2\right) \cdot l$		$\frac{1}{12}\bar{M} \cdot \left(3M_1 + 5M_2\right) \cdot l$
	$\frac{1}{12}\bar{M} \cdot \left(3M_1 + M_2\right) \cdot l$		$\frac{1}{12}\bar{M} \cdot \left(5M_1 + 3M_2\right) \cdot l$
	$\frac{1}{12}\bar{M} \cdot \left(M_1 + 3M_2\right) \cdot l$		

Continuation of table, see Page 260.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 259.

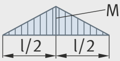
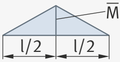
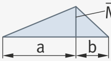
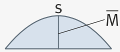
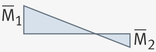
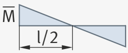
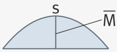
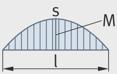
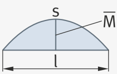
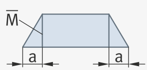
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$-\frac{1}{6} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{1}{6} M \cdot \bar{M} \cdot l$
	$-\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(1 - \frac{a}{l}\right)$
	0		
			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(3 - 3\frac{a}{l} + \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(2 - \frac{c^2}{d \cdot b}\right)$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(3 - 3\frac{b}{l} + \frac{b^2}{l^2}\right)$
	$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(\frac{3\,l}{4\,b} - \frac{a^2}{l \cdot b}\right)$		$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(1 + \frac{a}{l} - \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \left[\bar{M}_1 \cdot \left(1 + \frac{b}{l}\right) + \bar{M}_2 \cdot \left(1 + \frac{a}{l}\right)\right] \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(5 - \frac{b}{l} - \frac{b^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \left[\bar{M}_1 \cdot \left(1 + \frac{b}{l}\right) + \bar{M}_2 \cdot \left(1 + \frac{a}{l}\right)\right] \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(5 - \frac{a}{l} - \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(1 - 2\frac{a}{l}\right)$		

Continuation of table, see Page 261.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 260.

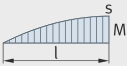
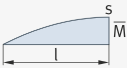
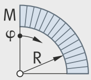
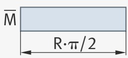
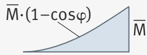
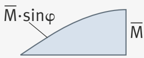
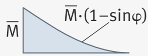
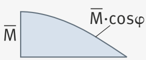
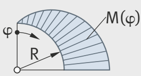
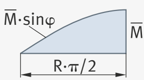
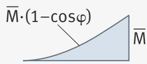
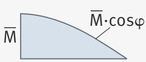
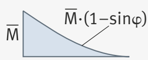
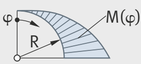
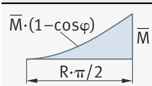
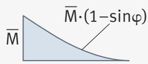
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{7}{48} M \cdot \bar{M} \cdot l$
	$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(\frac{3\,l}{4\,b} - \frac{a^2}{l \cdot b}\right)$		$\frac{7}{48} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{5}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{17}{48} M \cdot \bar{M} \cdot l$
	0		$\frac{17}{48} M \cdot \bar{M} \cdot l$
			
	$\frac{1}{5} M \cdot \bar{M} \cdot l$		$\frac{2}{15} M \cdot \bar{M} \cdot l$
	$\frac{1}{30} M \cdot \bar{M} \cdot l$		$\frac{3}{10} M \cdot \bar{M} \cdot l$
	$\frac{1}{5} M \cdot \bar{M} \cdot l$		
			
	$\frac{8}{15} M \cdot \bar{M} \cdot l$		$\frac{7}{15} M \cdot \bar{M} \cdot l$
	$\frac{7}{15} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l \cdot \left(1 - 2\frac{a^2}{l^2} + \frac{a^3}{l^3}\right)$

Continuation of table, see Page 262.

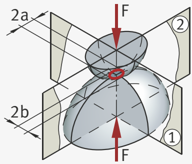
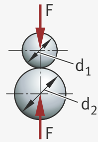
Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 261.

Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{8}{15} M \cdot \bar{M} \cdot l$		$\frac{1}{30} M \cdot \bar{M} \cdot l$
Constant and harmonic moment distributions on circular arc beams			
	$dx = R \cdot d\varphi$ M = const.		
	$M \cdot \bar{M} \cdot R \cdot \frac{\pi}{2}$		$M \cdot \bar{M} \cdot R \cdot \left(\frac{\pi}{2} - 1\right)$
	$M \cdot \bar{M} \cdot R$		$M \cdot \bar{M} \cdot R \cdot \left(\frac{\pi}{2} - 1\right)$
	$M \cdot \bar{M} \cdot R$		
	$M(\varphi) = \sin \varphi$		
	$\frac{1}{4} M \cdot \bar{M} \cdot R \cdot \pi$		$\frac{1}{2} M \cdot \bar{M} \cdot R$
	$\frac{1}{2} M \cdot \bar{M} \cdot R$		$M \cdot \bar{M} \cdot R \cdot \left(1 - \frac{\pi}{4}\right)$
	$M(\varphi) = 1 - \cos \varphi$		
	$M \cdot \bar{M} \cdot R \cdot \left(\frac{3}{4}\pi - 2\right)$		$M \cdot \bar{M} \cdot R \cdot \left(\frac{\pi}{2} - \frac{3}{2}\right)$

Hertzian contact and pressure

Calculating contact pairs The equations for calculating a number of important Hertzian contact pairs are:

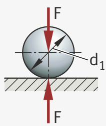
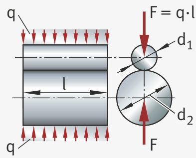
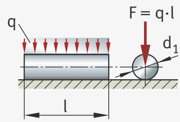
Contact type	General	Contact between curved surfaces	Point contact ball/ball
Hertzian pressure (general formula)	p_{max}	$p_{max} = \frac{3}{2} \cdot \frac{F}{\pi \cdot a\,b}$	$p_{max} = \frac{3}{2} \cdot \frac{F}{\pi \cdot a^2}$
Solid body combination Solid body 1, solid body 2[1)]	$\sum\rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22}$ Calculation of the curvature total, see Page 265.	①, ② Principal planes of curvature 	Ball/ball 
Main axes of the "elliptical" contact area	a, b a = major semi-axis b = minor semi-axis	$a = \xi \sqrt[3]{\frac{3F(1-\nu^2)}{E \cdot \sum\rho}}$ $b = \eta \sqrt[3]{\frac{3F(1-\nu^2)}{E \cdot \sum\rho}}$ $\xi, \eta = f(\cos\tau)$[2)]	$a = b = \sqrt[3]{\frac{3F(1-\nu^2)}{4E\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}}$
Maximum Hertzian pressure	p_{max}	$p_{max} = \frac{1}{\xi \cdot \eta} \sqrt[3]{\frac{3F \cdot E^2 \left(\sum\rho\right)^2}{8\pi^3 (1-\nu^2)^2}}$ $\xi, \eta = f(\cos\tau)$[2)]	$p_{max} = \sqrt[3]{\frac{6F \cdot E^2}{\pi^3 (1-\nu^2)^2} \left(\frac{1}{d_1}+\frac{1}{d_2}\right)^2}$
Convergence of both solid bodies	δ	$\delta = \frac{\psi}{\xi} \sqrt[3]{\frac{9F^2 \sum\rho\,(1-\nu^2)^2}{8E^2}}$ $\psi/\xi = f(\cos\tau)$[2)]	$\delta = \sqrt[3]{\frac{9F^2(1-\nu^2)^2}{2E^2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}$
Hertzian pressure for values from column "General"	p_{max} in N/mm^2 for $\nu_1 = \nu_2 = 0{,}3$ $E_1 = E_2 = 2{,}1 \cdot 10^5\ N/mm^2$ d, l in mm, F in N	$p_{max} = \frac{864}{\xi \cdot \eta} \sqrt[3]{F\left(\sum\rho\right)^2}$ $\xi \cdot \eta = f(\cos\tau)$[2)]	$p_{max} = 2176 \sqrt[3]{F\left(\frac{1}{d^1}+\frac{1}{d^2}\right)^2}$

Continuation of table, see Page 264.

1) If the solid bodies are composed of different materials with the elastic constants E_1 and ν_1 or E_2 and ν_2, the term $\frac{1-\nu^2}{E}$ is replaced in all equations by $\frac{1}{2}\left(\frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2}\right)$

2) Auxiliary value $\cos\tau$ see Page 265.

Continuation of table, Calculating contact pairs, from Page 263.

Contact type	Point contact ball/plane	Line contact	
Hertzian pressure (general formula)	$p_{max} = \frac{3}{2}\cdot\frac{F}{\pi\cdot a^2}$	$p_{max} = \frac{4}{\pi}\cdot\frac{F}{2b\cdot l} = \frac{2}{\pi}\cdot\frac{F}{b\cdot l}$	
Solid body combination Solid body 1, solid body 2[1)]	Ball/plane 	Cylinder/cylinder 	Cylinder/plane 
Main axes of the "elliptical" contact area	$a = b = \sqrt[3]{\frac{3F\left(1-\nu^2\right)d_1}{4E}}$	$a = l$ $b = \sqrt{\frac{4F\left(1-\nu^2\right)}{\pi\cdot E\cdot l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}}$	$a = l$ $b = \sqrt{\frac{4F\left(1-\nu^2\right)d_1}{\pi\cdot E\cdot l}}$ Contact area = rectangle
Maximum Hertzian pressure	p_{max} $= \sqrt[3]{\frac{6F\cdot E^2}{\pi^3\left(1-\nu^2\right)^2\cdot d_1^2}}$	p_{max} $= \sqrt{\frac{F\cdot E}{\pi\cdot l\left(1-\nu^2\right)}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}$	$p_{max} = \sqrt{\frac{F\cdot E}{\pi\cdot d_1\cdot l\cdot\left(1-\nu^2\right)}}$
Convergence of both solid bodies	$\delta = \sqrt[3]{\frac{9F^2\left(1-\nu^2\right)^2}{2E^2\cdot d_1}}$	$\delta = \frac{2F}{\pi l}\left[\frac{1-\nu_1^2}{E_1}\left(\ln\frac{d_1}{b}+0,407\right)\right.$ $\left.+\frac{1-\nu_2^2}{E_2}\left(\ln\frac{d_2}{b}+0,407\right)\right]$	$\delta = \frac{3,97}{10^5}\cdot\frac{F^{0,9}}{l^{0,8}}$ for steel/steel
Hertzian pressure for values from column "General"	$p_{max} = 2176\sqrt[3]{\frac{F}{d_1^2}}$	$p_{max} = 271\sqrt{\frac{F}{d_1\cdot l}\left(1+\frac{d_1}{d_2}\right)}$	$p_{max} = 271\sqrt{\frac{F}{d_1\cdot l}}$

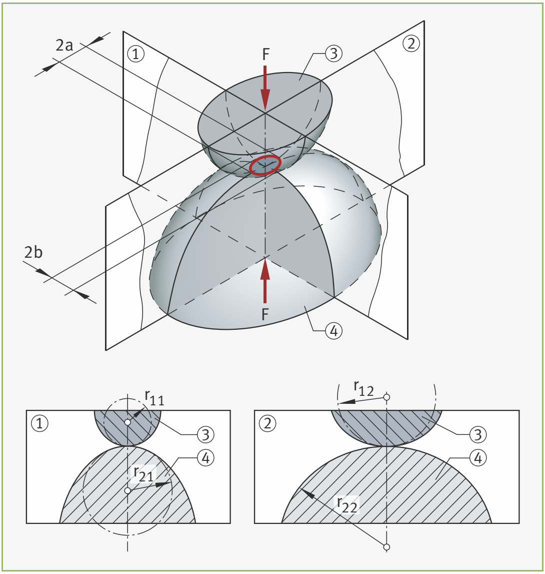
1) If the solid bodies are composed of different materials with the elastic constants E_1 and ν_1 or E_2 and ν_2, the term $\frac{1-\nu^2}{E}$ is replaced in all equations by $\frac{1}{2}\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)$

Hertzian coefficients for curved surfaces

Values for the coefficients ξ, η, $\xi \cdot \eta$, ψ/ξ according to Hertz are calculated below for contact between curved surfaces under load.
The Hertzian coefficients ξ, η and ψ/ξ are also designated μ, ν and $2K/(\pi\ \mu)$, see publication Ball and Roller Bearings WLP.

Figure 4
Contact between curved surfaces under load

① Principal curvature plane 1
② Principal curvature plane 2
③ Solid body 1
④ Solid body 2



Calculation
Equation 23

For the curvature ρ, the following applies:

$$\rho_{ij} = \frac{1}{r_{ij}} > 0 \quad \text{(convex)} \qquad \rho_{ij} = \frac{1}{r_{ij}} < 0 \quad \text{(concave)}$$

$$\sum \rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22} = \frac{1}{r_{11}} + \frac{1}{r_{12}} + \frac{1}{r_{21}} + \frac{1}{r_{22}}$$

and for the auxiliary value $\cos \tau$:

Equation 24

$$\cos \tau = \frac{|\rho_{11} - \rho_{12} + \rho_{21} - \rho_{22}|}{\sum \rho}$$

The axes a, b of the contact ellipse are calculated using:

Equation 25

$$a = \xi \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum \rho}} \qquad b = \eta \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum \rho}}$$

For the maximum Hertzian pressure, the following applies:

Equation 26

$$p_{max} = \frac{1}{\xi \cdot \eta} \sqrt[3]{\frac{3F \cdot E^2 \left(\sum \rho\right)^2}{8\pi^3 \left(1-\nu^2\right)^2}}$$

and for the convergence of solid bodies:

Equation 27

$$\delta = \frac{\psi}{\xi} \sqrt[3]{\frac{9F^2 \cdot \sum \rho \left(1-\nu^2\right)^2}{8E^2}}$$

Table: Hertzian coefficients

The coefficients ξ, η, $\xi \cdot \eta$, ψ/ξ according to Hertz for contact between curved surfaces under load are thus:

cos τ	ξ	η	ξ · η	ψ/ξ	cos τ	ξ	η	ξ · η	ψ/ξ
0,9995	23,95	0,163	3,91	0,171	0,9870	7,02	0,301	2,11	0,411
0,9990	18,53	0,185	3,43	0,207	0,9865	6,93	0,303	2,10	0,416
0,9985	15,77	0,201	3,17	0,230	0,9860	6,84	0,305	2,09	0,420
0,9980	14,25	0,212	3,02	0,249	0,9855	6,74	0,307	2,07	0,423
0,9975	13,15	0,220	2,89	0,266	0,9850	6,64	0,310	2,06	0,427
0,9970	12,26	0,228	2,80	0,279	0,9845	6,55	0,312	2,04	0,430
0,9965	11,58	0,235	2,72	0,291	0,9840	6,47	0,314	2,03	0,433
0,9960	11,02	0,241	2,65	0,302	0,9835	6,40	0,316	2,02	0,437
0,9955	10,53	0,246	2,59	0,311	0,9830	6,33	0,317	2,01	0,440
0,9950	10,15	0,251	2,54	0,320	0,9825	6,26	0,319	2,00	0,444
0,9945	9,77	0,256	2,50	0,328	0,9820	6,19	0,321	1,99	0,447
0,9940	9,46	0,260	2,46	0,336	0,9815	6,12	0,323	1,98	0,450
0,9935	9,17	0,264	2,42	0,343	0,9810	6,06	0,325	1,97	0,453
0,9930	8,92	0,268	2,39	0,350	0,9805	6,00	0,327	1,96	0,456
0,9925	8,68	0,271	2,36	0,356	0,9800	5,94	0,328	1,95	0,459
0,9920	8,47	0,275	2,33	0,362	0,9795	5,89	0,330	1,94	0,462
0,9915	8,27	0,278	2,30	0,368	0,9790	5,83	0,332	1,93	0,465
0,9910	8,10	0,281	2,28	0,373	0,9785	5,78	0,333	1,92	0,468
0,9905	7,93	0,284	2,25	0,379	0,9780	5,72	0,335	1,92	0,470
0,9900	7,76	0,287	2,23	0,384	0,9775	5,67	0,336	1,91	0,473
0,9895	7,62	0,289	2,21	0,388	0,9770	5,63	0,338	1,90	0,476
0,9890	7,49	0,292	2,19	0,393	0,9765	5,58	0,339	1,89	0,478
0,9885	7,37	0,294	2,17	0,398	0,9760	5,53	0,340	1,88	0,481
0,9880	7,25	0,297	2,15	0,402	0,9755	5,49	0,342	1,88	0,483
0,9875	7,13	0,299	2,13	0,407	0,9750	5,44	0,343	1,87	0,486

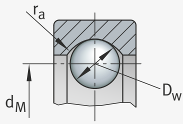
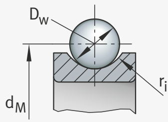
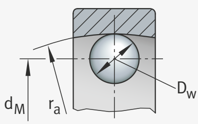
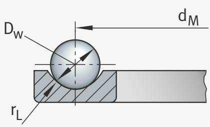
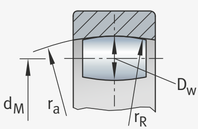
Continuation of table, see Page 267.

Continuation of table, Table: Hertzian coefficients, from Page 266.

cos τ	ξ	η	ξ · η	ψ/ξ	cos τ	ξ	η	ξ · η	ψ/ξ
0,9745	5,39	0,345	1,86	0,489	0,918	3,36	0,441	1,48	0,650
0,9740	5,35	0,346	1,85	0,491	0,916	3,33	0,443	1,47	0,653
0,9735	5,32	0,347	1,85	0,493	0,914	3,30	0,445	1,47	0,657
0,9730	5,28	0,349	1,84	0,495	0,912	3,27	0,448	1,46	0,660
0,9725	5,24	0,350	1,83	0,498	0,910	3,23	0,450	1,45	0,664
0,9720	5,20	0,351	1,83	0,500	0,908	3.20	0,452	1,45	0,667
0,9715	5,16	0,353	1,82	0,502	0,906	3,17	0,454	1,44	0,671
0,9710	5,13	0,354	1,81	0,505	0,904	3,15	0,456	1,44	0,674
0,9705	5,09	0,355	1,81	0,507	0,902	3,12	0,459	1,43	0,677
0,9700	5,05	0,357	1,80	0,509	0,900	3,09	0,461	1,42	0,680
0,9690	4,98	0,359	1,79	0,513	0,895	3,03	0,466	1,41	0,688
0,9680	4,92	0,361	1,78	0,518	0,890	2,97	0,471	1,40	0,695
0,9670	4,86	0,363	1,77	0,522	0,885	2,92	0,476	1,39	0,702
0,9660	4,81	0,365	1,76	0,526	0,880	2,86	0,481	1,38	0,709
0,9650	4,76	0,367	1,75	0,530	0,875	2,82	0,485	1,37	0,715
0,9640	4,70	0,369	1,74	0,533	0,870	2,77	0,490	1,36	0,721
0,9630	4,65	0,371	1,73	0,536	0,865	2,72	0,494	1,35	0,727
0,9620	4,61	0,374	1,72	0,540	0,860	2,68	0,498	1,34	0,733
0,9610	4,56	0,376	1,71	0,543	0,855	2,64	0,502	1,33	0,739
0,9600	4,51	0,378	1,70	0,546	0,850	2,60	0,507	1,32	0,745
0,9590	4,47	0,380	1,70	0,550	0,840	2,53	0,515	1,30	0,755
0,9580	4,42	0,382	1,69	0,553	0,830	2,46	0,523	1,29	0,765
0,9570	4,38	0,384	1,68	0,556	0,820	2,40	0,530	1,27	0,774
0,9560	4,34	0,386	1,67	0,559	0,810	2,35	0,537	1,26	0,783
0,9550	4,30	0,388	1,67	0,562	0,800	2,30	0,544	1,25	0,792
0,9540	4,26	0,390	1,66	0,565	0,750	2,07	0,577	1,20	0,829
0,9530	4,22	0,391	1,65	0,568	0,700	1,91	0,607	1,16	0,859
0,9520	4,19	0,393	1,65	0,571	0,650	1,77	0,637	1,13	0,884
0,9510	4,15	0,394	1,64	0,574	0,600	1,66	0,664	1,10	0,904
0,9500	4,12	0,396	1,63	0,577	0,550	1,57	0,690	1,08	0,922
0,9480	4,05	0,399	1,62	0,583	0,500	1,48	0,718	1,06	0,938
0,9460	3,99	0,403	1,61	0,588	0,450	1,41	0,745	1,05	0,951
0,9440	3,94	0,406	1,60	0,593	0,400	1,35	0,771	1,04	0,962
0,9420	3,88	0,409	1,59	0,598	0,350	1,29	0,796	1,03	0,971
0,9400	3,83	0,412	1,58	0,603	0,300	1,24	0,824	1,02	0,979
0,9380	3,78	0,415	1,57	0,608	0,250	1,19	0,850	1,01	0,986
0,9360	3,73	0,418	1,56	0,613	0,200	1,15	0,879	1,01	0,991
0,9340	3,68	0,420	1,55	0,618	0,150	1,11	0,908	1,01	0,994
0,9320	3,63	0,423	1,54	0,622	0,100	1,07	0,938	1,00	0,997
0,9300	3,59	0,426	1,53	0,626	0,050	1,03	0,969	1,00	0,999
0,928	3,55	0,428	1,52	0,630	0	1	1	1	1
0,926	3,51	0,431	1,51	0,634					
0,924	3,47	0,433	1,50	0,638					
0,922	3,43	0,436	1,50	0,642					
0,920	3,40	0,438	1,49	0,646					

Hertzian pressure in rolling bearings

The following auxiliary values are used to calculate Hertzian pressure in rolling bearings:

Deep groove ball bearings ball – outer ring 	$\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ [1] $r_a \approx 0{,}53 \cdot D_w$	$\sum\rho = \frac{2}{D_w} \cdot \left(2 - \frac{\gamma}{1+\gamma} - \frac{D_w}{2 \cdot r_a}\right)$ $\cos\tau = \frac{-\frac{\gamma}{1+\gamma} + \frac{D_w}{2 \cdot r_a}}{2 - \frac{\gamma}{1+\gamma} - \frac{D_w}{2 \cdot r_a}}$
Deep groove ball bearings ball – inner ring 	$\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ [1] $r_i \approx 0{,}52 \cdot D_w$	$\sum\rho = \frac{2}{D_w} \cdot \left(2 + \frac{\gamma}{1-\gamma} - \frac{D_w}{2 \cdot r_i}\right)$ $\cos\tau = \frac{\frac{\gamma}{1-\gamma} + \frac{D_w}{2 \cdot r_i}}{2 + \frac{\gamma}{1-\gamma} - \frac{D_w}{2 \cdot r_i}}$
Self-aligning ball bearings ball – outer ring 	$\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ [1] $r_a = \frac{d_M + D_w}{2}$	$\sum\rho = \frac{4}{D_w} \left(\frac{1}{1+\gamma}\right)$ $\cos\tau = 0$
Axial ball bearings ball – bearing ring 	$\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ [1] $r_L \approx 0{,}54 \cdot D_w$	$\sum\rho = \frac{2}{D_w} \left(2 - \frac{D_w}{2 \cdot r_L}\right)$ $\cos\tau = \frac{1}{4 \cdot \frac{r_L}{D_w} - 1}$
Barrel roller bearings, spherical roller bearings roller – outer ring 	$\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ [1] $r_a = \frac{d_M + D_w}{2}$	$\sum\rho = \frac{1 + \gamma \cdot \left(\frac{r_A}{r_R} - 1\right)}{1+\gamma}$ $\cos\tau = \frac{1 - \gamma \cdot \left(\frac{r_a}{r_R} - 1\right)}{1 + \gamma \cdot \left(\frac{r_a}{r_R} - 1\right)}$

[1] α is the nominal contact angle of the bearing.

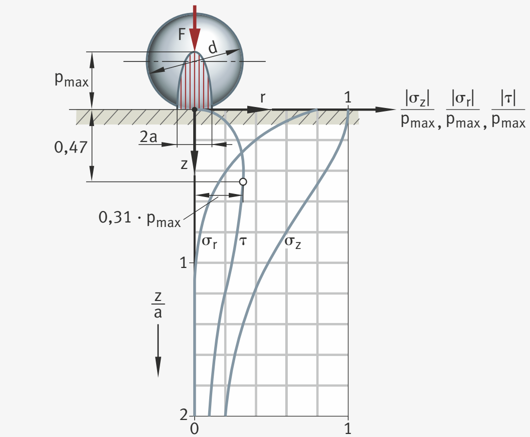
Stress state under Hertzian contact

According to Hertz's theory, stresses and deformations occur due to the effect of compressive forces where there is contact (point type or linear) between two solid bodies.

Point contact ball – plane

In the event of ball – plane point contact, the following relationships apply:

Figure 5
Point contact ball – plane



For the radius of the contact area, the following applies:

Equation 28

$$a = \sqrt[3]{\frac{3\,F\left(1-\nu^2\right)\cdot d}{4\,E}}$$

The stress state (for r = 0) is calculated using:

Equation 29

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\left(\frac{z}{a}\right)^2 + 1}$$

$$\frac{\sigma_r}{p_{max}} = -(1+\nu)\left[1 - \frac{z}{a}\arctan\left(\frac{a}{z}\right)\right] + \frac{1}{2\left[\left(\frac{z}{a}\right)^2 + 1\right]}$$

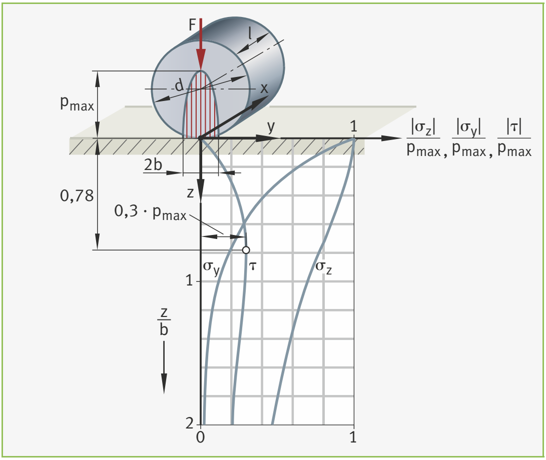
$$\frac{\tau}{p_{max}} = -\frac{3}{4}\cdot\frac{1}{\left(\frac{z}{a}\right)^2 + 1} + \frac{1+\nu}{2}\left[1 - \frac{z}{a}\arctan\left(\frac{a}{z}\right)\right]$$

Line contact roller – plane

Figure 6

Line contact roller – plane (planar stress state)

In the event of roller – plane line contact (**planar stress state**), the following relationships apply:



For the half width of the contact area, the following applies:

Equation 30

$$b = \sqrt{\frac{4\,F\left(1-\nu^2\right)\cdot d}{\pi \cdot E \cdot l}}$$

The stress state (for y = 0) is calculated using:

Equation 31

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

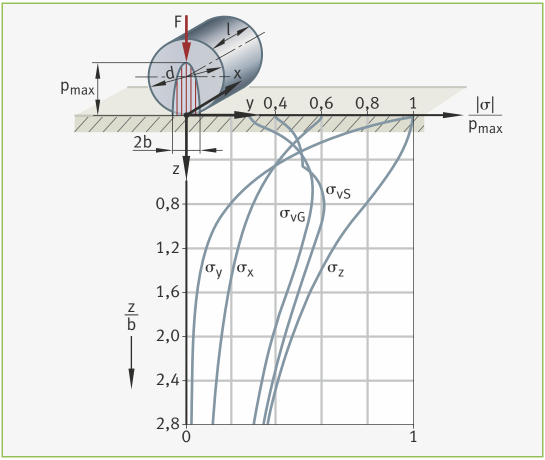
$$\frac{\sigma_y}{p_{max}} = 2\left(\frac{z}{b}\right) - \frac{1+2\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

$$\frac{\tau}{p_{max}} = \frac{z}{b} - \frac{\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

Line contact roller – plane

In the event of roller – plane line contact (**spatial stress state**), the following relationships apply:

Figure 7
Line contact roller – plane (spatial stress state)



The stress state (for x = y = 0) is calculated using:

Equation 32

$$\frac{\sigma_x}{p_{max}} = -2\nu\left[\sqrt{1+\left(\frac{z}{b}\right)^2} - \left(\frac{z}{b}\right)\right]$$

$$\frac{\sigma_y}{p_{max}} = -\left[\frac{1+2\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}} - 2\left(\frac{z}{b}\right)\right]$$

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

These equations represent the maximum stresses for the coordinates x = y = 0. They are based on the assumption of a plane deformation state ($\epsilon_x = 0$).

Equivalent stress

As strength hypotheses for calculating an equivalent stress, the following hypotheses have generally gained acceptance and proved effective:

- **shear stress hypothesis** according to **Tresca** and **St. Venant**
- **distortion energy hypothesis** according to **Hencky** and **von Mises**

Shear stress hypothesis

According to the shear stress hypothesis, it is assumed that the material begins to flow when the maximum shear stress at any one point reaches a critical value.

The equivalent stress is calculated as:

Equation 33

$$\sigma_{vS} = 2\tau_{max} = \max\begin{bmatrix}\sigma_z - \sigma_y \\ \sigma_z - \sigma_x \\ \sigma_y - \sigma_x\end{bmatrix}$$

Distortion energy hypothesis

By contrast, according to the distortion energy hypothesis, plastic deformation sets in when the distortion energy that can be absorbed elastically in a volume element is exceeded.

The equivalent stress is calculated as:

Equation 34

$$\sigma_{vG} = \sqrt{\frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2\right]}$$

with the principal normal stresses σ_x, σ_y, σ_z.

Material strength

According to both the shear stress hypothesis and the distortion energy hypothesis, the maximum equivalent stress within the material is:

Shear stress hypothesis	Distortion energy hypothesis
$\sigma_{vS\,max} = 0{,}60 \cdot p_{max}$ at a depth $z = 0{,}78 \cdot b$	$\sigma_{vG\,max} = 0{,}56 \cdot p_{max}$ at a depth $z = 0{,}71 \cdot b$

If the equivalent stress according to the shear stress hypothesis is assumed, the material strain is:

Equation 35

$$\sigma_{vS\,max} = 0{,}60 \cdot p_{max}$$

In order to prevent plastic deformation, according to the shear stress hypothesis, in the material under static loading (constant strength over the whole cross-section), the following must be fulfilled:

Equation 36

$$\sigma_{vS\,max} < R_{p0,2}$$

In a given material with the yield point R_e or substitute proof stress $R_{p0,2}$, this gives a permissible maximum Hertzian pressure of:

Equation 37

$$p_{max\,per} < 1{,}67 \cdot R_{p0,2}$$

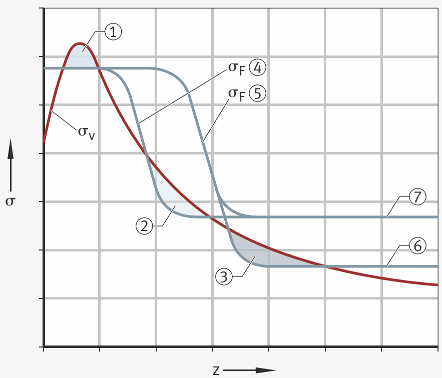
For case, flame or induction hardened materials, an adequate hardening depth must be ensured. The hardening depth according to DIN EN ISO 2639:2002 is the depth of hardened surface zone which has a hardness of at least 550 HV. The hardness curve to the core of the material must also be such that the strength and yield point of the material which can be derived from the hardness is above the equivalent stress curve at all points.

Figure 8 shows where deformation zones can form in the material when comparing the material strain with the yield point of the material:

Figure 8
Deformation zones

σ_V = equivalent stress
σ_F = yield stress
z = surface distance

①, ②, ③ Deformation zones
④ For small case hardening depth
⑤ For large case hardening depth
⑥ Low core strength
⑦ High core strength



- In zone ①, the yield point of the material is exceeded in the area of maximum stress in a material with constant strength, respectively through hardening, as well as surface layer hardening. This deformation occurs at sufficiently high Hertzian pressure in all materials and for all hardening processes.
- In zone ②, the material deforms plastically if the chosen hardening depth is too low.
- In zone ③, plastic deformation occurs if the hardness or the yield stress of the core material is too low.

A steep hardness gradient, which may occur particularly with flame or induction hardening, leads under the same nominal hardening depth to an expansion of the deformation zones.

Hardening depth

Case, flame or induction hardened raceways must have a surface hardness of 670 HV to 840 HV and a sufficiently large hardening depth (for case hardening: case hardening depth CHD; for flame or induction hardening: surface hardening depth SHD).

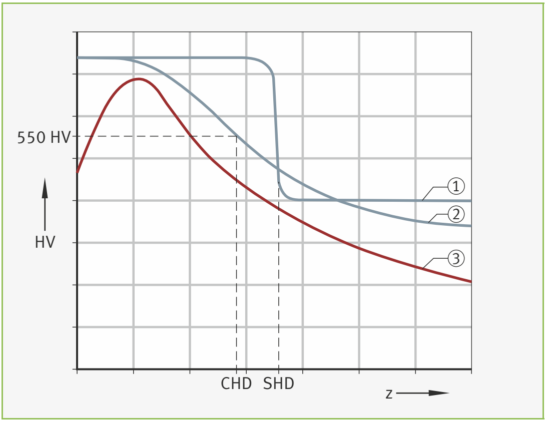
The hardness curves are shown in Figure 9; the required hardness curve is determined by converting the equivalent stress curve into Vickers hardness (see conversion table in the chapter Engineering materials, Page 317).

The following curves are produced for surface hardness:

Figure 9
Hardening depth

HV = Vickers hardness
z = distance from surface
CHD = case hardening depth
SHD = surface hardening depth

① Flame or induction hardening
② Case hardening
③ Required hardness



The required minimum hardening depth depends essentially on the rolling element diameter, material loading, core strength and the hardening process.

For raceways loaded up to the static load carrying capacity C_0, in which a Hertzian pressure of p_{max} = 4000 N/mm^2 is present for line contact, the hardening depths can be calculated from the following relationships.

The case hardening depth for **case hardening** is:

Equation 38

$$CHD \geq 0{,}052 \cdot D_w$$

D_w = rolling element diameter

The surface hardening depth for **flame or induction hardening** is:

Equation 39

$$SHD \geq 140 \cdot D_w / R_{p0,2}$$

Material selection for rolling bearing raceways

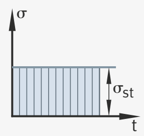
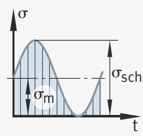
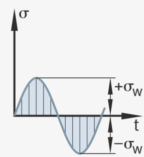
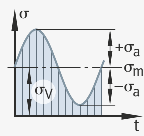
When selecting materials for rolling bearing raceways, it is necessary to bear in mind that in order to achieve the full static and dynamic load carrying capacity of the bearing location, an adequate case or surface hardness, adequate hardness zone formation, and a good degree of cleanliness corresponding to the usual standard for rolling bearing steels, are required. An adequate hardness is usually between 58 HRC (654 HV) and 64 HRC (800 HV); conversion of the hardness values takes place in accordance with DIN EN ISO 18265.
For all of the steels listed below, larger component cross-sections usually call for an increased alloy content in the steel.

The following are particularly suitable for rolling bearing rings and rolling elements:

- **through hardening steels**
  (for example 100Cr6 or 100CrMnSi6-4 to DIN EN ISO 683-17).
  Surface layer hardening, in particular induction surface layer hardening, of these steels or of country-specific equivalents such as SAE52100 is also feasible in special cases. Either martensitic or bainitic hardening is used.
- **case hardening steels**
  (for example 17MnCr5 to DIN EN ISO 683-17 or 16MnCr5 to DIN EN 10084). The necessary surface hardness and the hardness depth profile must be adjusted through the diffusion of carbon or carbon and nitrogen prior to actual martensitic hardening.
  Appropriate calculation tools such as Bearinx are available for determining the necessary hardness profiles.
  This process is more costly and energy-intensive than through hardening.
- **steels for induction hardening**
  (for example C56E2 to DIN EN ISO 683-17). This hardening method usually only hardens the raceway. The initial state recommended for the steel is quenched. The process requires coordination of the inductor, process parameters and design; preliminary tests are usually necessary. Slippage, i.e. areas that are heated twice and therefore become soft, can often be expected. Laser hardening is increasingly being used and, only rarely, flame hardening in the place of induction hardening.

Dynamic loading – geometrical stability

Component loading The stress distributions for component loading are as follows:

Load case	I static	II purely pulsating	III purely alternating	I + III general oscillating
				
Maximum stress	σ_{st} = const.	$\sigma_o = \sigma_{sch}$	$\sigma_o = +\sigma_W$	$\sigma_o = \sigma_m + \sigma_a$
Mean stress	–	$\sigma_m = \sigma_{sch}/2$	$\sigma_m = 0$	$\sigma_m = \sigma_V$ (preload)
Minimum stress	–	$\sigma_u = 0$	$\sigma_u = -\sigma_W$	$\sigma_u = \sigma_m - \sigma_a$
Strength characteristic of the material that is decisive for component calculation				
	Breaking strength R_m (brittle material)	Fatigue strength under pulsating stresses σ_{Sch}	Fatigue strength under reversed stresses σ_W	Amplitude strength σ_A
	Yield point R_e; $R_{p0,2}$ (tough material)	Fatigue limit characteristic σ_D (general) or low cycle fatigue strength for low cycle fatigue design, for example σ_{Sch-N}, σ_{W-N}		

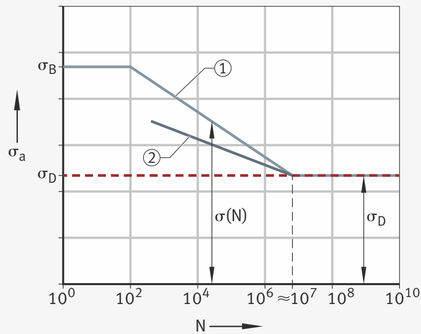
Wöhler's diagram

The following diagram shows Wöhler's diagram for the example of tension – compression.

Figure 10
Wöhler's diagram

σ_a = stress amplitude
N = load cycles
σ_D = fatigue limit
$\sigma(N)$ = low cycle fatigue strength
σ_B = stress at fracture

1 Wöhler curve
2 Damage line



If the load falls below the damage line, the material will not be subjected to any preliminary damage.

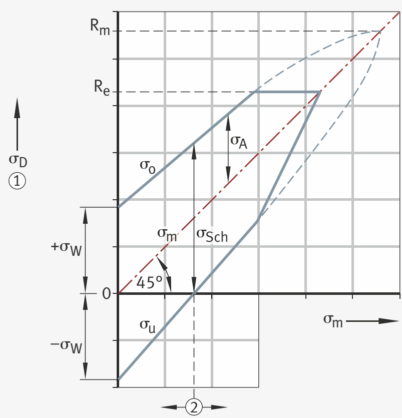
Fatigue strength diagram in accordance with Smith

Fatigue strength is illustrated in the diagram according to Smith.

Figure 11
Fatigue strength diagram in accordance with Smith

σ_D = fatigue limit
σ_m = mean stress
σ_W = fatigue strength under reversed stresses
σ_A = amplitude strength
σ_{Sch} = fatigue strength under pulsating stresses
σ_o = maximum stress
σ_u = minimum stress
R_m = breaking strength
R_e = yield point

1 Strength characteristics
2 Reverse stress domain, pulsating stress domain



Influence of size and surface

The influence of size and surface on component strength (≠ material strength) is illustrated in the diagrams in Figure 12 and Figure 13. The following applies:

Equivalent stress on the component		Permissible load		Geometrical stability of the shape
σ_v	$\leqq$	σ_{per}	$=$	$\dfrac{\sigma_D \cdot b_o \cdot b_d}{S_{min} \cdot \beta_k}$

where

σ_D = decisive fatigue limit value of the material

b_o = surface factor ($\leqq 1$)

b_d = size factor ($\leqq 1$)

β_k = fatigue notch factor ($\geqq 1$)

S_{min} = minimum safety (1,2 ... 2)

Figure 12
Influence of size

b_d = size factor
d = component diameter

① For bending and torsion
② For tension/compression

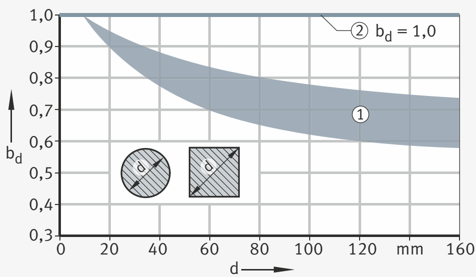
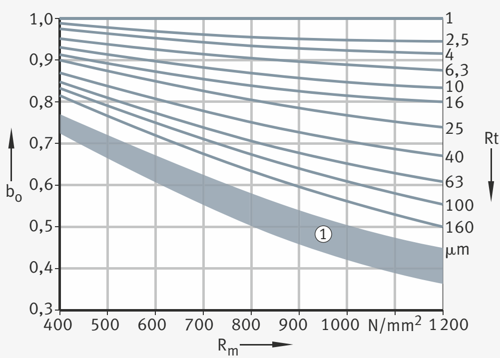


Figure 13
Influence of surface quality on fatigue strength

b_o = surface factor
Rt = roughness depth
R_m = breaking strength of the material

① Surfaces with rolling scale



Fatigue strength diagrams for general structural steels

Fatigue strength diagrams for general structural steels (DIN 17100 and DIN EN 10025) are illustrated below.

The following applies:

- load case I: stationary loading
- load case II: pure pulsating loading
- load case III: pure alternating loading

For cold drawn material (for example E295GC), the yield point values may be up to 50% higher, whereas the fatigue strength under pulsating and under completely reversed stress may only be assumed to be about 10% higher than the table values. Cold forming reduces the material's ability to stretch, and its sensitivity to cleavage fracture is higher.

Tensile and compressive loading

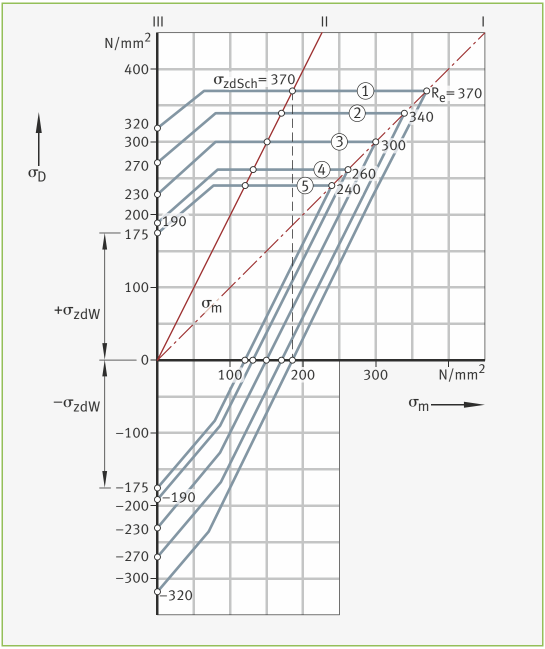
Under tensile and compressive loading we arrive at:

Figure 14

Fatigue strength of general structural steels: tensile and compressive loading

σ_D = fatigue limit
σ_m = mean stress
σ_{zdW} = fatigue strength under reversed tensile/compressive stresses
σ_{zdSch} = fatigue strength under pulsating tensile stresses
R_e = yield point

1 E360
2 E335
3 E295
4 S275
5 S235



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

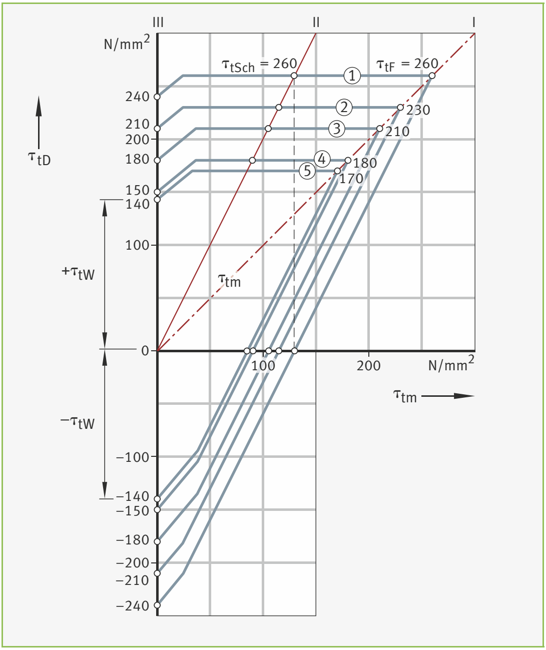
Torsional loading

Under torsional loading we arrive at:

Figure 15
Fatigue strength of general structural steels: torsional loading

τ_{tD} = torsional fatigue limit
τ_{tm} = torsional mean stress
τ_{tW} = fatigue strength under reversed torsional stresses
τ_{tSch} = fatigue strength under pulsating torsional stresses
τ_{tF} = yield point under torsion

1 E360
2 E335
3 E295
4 S275
5 S235



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

Bending loading

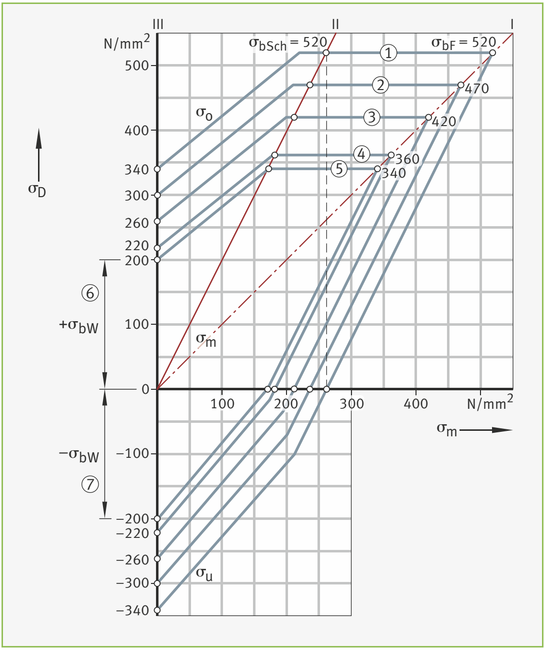
Under bending loading we arrive at:

Figure 16

Fatigue strength of general structural steels: bending loading

σ_D = fatigue strength
σ_m = mean stress
σ_{bW} = fatigue strength under reversed bending stresses
σ_o = maximum stress
σ_u = minimum stress
σ_{bSch} = fatigue strength under pulsating bending stresses
σ_{bF} = yield point under bending

① E360
② E335
③ E295
④ S275
⑤ S235
⑥ Tensile area
⑦ Compressive area



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

Engineering materials

A selection of significant engineering materials is presented on the following pages – starting with various steel grades (including their heat treatment), via cast iron, cast steel and non-ferrous metals through to plastics.

Steel

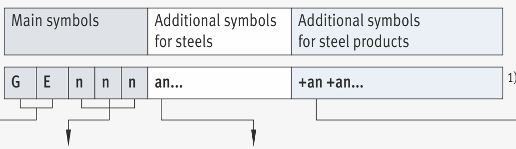
Steel grades

Various grades of steel are defined in standards (DIN, EN, ISO). A selection of steels is described in more detail below:

- unalloyed structural steels
- quenched and tempered steels
- case hardening steels
- stainless steels
- rolling bearing steels
- free-cutting steels

System of material designations

The standard DIN EN 10027-1:2017 gives short names for steels. The whole standard is very extensive. For this reason, the system of engineering steels is presented as an example.



Main symbols		Additional symbols		
Letter	Mechanical properties	For steel		For steel products
		Group 1	Group 2	
G = Cast steel (if necessary) E = Engineering steels	nnn = Defined minimum yield strength[2)]in MPa[3)] for the smallest thickness value	G = Other features, if necessary with one or two consecutive numbers or if notched bar impact properties are defined, in accordance with the following rules according to table Notched bar impact work, Page 283	C = Suitability for cold drawing	Symbols for the treated condition: see standard DIN EN 10027-1:2017, Table 18

1) n = number, a = letter, an = alphanumeric.

2) The term "yield strength" is defined, in accordance with the information in the relevant product standard, as the upper or lower yield strength (R_{eH} or R_{eL}) or the proof stress under non-proportional elongation (R_p) or proof stress under complete elongation (R_t).

3) 1 MPa = 1 N/mm^2.

Notched bar impact work The following table shows the notched bar impact work in accordance with DIN EN 10027-1:2017 Table 1, Group 1.

Notched bar impact work J (Joule)			Test temperature °C
27 J	40 J	60 J	
JR	KR	LR	+20
J0	K0	L0	0
J2	K2	L2	–20
J3	K3	L3	–30
J4	K4	L4	–40
J5	K5	L5	–50
J6	K6	L6	–60

Examples of short names Some examples of short names in accordance with EN 10027-1 are as follows:

Standard	Short name
EN 10025-2	E295
	E295GC
	E335
	E360

Standard	Short name
EN 10293	GE240
EN 10296-1	E355K2

Unalloyed structural steels According to the standard DIN EN 10025-2:2019, unalloyed structural steels are long and flat products made from hot-rolled, unalloyed, base and quality steels. They are characterised by their chemical composition and mechanical properties, see table, Mechanical properties, Page 284.

Unalloyed structural steels are used in building construction, civil engineering, bridge construction, hydraulic construction, tank and container construction and in vehicle and machine construction for example.

The steels referred to in this standard are not suitable for heat treatment. Stress relief annealing is permissible.

Mechanical properties The following table gives the mechanical properties of a number of unalloyed structural steels as an extract from the corresponding standard.

Steel grade designation		Tensile strength R_m[1] for nominal thickness values mm			Yield strength R_{eH}[1] for nominal thickness values mm					
Short name	Material number	< 3	> 3 ≦ 100	> 100 ≦ 150	≦ 16	> 16 ≦ 40	> 40 ≦ 63	> 63 ≦ 80	> 80 ≦ 100	> 100 ≦ 150
		MPa			min. MPa					
S185	1.0035	310 ... 540	290 ... 510	280 ... 500	185	175	175	175	175	165
S235JR S235J0 S235J2	1.0038 1.0114 1.0117	360 ... 510	360 ... 510	350 ... 500	235	225	215	215	215	195
S275JR S275J0 S275J2	1.0044 1.0143 1.0145	430 ... 580	410 ... 560	400 ... 540	275	265	255	245	235	225
S355JR S355J0 S355J2 S355K2	1.0045 1.0553 1.0577 1.0596	510 ... 680	470 ... 630	450 ... 600	355	345	335	325	315	295
E295[2]	1.0050	490 ... 660	470 ... 610	450 ... 610	295	285	275	265	255	245
E335[2]	1.0060	590 ... 770	570 ... 710	550 ... 710	335	325	315	305	295	275
E360[2]	1.0070	690 ... 900	670 ... 830	650 ... 830	360	355	345	335	325	305

For further mechanical and technological properties and information on the chemical composition of steels see DIN EN 10025.

[1] The values given in the table for the tensile test apply to longitudinal test pieces l. For strip, plate and wide flat steel with widths ≧ 600 mm, transverse test pieces t are applicable.

[2] These steel grades are not normally used for channels, angles and sections.

Quenched and tempered steels Standards DIN EN ISO 683-1:2018 and DIN EN ISO 683-2:2018 list the mechanical properties of steels in the quenched and tempered condition (+ QT).
Definition of the dimension limits does not mean that full martensitic quenching and subsequent tempering is possible up to the defined sampling point. The hardening depth is the result of the course of the end quenching curves.

Mechanical properties

The following table gives the mechanical properties of a number of unalloyed quenched and tempered steels as an extract from the corresponding standard.

Steel designation		Yield strength R_e[1] (0,2% proof stress) min. MPa			Tensile strength R_m MPa			Elongation at fracture[2] A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece)[3] KV min. J		
Short name	Material number	Diameter mm														
		> – ≤ 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100
C25 C25E C25R	1.0406 1.1158 1.1163	370	320	–	550 … 700	500 … 650	–	19	21	–	–	–	–	– 35 35	– 35 35	–
C30 C30E C30R	1.0528 1.1178 1.1179	400	350	300[4]	600 … 750	550 … 700	500 … 650[4]	18	20	21[4]	–	–	–	– 30 30	– 30 30	– 30[4] 30
C35 C35E C35R	1.0501 1.1181 1.1180	430	380	320	630 … 780	600 … 750	550 … 700	17	19	20	40	45	50	– 25 25	– 25 25	– 25 25
C40 C40E C40R	1.0511 1.1186 1.1189	460	400	350	650 … 800	630 … 780	600 … 750	16	18	19	35	40	45	– 20 20	– 20 20	– 20 20
C45 C45E C45R	1.0503 1.1191 1.1201	490	430	370	700 … 850	650 … 800	630 … 780	14	16	17	35	40	45	– 15 15	– 15 15	– 15 15
C50 C50E C50R	1.0540 1.1206 1.1241	520	460	400	750 … 900	700 … 850	650 … 800	13	15	16	–	–	–	–	–	–
C55 C55E C55R	1.0535 1.1203 1.1209	550	490	420	800 … 950	750 … 900	700 … 850	12	14	15	30	35	40	–	–	–
C60 C60E C60R	1.0601 1.1221 1.1223	580	520	450	850 … 1000	800 … 950	750 … 900	11	13	14	25	30	35	–	–	–
28Mn6	1.1170	590	490	440	800 … 950	700 … 850	650 … 800	13	15	16	40	45	50	25	30	30

[1] R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.

[2] Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).

[3] Minimum notched bar impact work must be agreed if testing with a Charpy U-notch test piece is required.

[4] Valid for diameters up to 63 mm or for thicknesses up to 35 mm.

The following table gives the mechanical properties of a number of alloyed quenched and tempered steels as an extract from the corresponding standard.

Steel designation		Yield strength R_e[1) (0,2% proof stress) min. MPa			Tensile strength R_m MPa			Elongation at fracture[2) A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece)[3) KV min. J		
Short name	Material number	Diameter mm														
		> – ≦ 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100
34Cr4 34CrS4	1.7033 1.7037	700	590	460	900 … 1100	800 … 950	700 … 850	12	14	15	35	40	45	–	40	40
37Cr4 37CrS4	1.7034 1.7038	750	630	510	950 … 1150	850 … 1000	750 … 900	11	13	14	35	40	40	–	35	35
41Cr4 41CrS4	1.7035 1.7039	800	660	560	1000 … 1200	900 … 1100	800 … 950	11	12	14	30	35	40	–	35	35
25CrMo4 25CrMoS4	1.7218 1.7213	700	600	450	900 … 1100	800 … 950	700 … 850	12	14	15	50	55	60	–	50	50
34CrMo4 34CrMoS4	1.7220 1.7226	800	650	550	1000 … 1200	900 … 1100	800 … 950	11	12	14	45	50	55	–	40	45
42CrMo4 42CrMoS4	1.7225 1.7227	900	750	650	1100 … 1300	1000 … 1200	900 … 1100	10	11	12	40	45	50	–	35	35
50CrMo4	1.7228	900	780	700	1100 … 1300	1000 … 1200	900 … 1100	9	10	12	40	45	50	–	30	30

Continuation of table, see Page 287.

1) R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.
2) Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).
3) Minimum notched bar impact work must be agreed if testing with a Charpy U-notch test piece is required.

Continuation of table, Mechanical properties (quenched and tempered steels), from Page 286.

Steel designation		Yield strength R_e[1) (0,2% proof stress) min. MPa			Tensile strength R_m MPa			Elongation at fracture[2) A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece)[3) KV min. J		
Short name	Material number	Diameter mm														
	> / ≦	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100
36CrNiMo4	1.6511	900	800	700	1100 … 1300	1000 … 1200	900 … 1100	10	11	12	–	–	–	–	–	–
34CrNiMo6	1.6582	1000	900	800	1200 … 1400	1100 … 1300	1000 … 1200	9	10	11	40	45	50	–	45	45
30CrNiMo8	1.6580	850	850	800	1030 … 1230	1030 … 1230	980 … 1800	12	12	12	40	40	45	–	30	35
51CrV4	1.8159	900	800	700	1100 … 1300	1000 … 1200	900 … 1100	9	10	12	40	45	50	–	30	30

1) R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.
2) Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).
3) Minimum notched bar impact work must be agreed if testing with a Charpy U-notch test piece is required.

Case hardening steels

According to the standard DIN EN ISO 683-3:2022, case hardening steels are structural steels with a relatively low carbon content.
They are used for components whose surface zones are usually carburised or carbonitrided prior to hardening.

After hardening, these steels exhibit a high degree of hardness in the surface zone and good resistance to wear. The core zone primarily exhibits a high degree of toughness, however, this requires coordination of the cross-section to be hardened, the hardening process and the steel composition.

The standard DIN EN ISO 683-3 applies to semi-finished products, for example: cogs, roughed slabs, billets, hot-rolled wire, hot-rolled or forged steel bar (round, rectangular, hexagonal, octagonal and flat steel), hot-rolled wide flat steel, hot or cold-rolled sheet and strip, open die and drop forgings.

Brinell hardness The following table gives the Brinell hardness of a number of case hardening steels in various treated conditions as an extract from the corresponding standard.

Steel designation		Hardness in treated condition[1)]			
Short name	Material number	+S[2)] max. HBW	+A[3)] max. HBW	+TH[4)] HBW	+FP[5)] HBW
C10E	1.1121	–	131	–	–
C10R	1.1207	–	131	–	–
C15E	1.1141	–	143	–	–
C15R	1.1140	–	143	–	–
17Cr3	1.7016	–	174	–	–
17CrS3	1.7014	–	174	–	–
28Cr4	1.7030	255	217	166 ... 217	156 ... 207
28CrS4	1.7036	255	217	166 ... 217	156 ... 207
16MnCr5	1.7131	–	207	156 ... 207	140 ... 187
16MnCrS5	1.7139	–	207	156 ... 207	140 ... 187
20MnCr5	1.7147	255	217	170 ... 217	152 ... 201
20MnCrS5	1.7149	255	217	170 ... 217	152 ... 201
20MoCr4	1.7321	255	207	156 ... 207	140 ... 187
20MoCrS4	1.7323	255	207	156 ... 207	140 ... 187
20NiCrMo2-2	1.6523	–	212	161 ... 201	149 ... 194
20NiCrMoS2-2	1.6526	–	212	161 ... 201	149 ... 194
17NiCrMo6-4	1.6566	255	229	179 ... 229	149 ... 201
17NiCrMoS6-4	1.6569	255	229	179 ... 229	149 ... 201

1) Hardness requirements for the products supplied in the following conditions.
2) Treated to improve shearability.
3) Soft-annealed.
4) Treated for strength.
5) Treated to ferrite-pearlite structure.

Stainless steels

The standard DIN EN 10088:2014 gives the chemical composition and mechanical properties of stainless steels.

Stainless steels are particularly resistant to chemically aggressive substances. They contain at least 10,5% Cr and no more than 1,2% C. The property "corrosion-resistant" requires a heat treatment that is adapted to the steel. This applies in particular to martensitic-hardened steels. In accordance with their essential use characteristics, they are further subdivided into:

- Corrosion-resistant steels:
  material numbers 1.40xx to 1.46xx
- Heat-resistant steels:
  material numbers 1.47xx and 1.48xx
- Creep-resistant steels:
  material numbers 1.49xx

Stainless steels can also be classified in accordance with their microstructure into:

- Ferritic steels:
  good suitability for welding, creep-resistant, special magnetic properties, poor suitability for machining by cutting, suitable for cold forming, not resistant to intercrystalline corrosion, E = 220 000 MPa
- Martensitic steels:
  hardenable, good suitability for machining by cutting, high strength, magnetic, weldable under certain conditions, E = 216 000 MPa
- Precipitation hardening steels:
  hardenable by precipitation hardening, suitability for machining by cutting dependent on hardness, magnetic, E = 200 000 MPa
- Austenitic steels:
  good suitability for welding, good suitability for cold forming, difficult to machine by cutting, non-magnetic, E = 200 000 MPa
- Austenitic-ferritic steels (duplex steels):
  resistant to stress corrosion cracking, high erosion resistance and high fatigue strength, E = 200 000 MPa

Mechanical properties The following table gives the mechanical properties of a number of stainless steels as an extract from the corresponding standard.

Steel designation		Elongation at fracture A	Tensile strength for d_N[1) R_{mN}	Yield strength, 0,2% proof stress for d_N[1) R_{eN}, $R_{p0,2N}$	Examples of application
Short name	Material number	min. %	min. MPa	min. MPa	
Stainless steels in accordance with DIN EN 10088-3 (semi-finished products, bars and profiles)					Characterised by better resistance to chemically aggressive substances (compared with low-alloy steels); the resistance is based on the formation of covering layers due to the chemical attack
Treated condition	Ferritic steels: annealed (+A)				
	Martensitic steels: quenched and tempered (+QT, for example QT700)				
	Austenitic and austenitic-ferritic steels: solution annealed (+AT)				
Practically no influence on size due to technology					
Ferritic steels					
X2CrMoTiS18-2	1.4523	15	430	280	Acid-resistant parts in the textile industry
X6CrMoS17	1.4105	20	430	250	Free-cutting steels; bolts, fasteners
X6Cr13	1.4000	20	400	230	Chip carriers, cutlery, interior fittings
X6Cr17	1.4016	20	400	240	Connectors, deep drawn formed parts
Martensitic steels					
X20Cr13	1.4021	13	700	500	Armatures, flanges, springs, turbine parts
X39CrMo17-1	1.4122	12	750	550	Tubes, shafts, spindles, wear parts
X14CrMoS17	1.4104	12	650	500	Free-cutting steel; turned parts, apparatus fittings
X12CrS13	1.4005	12	650	450	Connectors, cutting tools, components subjected to wear
X3CrNiMo13-4	1.4313	15	780	620	
X17CrNi16-2	1.4057	14	800	600	

Continuation of table, see Page 291.

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

1) d_N: reference dimension (diameter, thickness) of semi-finished product in accordance with the relevant material standard.
R_{mN}, R_{eN}, $R_{p0,2N}$: standard values for tensile strength, yield strength and 0,2% proof stress measured relative to d_N.
Guide values for fatigue strength: $\sigma_{bW} \approx 0{,}5 \cdot R_m$, $\sigma_{zdW} \approx 0{,}4 \cdot R_m$, $\tau_{tW} \approx 0{,}3 \cdot R_m$.

Continuation of table, Mechanical properties (stainless steels), from Page 290.

Steel designation		Elongation at fracture A	Tensile strength for d_N[1)] R_m	Yield strength, 0,2% proof stress for d_N[1)] R_{eN}, $R_{p0,2N}$	Examples of application
Short name	Material number	min. %	min. MPa	min. MPa	
Austenitic steels					
X5CrNi18-10	1.4301	45	500	190	Universal use; building, vehicle construction, foodstuffs industry
X8CrNiS18-9	1.4305	35	500	190	Free-cutting steel; machine and connecting elements
X6CrNiTi18-10	1.4541	40	500	190	Household goods, photography industry, sanitary use
X2CrNiMo17-12-2 X2CrNiMoN17-13-3	1.4404 1.4429	40 40	500 580	200 280	Offshore engineering, pressure vessels, welded construction parts; pins, shafts
X2CrNiMo17-12-2	1.4401	40	500	200	Bleaching equipment, foodstuffs, oil and dyeing industry
X6CrNiMoTi17-12-2	1.4571	40	500	200	Containers (tanker trucks), heating vessels, synthetic resin and rubber industry
All austenitic grades cold hardened					
Tensile strength step	C700 C800	20 12	700 800	350 500	Load-bearing components
Austenitic-ferritic steels (duplex steels)					
X2CrNiMoN22-5-3	1.4462	25	650	450	Components for high chemical and mechanical loading; water and wastewater engineering, offshore engineering, pulp and chemicals industry, tank construction, centrifuges, conveying equipment
X2CrNiN23-4	1.4362	25	600	400	
X2CrNiMoCuWN25-7-4	1.4501	25	730	530	

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1)] d_N: reference dimension (diameter, thickness) of semi-finished product in accordance with the relevant material standard.
R_{mN}, R_{eN}, $R_{p0,2N}$: standard values for tensile strength, yield strength and 0,2% proof stress measured relative to d_N.
Guide values for fatigue strength: $\sigma_{bW} \approx 0{,}5 \cdot R_m$, $\sigma_{zdW} \approx 0{,}4 \cdot R_m$, $\tau_{tW} \approx 0{,}3 \cdot R_m$.

Rolling bearing steels According to the standard DIN EN ISO 683-17:2015, rolling bearing steels are steels for components of rolling bearings. They are subjected first and foremost to high local alternating stresses and wear during operation. In the used condition they exhibit a hardening microstructure (at least in the surface zone).

Hardness The following table gives the hardness in the soft annealed delivered condition as an extract from the corresponding standard. In the final state after martensitic or bainitic hardening, the surface hardness is usually above 58 HRC.

Steel designation		Hardness in the delivered condition						Previous designation
Short name	Material number	+S max. HBW	+A max. HBW	+HR HBW	+AC[1] max. HBW	+AC[1] +C max. HBW	+FP HBW	
Through hardening rolling bearing steels								
100Cr6	1.3505	[2]	–	–	207	241[3][4][5]	–	100 Cr 6
100CrMnSi6-4	1.3520	[2]	–	–	217	251[4][5]	–	100 CrMn 6
100CrMo7	1.3537	[2]	–	–	217	251[4][5]	–	100 CrMo 7
100CrMo7-3	1.3536	[2]	–	–	230	–	–	100 CrMo 7 3
100CrMnMoSi8-4-6	1.3539	[2]	–	–	230	–	–	100 CrMnMo 8
Case hardening rolling bearing steels								
17MnCr5	1.3521	[6]	207	156 ... 207	170	[7]	140 ... 187	17 MnCr 5
19MnCr5	1.3523	255	217	170 ... 217	179	[7]	152 ... 201	19 MnCr 5
18NiCrMo14-6	1.3533	255	–	–	241	[7]	–	17 NiCrMo 14

Continuation of table, see Page 293.

[1] For case hardening steels, this condition is applied if cold forming operations are intended. For through hardening, stainless and high-temperature rolling bearing steels, this condition is also used if the steel is processed further by machining operations.

[2] If this condition is necessary, the maximum hardness value and the requirements concerning the structure are to be agreed upon when the enquiry and order are placed.

[3] The hardness of wire for needle roller bearings should not exceed 331 HBW.

[4] The hardness of cold-finished tubes should not exceed 321 HBW.

[5] The hardness value for bright steel products with a diameter $<$ 13 mm should be $<$ 320 HBW.

[6] Under suitable conditions, this grade is shearable in the untreated condition.

[7] Depending on the degree of cold forming, the values may exceed those for the condition +AC by up to approx. 50 HBW.

Continuation of table, Hardness, from Page 292.

Steel designation		Hardness in the delivered condition						Previous designation
Short name	Material number	+S max. HBW	+A max. HBW	+HR HBW	+AC[1] max. HBW	+AC[1] +C max. HBW	+FP HBW	
Induction hardening rolling bearing steels								
C56E2	1.1219	255[2]	229	–	–	–	–	Cf 54
43CrMo4	1.3563	255[2]	241	–	–	–	–	43 CrMo 4
Stainless rolling bearing steels								
X47Cr14	1.3541	[3]	–	–	248	[4]	–	X 45 Cr 13
X108CrMo17	1.3543	[3]	–	–	255	[4]	–	X 102 CrMo 17
X89CrMoV18-1	1.3549	[3]	–	–	255	[4]	–	X 89 CrMoV 18 1
High-temperature rolling bearing steels								
80MoCrV42-16	1.3551	[3]	–	–	248	[4]	–	80 MoCrV 42 16
X82WMoCrV6-5-4	1.3553	[3]	–	–	248	[4]	–	X 82 WMoCrV 6 5 4
X75WCrV18-4-1	1.3558	[3]	–	–	269	[4]	–	X 75 WCrV 18 4 1

[1] For case hardening steels, this condition is applied if cold forming operations are intended.
For through hardening, stainless and high-temperature rolling bearing steels, this condition is also used if the steel is processed further by machining operations.

[2] Depending on the chemical composition of the cast and the dimensions, condition +A may be necessary.

[3] In general, shearability will only apply in condition +AC.

[4] Depending on the degree of cold forming, the values may exceed those for the condition +AC by up to approx. 50 HBW.

Free-cutting steels According to the standard DIN EN ISO 683-4:2018, free-cutting steels are characterised by good cutting properties and good chip brittleness. These are essentially achieved by higher sulphur contents and with further additives (for example, lead) where required.

Bright free-cutting steels differ from the hot formed free-cutting steels by virtue of the fact that they have obtained a smooth, bright surface and a significantly higher dimensional accuracy through cold forming (drawing) or machining (pre-turning, rough grinding).

Mechanical properties The following table gives the mechanical properties of a number of free-cutting steels as an extract from the corresponding standard.

Steel designation		Diameter d		Untreated		Quenched and tempered		
Short name	Material number	over mm	incl. mm	Hardness[1)] max. HBW	Tensile strength R_m MPa	Yield strength R_e min. MPa	Tensile strength R_m MPa	Elongation A min. %
Free-cutting steels not intended for heat treatment								
11SMn30 11SMnPb30 11SMn37 11SMnPb37	1.0715 1.0718 1.0736 1.0737	5	10	169	380 ... 570	–	–	–
		10	16	169	380 ... 570	–	–	–
		16	40	169	380 ... 570	–	–	–
		40	63	169	370 ... 570	–	–	–
		63	100	154	360 ... 520	–	–	–
Case hardening steels								
10S20 10SPb20	1.0721 1.0722	5	10	156	360 ... 530	–	–	–
		10	16	156	360 ... 530	–	–	–
		16	40	156	360 ... 530	–	–	–
		40	63	156	360 ... 530	–	–	–
		63	100	146	350 ... 490	–	–	–
15SMn13 17SMn20	1.0725	5	10	181	430 ... 610	–	–	–
		10	16	178	430 ... 600	–	–	–
		16	40	178	430 ... 600	–	–	–
		40	63	172	430 ... 580	–	–	–
		63	100	160	420 ... 540	–	–	–

Continuation of table, see Page 295.

1) In cases of dispute, the tensile strength values are decisive. The hardness values are for information only.

Continuation of table, Mechanical properties (free-cutting steels), from Page 294.

Steel designation		Diameter d		Untreated		Quenched and tempered		
Short name	Material number	over mm	incl. mm	Hardness[1] max. HBW	Tensile strength R_m MPa	Yield strength R_e min. MPa	Tensile strength R_m MPa	Elongation A min. %
Quenched and tempered steels								
35S20 35SPb20	1.0726 1.0756	5	10	210	550 … 720	430	630 … 780	15
		10	16	204	550 … 700	430	630 … 780	15
		16	40	198	520 … 680	380	600 … 750	16
		40	63	196	520 … 670	320	550 … 700	17
		63	100	190	500 … 650	320	550 … 700	17
36SMn14 36SMnPb14	1.0764 1.7065	5	10	225	580 … 770	480	700 … 850	14
		10	16	225	580 … 770	460	700 … 850	14
		16	40	219	560 … 750	420	670 … 820	15
		40	63	216	560 … 740	400	640 … 700	16
		63	100	216	550 … 740	360	570 … 720	17
38SMn28 38SMnPb28	1.0760 1.0761	5	10	228	580 … 780	480	700 … 850	15
		10	16	219	580 … 750	460	700 … 850	15
		16	40	213	560 … 730	420	700 … 850	15
		40	63	213	560 … 730	400	700 … 850	16
		63	100	204	550 … 700	380	630 … 800	16

[1] In cases of dispute, the tensile strength values are decisive. The hardness values are for information only.

Cast iron and cast steel

Flake graphite cast iron

According to the standard DIN EN 1561:2012, flake graphite cast iron is a casting alloy based on iron-carbon, where the carbon is chiefly present in the form of flake graphite.

The properties of flake graphite cast iron are dependent on the form and distribution of the graphite and on the metallic matrix.

Tensile strength The following table gives the tensile strength of a number of grades of flake graphite cast iron as an extract from the corresponding standard.

Material designation		Decisive wall thickness value		Tensile strength R_m			Previous designation
				Values that must be maintained[1)]		Expected values in casting[4)]	
Short designation	Material number	over mm	incl. mm	in test piece cast separately[2)] MPa	in integrally cast test piece[3)] min. MPa	min. MPa	
EN-GJL-100	5.1100	5	40	–	100	–	GG-10 EN-JL 1010
EN-GJL-150	5.1200	2,5[5)]	50	150 ... 250	150	135	GG-15 EN-JL 1020
		50	100		130	120	
		100	200		110	110	
EN-GJL-200	5.1300	2,5[5)]	50	200 ... 300	200	180	GG-20 EN-JL 1030
		50	100		180	160	
		100	200		160	145	
EN-GJL-250	5.1301	5[5)]	50	250 ... 350	250	225	GG-25 EN-JL 1040
		50	100		220	200	
		100	200		200	185	
EN-GJL-300	5.1302	10[5)]	50	300 ... 400	300	270	GG-30 EN-JL 1050
		50	100		260	245	
		100	200		240	220	
EN-GJL-350	5.1303	10[5)]	50	350 ... 450	350	320	GG-35 EN-JL 1060
		50	100		310	290	
		100	200		280	260	

1) If proof of tensile strength has been agreed for an order, the nature of the test piece must be specified in the order.

2) The values refer to test pieces with an unfinished casting diameter of 30 mm.
If nothing can be defined for a specific wall thickness range, this is identified by a dash.

3) The values refer to samples that have been machined from cast test pieces.

4) Guide values for the tensile strength of samples created using test pieces taken from the casting.
If nothing can be defined for a specific wall thickness range, this is identified by a dash.
The values are for information purposes.

5) This dimension is included as the lower limit of the wall thickness range.

Spheroidal graphite cast iron According to the standard DIN EN 1563:2019, spheroidal graphite cast iron is a casting material based on iron-carbon, where the carbon is chiefly present in the form of spheroidal graphite particles.
Spheroidal graphite cast iron is also known as ductile cast iron.

Mechanical properties The following table (extract from the standard) lists the mechanical properties of a number of spheroidal graphite cast irons, measured on samples machined from test pieces.

Material designation		Decisive wall thickness value		Guaranteed properties			Previous designation
Designation	Material number	over mm	incl. mm	Tensile strength R_m min. MPa	0,2% proof stress $R_{p0,2}$ min. MPa	Elongation at fracture A min. %	
Ferritic to pearlitic grades							
EN-GJS-350-22-LT (LT: for low temperatures)	5.3100	–	30	350	220	22	GGG-35.3 EN-JS1015
		30	60	330	210	18	
		60	200	320	200	15	
EN-GJS-350-22-RT (RT: for room temperature)	5.3101	–	30	350	220	22	EN-JS1014
		30	60	330	220	18	
		60	200	320	210	15	
EN-GJS-400-18-LT (LT: for low temperatures)	5.3103	–	30	400	240	18	GGG-40.3 EN-JS1025
		30	60	380	230	15	
		60	200	360	220	12	
EN-GJS-400-18-RT (RT: for room temperature)	5.3104	–	30	400	250	18	EN-JS1024
		30	60	390	250	15	
		60	200	370	240	12	
EN-GJS-400-15	5.3106	–	30	400	250	15	GGG-40 EN-JS1030
		30	60	390	250	14	
		60	200	370	240	11	
EN-GJS-600-3	5.3201	–	30	600	370	3	GGG-60 EN-JS1060
		30	60	600	360	2	
		60	200	550	340	1	

Continuation of table, see Page 298.

Continuation of table, Mechanical properties (Spheroidal graphite cast iron), from Page 297.

Material designation		Decisive wall thickness value		Guaranteed properties			Previous desig-nation
Designation	Material number	over mm	incl. mm	Tensile strength R_m min. MPa	0,2% proof stress $R_{p0,2}$ min. MPa	Elongation at fracture A min. %	
Solid-solution strengthened ferritic grades							
EN-GJS-450-18	5.3108	–	30	450	350	18	–
		30	60[1]	430	340	14	
EN-GJS-500-14	5.3109	–	30	500	400	14	–
		30	60[1]	480	390	12	
EN-GJS-600-10	5.3110	–	30	600	470	10	–
		30	60[1]	580	450	8	

[1] Properties for wall thicknesses › 60 mm must be agreed between the manufacturer and buyer.

The following table (extract from the standard) shows minimum values for impact energy measured on V-notch test pieces, which were manufactured from specimens by means of machining.

Material designation		Decisive wall thickness value		Minimum values for impact energy					
				Room temperature RT (23 ± 5) °C		Low temperatures LT (–20 ± 2) °C		Low temperatures LT (–40 ± 2) °C	
Short designation	Material number	over mm	incl. mm	Mean value[1] J	Individual value J	Mean value[1] J	Individual value J	Mean value[1] J	Individual value J
EN-GJS-350-22-LT (LT: for low temperatures)	5.3100	–	30	–	–	–	–	12	9
		30	60	–	–	–	–	12	9
		60	200	–	–	–	–	10	7
EN-GJS-350-22-RT (RT: for room temperature)	5.3101	–	30	17	14	–	–	–	–
		30	60	17	14	–	–	–	–
		60	200	15	12	–	–	–	–
EN-GJS-400-18-LT (LT: for low temperatures)	5.3103	–	30	–	–	12	9	–	–
		30	60	–	–	12	9	–	–
		60	200	–	–	10	7	–	–
EN-GJS-400-18-RT (RT: for room temperature)	5.3104	–	30	14	11	–	–	–	–
		30	60	14	11	–	–	–	–
		60	200	12	9	–	–	–	–

[1] Mean value from 3 tests.

Integrally cast test pieces

The properties of an integrally cast test piece cannot accurately reproduce the properties of the actual casting. Improved approximate values can, however, be achieved in this instance compared with a test piece that is cast separately.

The following table (extract from the standard) lists guide values for the mechanical properties of a number of spheroidal graphite cast irons, measured on samples taken from the casting.

Material designation		Decisive wall thickness value		Guaranteed properties			Previous designation
Short designation	Material number	over mm	incl. mm	Tensile strength R_m min. MPa	0,2% proof stress $R_{p0,2}$ min. MPa	Elongation at fracture A min. %	
Ferritic to pearlitic grades							
EN-GJS-350-22C-LT (LT: for low temperatures)	5.3100	–	30	350	220	22	GGG-35.3 EN-JS1015
		30	60	330	210	18	
		60	200	320	200	15	
EN-GJS-350-22C-RT (RT: for room temperature)	5.3101	–	30	350	220	22	EN-JS1029
		30	60	330	220	18	
		60	200	320	210	15	
EN-GJS-400-18C-LT (LT: for low temperatures)	5.3103	–	30	400	240	18	GGG-40.3 EN-JS1025
		30	60	380	230	15	
		60	200	360	220	12	
EN-GJS-400-18C-RT (RT: for room temperature)	5.3104	–	30	400	250	18	EN-JS1059
		30	60	390	250	15	
		60	200	370	240	12	
EN-GJS-400-15C	5.3106	–	30	400	250	15	GGG-40 EN-JS1030
		30	60	390	250	14	
		60	200	370	240	11	
EN-GJS-600-3C	5.3201	–	30	600	370	3	GGG-60 EN-JS1060
		30	60	600	360	2	
		60	200	550	340	1	

Continuation of table, see Page 300.

Continuation of table, Integrally cast test pieces (Spheroidal graphite cast iron), from Page 299.

Material designation		Decisive wall thickness value		Guaranteed properties			Previous designation
Short designation	Material number	over mm	incl. mm	Tensile strength R_m min. MPa	0,2% proof stress $R_{p0,2}$ min. MPa	Elongation at fracture A min. %	
Solid-solution strengthened ferritic grades							
EN-GJS-450-18C	5.3108	–	30	350	440	16	–
		30	60	340	420	16	
		60	200	Guide values must be made available by the manufacturer			
EN-GJS-500-14C	5.3109	–	30	400	480	12	–
		30	60	390	460	10	
		60	200	Guide values must be made available by the manufacturer			
EN-GJS-600-10C	5.3110	–	30	450	580	8	–
		30	60	430	560	6	
		60	200	Guide values must be made available by the manufacturer			

If a higher material strength is required for a cast construction, this can be achieved with an ADI grade (austempered ductile (cast) iron). The heat treatment required for this purpose comprises either intermediate transformation (austempering) or quenching by means of hardening and tempering.

The most common ADI grades offer the following properties in accordance with DIN EN 1564:2012:

Material designation	Hardness		Yield point R_e	Tensile strength R_m	Elongation at fracture A
Short name	min. HBW	max. HBW	min. MPa	min. MPa	min. %
EN-GJS-800-8	260	320	500	800	8
EN-GJS-1000-5	300	360	700	1000	5
EN-GJS-1200-2	340	440	850	1200	2
EN-GJS-1400-1	380	480	1100	1400	1

Cast steel for general applications

In accordance with the standard DIN EN 10293:2015, the mechanical and magnetic properties of a number of cast steel grades are listed in the following table as an extract.

Cast steel grade		Heat treatment symbol[1)]	Decisive wall thickness value		Tensile strength R_m	0,2% proof stress $R_{p0,2}$	Elongation at fracture[2)] A	Notched bar impact work KV	Magnetic induction[4)] at a field strength of		
Short name	Material number		over mm	incl. mm	MPa	min. MPa	min. %	at RT[3)] min. J	25 A/cm min. T	50 A/cm min. T	100 A/cm min. T
GE200	1.0420	+N	–	300	380 ... 530	200	25	27	1,45	1,60	1,75
GE240	1.0446	+N	–	300	450 ... 600	240	22	27	1,40	1,55	1,70
GE300	1.0558	+N	–	30	600 ... 750	300	15	27	1,30	1,50	1,65
			30	100	520 ... 670	300	18	31			
GS200	1.0449	+N	–	100	380 ... 530	200	25	35	–	–	–
GS240	1.0455	+N	–	100	450 ... 600	240	22	31	–	–	–

1) +N means: normalising.
2) Elongation at fracture: initial length $L_0 = 5d_0$.
3) RT stands for: room temperature (23 ± 5) °C.
4) These values apply only by arrangement.

Creep-resistant cast steel

In accordance with the standard DIN EN 10213:2016, the mechanical properties for a number of creep-resistant cast steels are listed in the following table as an extract.

Cast steel grade		Heat treatment symbol[1]	Tensile strength R_m	0,2% proof stress $R_{p0,2}$ at a temperature of							Elongation at fracture A	Notched bar impact work KV
Short name	Material number			20 °C	200 °C	300 °C	350 °C	400 °C	450 °C	500 °C		
			MPa	MPa							%	J
GP240GH	1.0619	+N	420 ... 600	240	175	145	135	130	125	–	22	27
		+QT	420 ... 600	240	175	145	135	130	125	–	22	40
GP280GH	1.0625	+N	480 ... 640	280	220	190	170	160	150	–	22	27
		+QT	480 ... 640	280	220	190	170	160	150	–	22	35
G20Mo5	1.5419	+QT	440 ... 590	245	190	165	155	150	145	135	22	27
G17CrMo5-5	1.7357	+QT	490 ... 690	315	250	230	215	200	190	175	20	27
G17CrMo9-10	1.7379	+QT	590 ... 740	400	355	345	330	315	305	280	18	40
G12CrMoV5-2	1.7720	+QT	510 ... 660	295	244	230	–	214	–	194	17	27
G17CrMoV5-10	1.7706	+QT	590 ... 780	440	385	365	350	335	320	300	15	27
GX15CrMo5	1.7365	+QT	630 ... 760	420	390	380	–	370	–	305	16	27
GX8CrNi12	1.4107	+QT1	540 ... 690	355	275	265	–	255	–	–	18	45
		+QT2	600 ... 800	500	410	390	–	370	–	–	16	40
GX4CrNi13-4	1.4317	+QT	760 ... 960	550	485	455	440	–	–	–	15	27
GX23CrMoV12-1	1.4931	+QT	740 ... 880	540	450	430	410	390	370	340	15	27
GX4CrNiMo16-5-1	1.4405	+QT	760 ... 960	540	485	455	–	–	–	–	15	60

[1] +N means: normalising.
+QT means: quenching and tempering (hardening in air or liquid + tempering).
If alternative heat treatments are available, the required alternative should be specified in the order, for example GX8CrNi12 +QT1 or 1.4107 +QT.

Malleable cast iron

According to the standard DIN EN 1562:2019, malleable cast iron is an iron-carbon cast material whose castings solidify largely without graphite if the design is appropriate for the material involved.

Depending on the type of heat treatment used for the unfinished casting, we arrive at:

- blackheart malleable cast iron (non-decarburised)
- whiteheart malleable cast iron (decarburised)

After annealing treatment, the iron carbide (cementite) present in the structure disappears completely. With the exception of fully decarburised, whiteheart malleable cast iron, both groups contain free carbon in the form of graphite, known as temper carbon. Both groups have material grades with structures that can range from ferrite to pearlite and/or other transformation products of austenite.
The chemical composition of the unfinished malleable cast iron and the nature of the temperature-dependent and time-dependent annealing process define the structural constitution and consequently the properties of the material.

The materials are designated in terms of tensile strength and elongation. This takes place relative to a 12 mm diameter test piece for whiteheart malleable cast iron and 12 mm or 15 mm diameter test pieces for blackheart malleable cast iron. However, comparative values of tensile strength and elongation after fracture are also given for other test piece diameters.

If welding work is to be carried out during the course of production or when using malleable iron castings, this must be agreed between the purchaser and the manufacturer of the casting. Heat treatment is required after repair welding.

Distortion as the result of heat treatment must be eliminated by straightening. Hot straightening or stress relief annealing can be agreed upon in special cases.

Malleable cast iron is easy to machine. The suitability of individual grades depends on the respective structural constitution.

The following shrinkage dimensions are valid for model production:

- 1% to 2% for whiteheart malleable cast iron
- 0% to 1,5% for blackheart malleable cast iron

The average density of the material is 7,4 kg/dm^3.

Mechanical properties The following table gives the mechanical properties of malleable cast iron as an extract from the corresponding standard.

Material designation		Decisive wall thickness value		Nominal diameter of the sample d	Tensile strength R_m	0,2% proof stress $R_{p0,2}$	Elongation at fracture $A_{3,4}$	Brinell hardness[1]	Minimum values for impact energy		Previous designation
Short designation	Material number	over	incl.						Average from 3 test pieces	Individual value	
		mm	mm	mm	min. MPa	min. MPa	min. %	max. HBW	min. J	min. J	
Whiteheart malleable (decarburised) cast iron											
EN-GJMW-350-4[2]	5.4200	–	3	6	270	–	10	230	–	–	GTW-35-04 EN-JM1010
		3	5	9	310	–	5	230	–	–	
		5	7	12	350	–	4	230	–	–	
		7	–	15	360	–	3	230	–	–	
EN-GJMW-360-12[2]	5.4201	–	3	6	280	–[3]	16	200	14	10	GTW-S-38-12 EN-JM1020
		3	5	9	320	170	15	200	14	10	
		5	7	12	360	190	12	200	14	10	
		7	–	15	370	200	8	200	14	10	
EN-GJMW-400-5[2]	5.4202	–	3	6	300	–[3]	12	220	7	5	GTW-40-05 EN-JM1030
		3	5	9	360	200	8	220	7	5	
		5	7	12	400	220	5	220	7	5	
		7	–	15	420	230	4	220	7	5	
EN-GJMW-450-7[2]	5.4203	–	3	6	330	–[3]	12	220	10	7	GTW-45-07 EN-JM1040
		3	5	9	400	230	10	220	10	7	
		5	7	12	450	260	7	220	10	7	
		7	–	15	480	280	4	220	10	7	
EN-GJMW-550-4	5.4204	–	3	6	–	–[3]	–	250	–	–	–
		3	5	9	490	310	5	250	–	–	
		5	7	12	550	340	4	250	–	–	
		7	–	15	570	350	3	250	–	–	

Continuation of table, see Page 305.

1) The values are for information purposes.

2) Material approved for pressure equipment in accordance with European legislation, see standard.

3) Values and methods must be agreed.

Continuation of table, Mechanical properties (malleable cast iron), from Page 304.

Material designation		Decisive wall thickness value		Nominal diameter of the sample d	Tensile strength R_m	0,2% proof stress $R_{p0,2}$	Elongation at fracture $A_{3,4}$	Brinell hardness[1)]	Minimum values for impact energy		Previous designation
Short designation	Material number	over mm	incl. mm	mm	min. MPa	min. MPa	min. %	max. HBW	Average from 3 test pieces min. J	Individual value min. J	
Blackheart malleable (non-decarburised) cast iron											
EN-GJMB-300-6[2)]	5.4100	–	–	12 or 15	300	–	6	150	–	–	–
EN-GJMB-350-10[3)]	5.4101	–	–	12 or 15	350	200	10	150	14	10	GTS-35-10 EN-JM1130
EN-GJMB-450-6[3)]	5.4205	–	–	12 or 15	450	270	6	150 ... 200	10	7	GTS-45-06 EN-JM1140
EN-GJMB-500-5	5.4206	–	–	12 or 15	500	300	5	165 ... 215	–	–	–
EN-GJMB-550-4	5.4207	–	–	12 or 15	550	340	4	180 ... 230	–	–	GTS-55-04 EN-JM1160
EN-GJMB-600-3	5.4208	–	–	12 or 15	600	390	3	195 ... 290	–	–	–
EN-GJMB-650-2[3)]	5.4300	–	–	12 or 15	650	430	2	210 ... 260	5	3,5	GTS-65-02 EN-JM1180
EN-GJMB-700-2[4)5)]	5.4301	–	–	12 or 15	700	530	2	240 ... 290	–	–	GTS-70-02 EN-JM1190
EN-GJMB-800-1[5)]	5.4302	–	–	12 or 15	800	600	1	270 ... 320	–	–	–

1) The values are for information purposes.
2) Not suitable for pressure equipment applications.
3) Material approved for pressure equipment in accordance with European legislation, see standard.
4) Quenched in oil and subsequently tempered.
5) When this grade is quenched in air, the 0,2 % proof stress must be at least 430 MPa.

Heat treatment of steel

Heat treatment processes – hardening

Hardening refers to the release of carbon from carbides against the equilibrium state (soft annealed state). This requires heating to a temperature range above 800 °C (depending on the composition), during which the carbides partially dissolve. Either bainite and pearlite or martensite are formed from the released carbon through the appropriate temperature control.

Martensite formation requires the released carbon to be held in a solution until it has cooled to at least room temperature. For the formation of bainite and pearlite, the released carbon must be separated out into fine or extremely fine iron carbide components.

Due to the changes in the structure, heat treatment always results in a dimension change, which is often also accompanied by distortion.

The most important heat treatment processes in the field of "hardening" are presented below.

Hardening over the entire cross-section

During the hardening process, the material is heated up to the hardening or austenitising temperature first, in order to produce the austenitic structure and partially dissolve carbides. This temperature is then maintained for a defined period, after which the material is cooled or quenched to room temperature or below at a sufficient speed.
The appropriate speed can be found in the time-temperature-transformation curve for the respective steel.
The speed must be chosen so as to avoid undesirable transformations. Depending on the composition, hardening may be complete and austenite may remain (residual austenite).

Tempering

Tempering is a heat treatment process which is applied in order to make the martensitic-hardened and relatively brittle material tougher.
This involves increasing the temperature in the region of +160 °C to +650 °C with an adequate holding period and subsequent cooling to room temperature.

Tempering reduces the hardness, the strength decreases and the ductility and toughness increase. Depending on the type of steel, any residual austenite present is converted at temperatures of and above +200 °C.

During the tempering of steels, dimensional changes and visually different temper colours are again produced as a function of the tempering temperature, see table, Annealing and temper colours, Page 310.

Quenching and tempering

The combination of hardening and tempering at a temperature in excess of +500 °C is referred to as quenching and tempering. Quenching and tempering is intended to produce an optimum ratio between strength and toughness.

Surface layer hardening

In the case of surface layer hardening, austenitisation and hardening are restricted to the surface layer of the workpiece. In most cases, the material is heated by electric induction (medium or high-frequency alternating current) or by lasers. The material is quenched by dipping or spraying.

Through the surface layer hardening of components that have already been quenched and tempered, a high basic strength can be combined with a high surface hardness in areas that are subjected to particularly high loads. The thickness of the hardened surface layer results from the hardness profile as the surface hardening depth SHD in mm (synonym: surface layer hardening depth); cf. standard DIN EN 10328:2016.

Case hardening

Case hardening (carburising, carbonitriding) involves carburising or carbonitriding followed by hardening. This hardening either takes place directly afterwards (direct hardening), or after the material concerned has been subjected to intermediate cooling and re-heated to an appropriate hardening temperature (single hardening).

Depending on the required use characteristics and the subsequent machining requirements, the material is tempered, or deep-cooled and tempered, after hardening.

Case hardening gives the surface layer of workpieces a significantly higher level of hardness and the entire workpiece improved mechanical properties. To this end, the surface layer is enriched with carbon (carburising) or carbon and nitrogen (carbonitriding) before hardening. In comparison with carburisation, additional enrichment with nitrogen results in increased hardenability (by changing the transformation behaviour in the surface layer) and, after hardening, in improved annealing resistance.

Bainitic hardening

During bainitic hardening (isothermal transformation in the bainite stage), the material is first heated to and maintained at the austenitising temperature. The material is cooled to a temperature of between +200 °C and +350 °C, depending on the steel, and kept at this temperature until the steel structure has transformed to bainite. The material is then cooled to room temperature. In this state, the hardness is less than that of martensite, but toughness is increased.
One variant of production is to only partially convert the material into bainite, then cool and temper it.

Bainitic hardening is regarded as an alternative to martensitic hardening if a high level of toughness is required without a high level of hardness, and if distortion and dimensional changes have to be minimised. Bainitic-hardened workpieces have high temperature stability.

Heat treatment processes – annealing

Annealing involves heating a workpiece to a temperature above 600 °C and up to 1 300 °C, depending on the purpose of the annealing process. The aim of this is mainly to influence and optimise the processing characteristics of the material.

Annealing treatment consists of heating the material up to the required annealing temperature, holding the material at this temperature for a sufficiently long period and cooling it to a temperature that is appropriate to the respective objective of the annealing treatment.

The most important heat treatment processes in the field of "annealing" are presented below.

Stress relief annealing

Intrinsic stresses resulting from structural transformations or cold deformations can occur in workpieces. These are caused by irregular heating or cooling processes. In order to reduce these intrinsic stresses in workpieces, tools or blanks (as a result of plastic deformations), stress relief annealing is carried out at temperatures of between +600 °C and +650 °C. After an annealing time of 0,5 h, the material should be cooled as slowly as possible to ensure that no new stresses arise.

Soft annealing

In order to improve the deformability of C steels and facilitate machining, the material is soft annealed at temperatures in the range of Ac_1. This also applies to workpieces that have been strengthened by hardening, precipitation hardening or cold forming. The temperature depends on the material (this is +650 °C to +800 °C for steel or a value lower than this for non-ferrous metals).

If a specific structural state, characterised by spheroidising of the carbides, is to be achieved, then "annealing to spheroidised cementite" (abbreviation: GKZ annealing) is applied. The spheroidal shape of the cementite can also be achieved by austenitisation and controlled cooling.

Recrystallisation annealing

The possibility of cold forming a material is limited by the increase in hardness and the decrease in formability with the strain caused by deformation.

Recrystallisation annealing is applied to formed workpieces in order to eliminate any strain hardening that may have occurred and bring about a new formation of the grains. This re-enables or facilitates subsequent forming.

The temperature depends on the degree of deformation and, in the case of steel, is generally around +550 °C to +730 °C.

Normalising

Normalising is carried out at austenitisation temperature, in other words at a temperature a little above Ac_3 (in the case of hypereutectoid steels, above Ac_1). After an adequate holding period, the material is cooled at an appropriate rate so that a structure consisting of ferrite and pearlite is created at room temperature.

Normalising is used to refine a coarse-grained structure (for example in steel castings and welds) and to achieve as homogeneous a ferrite-pearlite distribution as possible. It should be applied instead of recrystallisation annealing if a coarse-grained structure is to be feared in the case of subcritically deformed workpieces.

If an excessive austenitisation temperature is chosen, the γ-mixed crystals grow, which also leads to a coarse-grained structure after transformation. An excessively slow cooling process can also result in a coarse ferrite grain.

Homogenising

Homogenising takes place at temperatures of between 1 030 °C and 1 300 °C. It serves to eliminate segregation zones in ingots and strands.

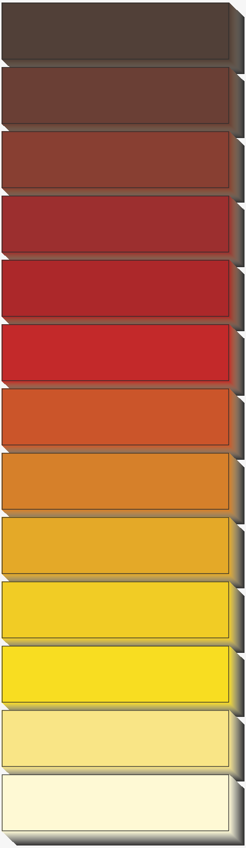
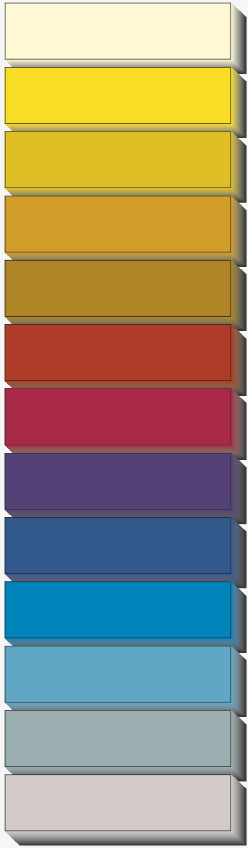
If the material is not subjected to hot forming after homogenising, it must be normalised in order to eliminate the coarse grain. Homogenising is rarely used in steel production today.

Precipitation hardening

Precipitation hardening consists of solution annealing, quenching and ageing above room temperature (hot ageing).
Ageing results in the segregation and separation of intermetallic compounds made up of specific solution elements dissolved in the base material. This changes material properties, for example hardness, strength, ductility and toughness. Precipitation hardening is predominantly used with high-alloy austenitic steels.

Annealing and temper colours

The following table shows the characteristic annealing and temper colours which occur at specific temperatures during the heat treatment of steel.

Annealing temperature °C	Annealing colours for steels	Tempering temperature °C	Temper colours for unalloyed tool steels[1]
550		200	
630		220	
680		230	
740		240	
780		250	
810		260	
850		270	
900		280	
950		290	
1000		300	
1100		320	
1200		340	
≧1300		360	

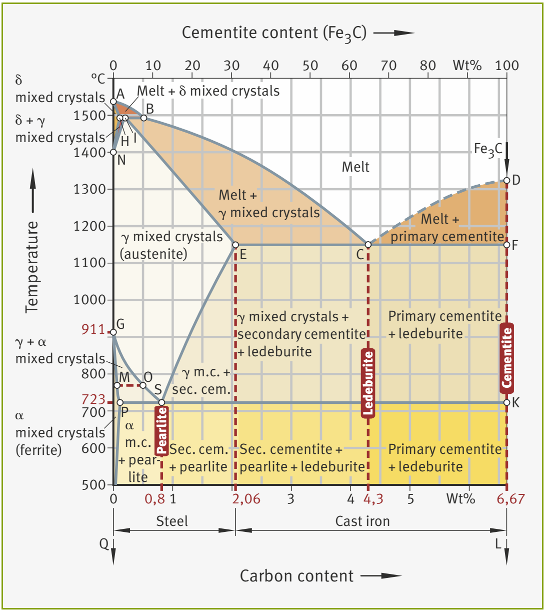
[1] In the case of high alloy steels, these temper colours only occur at higher temperatures. The temper colour is also influenced by the tempering time: longer tempering at a lower temperature results in the same temper colour as shorter tempering at a higher temperature.

Iron-carbon phase diagram

If we show the states of different iron grades as a function of their carbon content and the temperature, this gives the iron-carbon phase diagram.

Figure 1

Iron-carbon metastable system



The structure uniformly solidifying at the lowest solidification point of all melts (point C) is known as the eutectic point.

Iron crystal lattice

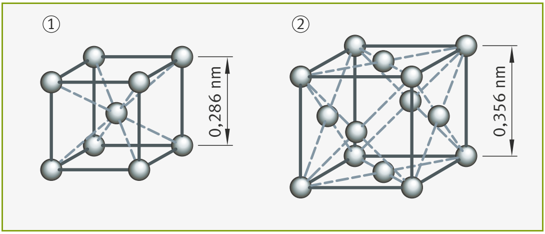
The elementary cell of iron has the following structure in the crystal lattice:

- body-centred cubic: α-iron (ferrite)
- face-centred cubic: γ-iron (austenite)

Figure 2

Crystal lattic structure

① Body-centred cubic crystal lattice
② Face-centred cubic crystal lattice

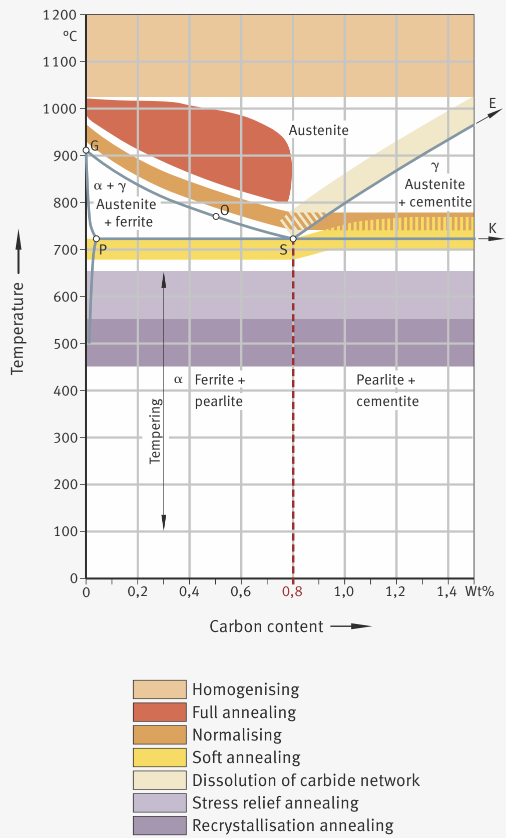


Heat treatment of steel

Heat treatment (hardening, annealing) gives the steel the required properties.

Figure 3

Heat treatment of steel



Case hardening The usual temperatures and heat treatments associated with the case hardening of case hardening steels are listed in the following tables.

Temperatures for case hardening According to the standard DIN EN ISO 683-3, the following temperatures are defined for the case hardening of case hardening steels:

Steel designation		Austenitising temperature for end quench test[1]	Carburis-ing tem-perature[2]	Direct or single hardening temperature[3)4)]	Double hardening		Temper-ing[5)]
Short name	Material number				Core hard-ness tem-perature[4)]	Surface hard-ness tem-perature[4)]	
		°C	°C	°C	°C	°C	°C
Unalloyed steels							
C10E C10R	1.1121 1.1207	–	880 ... 980	830 ... 870	880 ... 920	780 ... 820	150 ... 200
C15E C15R	1.1141 1.1140	–	880 ... 980	830 ... 870	880 ... 920	780 ... 820	150 ... 200
C16E C16R		–	880 ... 980	830 ... 870	880 ... 920	780 ... 820	150 ... 200
Alloyed steels							
17Cr3 17CrS3	1.7016 1.7014	880 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
20Cr4 20CrS4		900 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
28Cr4 28CrS4	1.7030 1.7036	850 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
16MnCr5 16MnCrS5	1.7131 1.7139	900 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
20MnCr5 20MnCrS5	1.7147 1.7149	900 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
20MoCr4 20MoCrS4	1.7321 1.7323	910 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
20NiCrMo2-2 20NiCrMoS2-2	1.6523 1.6526	900 ± 5	880 ... 980	820 ... 860	860 ... 900	780 ... 820	150 ... 200
17NiCrMo6-4 17NiCrMoS6-4	1.6566 1.6569	900 ± 5	880 ... 980	810 ... 850	830 ... 870	780 ... 820	150 ... 200

The temperatures specified for carburising, direct or single hardening, core hardening and surface hardening are guide values; the temperatures actually selected should be chosen with a view to meeting the set requirements.

1) Austenitisation duration (guide value): no less than 30 min.

2) The carburising temperature depends on the chemical composition of the steel, the mass of the product and the carburising agent. In direct hardening of steels, a temperature of 950 °C is generally not exceeded. For special processes, e.g. those performed in a vacuum, higher temperatures (e.g. 1020 °C to 1050 °C) are not uncommon.

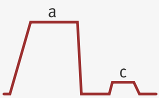
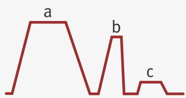
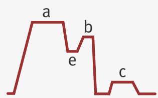
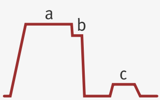
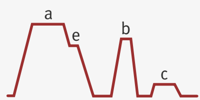
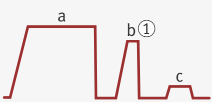
3) The type of quenching agent is determined, for example, by the shape of the product, the cooling conditions and the fill level of the furnace.

4) Steels which undergo single hardening and are at risk of distortion should be quenched at a temperature between the core hardness temperature and surface hardness temperature.

5) Tempering duration not less than 1 h (guide value).

Heat treatment during case hardening

The usual heat treatments associated with case hardening correspond to the following processes:

A **Direct hardening or double hardening**	**B** **Single hardening**	**C** **Hardening after isothermal transformation**
 Direct hardening from carburising temperature	 Single hardening from core or surface hardness temperature	 Hardening after isothermal transformation in the pearlite stage (e)
 Direct hardening after cooling to hardening temperature	 Single hardening after intermediate annealing (d) (soft annealing)	 Hardening after isothermal transformation in the pearlite stage (e) and cooling to room temperature
 Double hardening ① Surface zone		

a = carburising temperature.

b = hardening temperature.

c = tempering temperature.

d = intermediate annealing (soft annealing) temperature.

e = transformation temperature in the pearlite stage.

Heat treatment of rolling bearing steels

The following table shows common temperatures encountered in heat treatment.

Steel designation		Hardening temperature for end quench test	Normal-ising	Preheat-ing tem-perature	Harden-ing in oil[1]	Harden-ing in water[1]	Tem-pering	Previous designation
Short name	Material number	°C ± 5 °C	°C	°C	°C	°C	°C	
Through hardening rolling bearing steels								
100Cr6	1.3505	–	–	–	830 ... 870	–	› 150	100 Cr 6
100CrMnSi6-4	1.3520	–	–	–	830 ... 870	–	› 150	100 CrMn 6
100CrMo7	1.3537	–	–	–	840 ... 880	–	› 150	100 CrMo 7
100CrMo7-3	1.3536	–	–	–	840 ... 880	–	› 150	100 CrMo 7 3
100CrMoSi8-4-6	1.3539	–	–	–	840 ... 880	–	› 150	100 CrMnMo 8
Case hardening rolling bearing steels								
17MnCr5	1.3521	900	–	–	810 ... 840	–	› 150	17 MnCr 5
19MnCr5	1.3523	900	–	–	810 ... 840	–	› 150	19 MnCr 6
18NiCrMo14-6	1.3533	830	–	–	780 ... 820	–	› 150	17 NiCrMo 14

Continuation of table, see Page 316.

These are guide values.
From an operational perspective, the temperatures and other conditions should be selected in such a way that the required properties are achieved.

[1] A hot salt bath is used in place of oil.
Water with polymer additives is also used for case hardening steels.

Continuation of table, Heat treatment of rolling bearing steels, from Page 315.

Steel designation		Hardening temperature for end quench test	Normalising	Preheating temperature	Hardening in oil[1]	Hardening in water[1]	Tempering	Previous designation
Short name	Material number	°C ± 5 °C	°C	°C	°C	°C	°C	
Induction hardening rolling bearing steels								
C56E2	1.1219	840	830 ... 860	–	815 ... 845	805 ... 835	› 150	Cf 54
43CrMo4	1.3563	850	840 ... 880	–	830 ... 860	820 ... 850	› 150	43 CrMo 4
Stainless rolling bearing steels								
X47Cr14	1.3541	–	–	–	1020 ... 1070	–	› 100	X 45 Cr 13
X108CrMo17	1.3543	–	–	–	1030 ... 1080	–	› 100	X 102 CrMo 17
X89CrMoV18-1	1.3549	–	–	–	1030 ... 1080	–	› 100	X 89 CrMoV 18 1
High-temperature rolling bearing steels								
80MoCrV42-16	1.3551	–	–	750 ... 875	1070 ... 1120	–	500 ... 580[2]	80 MoCrV 42 16
X82WMoCrV6-5-4	1.3553	–	–	750 ... 875	1180 ... 1230	–	500 ... 580[2]	X 82 WMoCrV 6 5 4
X75WCrV18-4-1	1.3558	–	–	750 ... 875	1220 ... 1270	–	500 ... 580[2]	X 75 WCrV 18 4 1

With the exception of the hardening temperatures for the end quench test, these are guide values. From an operational perspective, the temperatures and other conditions should be selected in such a way that the required properties are achieved.

[1] Stainless rolling bearing steels and high-temperature rolling bearing steels are usually hardened in vacuum hardening furnaces.

[2] Tempering duration 2 h, recommended repetition 2 – 3 times.

Vickers hardness, Brinell hardness, Rockwell hardness, tensile strength

The data for Vickers hardness, Brinell hardness, Rockwell hardness and tensile strength cannot be converted directly into one another.
Therefore comparative values are given in the following (conversion) table:

Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness			Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness		
MPa	HV	HBW	HRB	HRC	HRA	MPa	HV	HBW	HRB	HRC	HRA
255	80	76,0	–	–	–	640	200	190	91,5	–	–
270	85	80,7	41,0	–	–	660	205	195	92,5	–	–
285	90	85,5	48,0	–	–	675	210	199	93,5	–	–
305	95	90,2	52,0	–	–	690	215	204	94,0	–	–
320	100	95,0	56,2	–	–	705	220	209	95,0	–	–
335	105	99,8	–	–	–	720	225	214	96,0	–	–
350	110	105	62,3	–	–	740	230	219	96,7	–	–
370	115	109	–	–	–	755	235	223	–	–	–
385	120	114	66,7	–	–	770	240	228	98,1	20,3	60,7
400	125	119	–	–	–	785	245	233	–	21,3	61,2
415	130	124	71,2	–	–	800	250	238	99,5	22,2	61,6
430	135	128	–	–	–	820	255	242	–	23,1	62,0
450	140	133	75,0	–	–	835	260	247	(101)	24,0	62,4
465	145	138	–	–	–	850	265	252	–	24,8	62,7
480	150	143	78,7	–	–	865	270	257	(102)	25,6	63,1
495	155	147	–	–	–	880	275	261	–	26,4	63,5
510	160	152	81,7	–	–	900	280	266	(104)	27,1	63,8
530	165	156	–	–	–	915	285	271	–	27,8	64,2
545	170	162	85,0	–	–	930	290	276	(105)	28,5	64,5
560	175	166	–	–	–	950	295	280	–	29,2	64,8
575	180	171	87,1	–	–	965	300	285	–	29,8	65,2
595	185	176	–	–	–	995	310	295	–	31,0	65,8
610	190	181	89,5	–	–	1030	320	304	–	32,2	66,4
625	195	185	–	–	–	1060	330	315	–	33,3	67,0

Continuation of table, see Page 318.

The numbers in parentheses are hardness values that lie outside the range of the standardised hardness testing methods but which, in practical terms, are frequently used as approximated values.

1) Test force: $F \geqq 98$ N.

2) Degree of load: $0,102F/D^2 = 30$ MPa.

Continuation of table, Vickers hardness, Brinell hardness, Rockwell hardness, tensile strength, from Page 317.

Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness			Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness	
MPa	HV	HBW	HRB	HRC	HRA	MPa	HV	HBW	HRC	HRA
1095	340	323	–	34,4	67,6	1920	580	(551)	54,1	78,0
1125	350	333	–	35,5	68,1	1955	590	(561)	54,1	78,4
1155	360	342	–	36,6	68,7	1995	600	(570)	55,2	78,6
1190	370	352	–	37,7	69,2	2030	610	(580)	55,7	78,9
1220	380	361	–	38,8	69,8	2070	620	(589)	56,3	79,2
1255	390	371	–	39,8	70,3	2105	630	(599)	56,8	79,5
1290	400	380	–	40,8	70,8	2145	640	(608)	57,3	79,8
1320	410	390	–	41,8	71,4	2180	650	(618)	57,8	80,0
1350	420	399	–	42,7	71,8	–	660	–	58,3	80,3
1385	430	409	–	43,6	72,3	–	670	–	58,8	80,6
1420	440	417	–	44,5	72,8	–	680	–	59,2	80,8
1455	450	428	–	45,3	73,3	–	690	–	59,7	81,1
1485	460	437	–	46,1	73,6	–	700	–	60,1	81,3
1520	470	447	–	46,9	74,1	–	720	–	61,0	81,8
1555	480	(456)	–	47,7	74,5	–	740	–	61,8	82,2
1595	490	(466)	–	48,4	74,9	–	760	–	62,5	82,6
1630	500	(475)	–	49,1	75,3	–	780	–	63,3	83,0
1665	510	(485)	–	49,8	75,7	–	800	–	64,0	83,4
1700	520	(494)	–	50,5	76,1	–	820	–	64,7	83,8
1740	530	(504)	–	51,1	76,4	–	840	–	65,3	84,1
1775	540	(513)	–	51,7	76,7	–	860	–	65,9	84,4
1810	550	(523)	–	52,3	77,0	–	880	–	66,4	84,7
1845	560	(532)	–	53,0	77,4	–	900	–	67,0	85,0
1880	570	(542)	–	53,6	77,8	–	920	–	67,5	85,3
						–	940	–	68,0	85,6

The numbers in parentheses are hardness values that lie outside the range of the standardised hardness testing methods but which, in practical terms, are frequently used as approximated values.

1) Test force: $F \geqq 98$ N.

2) Degree of load: $0{,}102F/D^2 = 30$ MPa.

Non-ferrous metals

Non-ferrous metal grades

The following tables compile a small selection of non-ferrous metals (NF metals) for general machine building, covering:

- copper alloys
- aluminium alloys
- magnesium alloys

In the case of cast components it should be noted that, according to many standards, the mechanical properties of materials apply solely to "test bars cast separately".

Copper alloys

The following table gives the mechanical properties of a number of copper alloys as an extract from the corresponding standards.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Wrought copper-zinc alloys in accordance with DIN EN 12163							
CuBe2	CW101C	R420 R600 R1150	2 ... 80 25 ... 80 2 ... 80 Round rods	35 10 2	420 600 1150	140 480 1000	E = 122 000 MPa; good solderability, optimum precipitation hardening time; springs of all types, membranes, tension bands, non-magnetic design parts, bearing blocks, worm and spur gears, turned clockmaking parts, injection moulding dies, spark-proof tools
CuCr1Zr CuCr1	CW106C CW105C	R200 R400 R470	8 ... 80 50 ... 80 4 ... 25 Round rods	30 12 7	200 400 470	60 310 380	E = 120 000 MPa; high electrical conductivity, high softening temperature and creep rupture strength, negligible weldability and solderability, high temperature resistance, precipitation hardenable; continuous casting moulds, springs and contacts for conduction of current, electrodes for resistance welding, continuously cast profiles

Continuation of table, see Page 320.

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 319.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Wrought copper-zinc based multiple element alloys in accordance with DIN EN 12163							
CuZn31Si1	CW708R	R460 R530	5 ... 40 5 ... 14 Round rods	22 12	460 530	250 330	E = 109 000 MPa; good sliding characteristics even under heavy loads, suitable for cold forming, suitable for soldering and welding under certain conditions; bearing bushes, sliding elements, guide systems, drop forged parts
Wrought copper-tin alloys in accordance with DIN EN 12163							
CuSn6	CW452K	R340 R400 R470 R550	2 ... 60 2 ... 40 2 ... 12 2 ... 6 Round rods	45 26 15 8	340 400 470 550	230 250 350 500	E = 118 000 MPa; highly suitable for cold forming, good suitability for welding and soldering, resistant to seawater and industrial atmospheres; springs of all types, hosepipes and coiled pipes, membranes, mesh and sieve wire, gears, bushes, parts for the chemicals industry
Wrought copper-zinc-lead alloys in accordance with DIN EN 12164							
CuZn37Mn3 Al2PbSi[3]	CW713R	R540 R590 R620	6 ... 80 6 ... 50 15 ... 50 Round rods	15 12 8	540 590 620	280 320 350	E = 93 000 MPa; high strength, high wear resistance, good resistance to atmospheric corrosion, resistant to oil corrosion; design parts in machine building, plain bearings, valve guides, gearbox parts, piston rings

Continuation of table, see Page 321.

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

[3] Pb-alloyed materials should be replaced as soon as possible.

Continuation of table, Copper alloys, from Page 320.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Wrought copper-aluminium alloys in accordance with DIN EN 12163							
CuAl10Fe3Mn2	CW306G	R590 R690	10 ... 80 10 ... 50 Round rods	12 6	590 690	330 510	E = 120 000 MPa; high fatigue strength under reversed stresses even under corrosive conditions, good corrosion resistance, resistant to seawater, resistant to scaling, erosion and cavitation, creep-resistant; design parts for chemical apparatus construction, scale-resistant parts, screws, shafts, gears, valve seats
Wrought copper-nickel alloys in accordance with DIN EN 12163							
CuNi10Fe1Mn	CW352H	R280 R350	10 ... 80 2 ... 20 Round rods	30 10	280 350	90 150	E = 134 000 MPa; excellent resistance to erosion, cavitation and corrosion, insensitive to stress corrosion cracking, tendency to pitting under foreign deposition, good suitability for cold forming and soldering; pipework, brake pipes, plates and bases for heat exchangers, condensers, apparatus construction, freshwater processing plant

Continuation of table, see Page 322.

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 321.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Cast copper-tin alloys (cast tin bronze) in accordance with DIN EN 1982							
CuSn12-C	CC483K	GS GM GC GZ	–	7 5 6 5	260 270 300 280	140 150 150 150	E = 94 000 MPa to 98 000 MPa; standard alloy with good sliding and wear characteristics together with good corrosion resistance, very good emergency running characteristics; bushes, sliding elements, sliding strips, bearing shells
Cast copper-zinc alloys in accordance with DIN EN 1982							
CuZn37Al1-C	CC766S	GM	–	25	450	170	E = 100 000 MPa; moderate strength; design and conducting material in machine building and precision engineering, gravity diecast parts for machine building and electrical engineering
Cast copper-tin-zinc-(lead) alloys (gunmetal) and cast copper-tin-lead alloys (cast tin-lead bronze) in accordance with DIN EN 1982							
CuSn7Zn4Pb7-C	CC483K	GS GM GC, GZ	–	15 12 12	230 230 260	120 120 120	E = 95 000 MPa; standard sliding material with excellent emergency running characteristics, moderate strength and hardness; plain bearings for hardened and unhardened shafts, sliding plates and strips, printing rollers, propeller shaft sleeves

Continuation of table, see Page 323.

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 322.

Material designation		Condition[1)]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2)]	0,2% proof stress $R_{p0,2N}$[2)]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Cast copper-aluminium alloys (cast aluminium bronze) in accordance with DIN EN 1982							
CuAl10Fe5Ni5-C	CC333G	GS GM GZ GC	–	13 7 13 13	600 650 650 650	250 280 280 280	E = 120 000 MPa; very good fatigue strength under reversed stresses even under corrosive conditions (seawater), high resistance to cavitation and erosion, long term loading up to 250 °C, weldable to S235; ships' propellers, stern tubes, impellers, pump housings

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

1) Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

2) The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Aluminium alloys The following table gives the mechanical properties of a number of aluminium alloys as an extract from the corresponding standards.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Aluminium and wrought aluminium alloys, not precipitation hardened, in accordance with DIN EN 485-2, DIN EN 754-2, DIN EN 755-2[3]							
ENAW-(Al99,5)	ENAW-1050A	O/H111 H14 H18	≦50 ≦25 ≦3 Sheets	>20 2 ... 6 2	65 105 140	20 85 120	E = 70 000 MPa; good corrosion resistance, highly suitable for cold and hot forming, good suitability for welding and soldering, poor machinability, surface protection by means of anodising; apparatus, containers, pipes for foodstuffs and beverages, deep drawn, pressed and sheet metal formed parts, busbars, overhead power lines, packaging
Wrought aluminium alloys, precipitation hardenable, in accordance with DIN EN 485-2, DIN EN 754-2, DIN EN 755-2							
ENAW-AlZn4,5Mg1	ENAW-7020	T6	≦40 Profiles	10	350	290	E = 70 000 MPa; construction alloys of series 7000 with very high strength and low resistance, good cold forming suitability in the soft condition (O), self-hardening by fusion welding process, but sensitive to notching and ageing; profiles, tubes and sheets for welded supporting structures in building construction, vehicle engineering and machine building
Cast aluminium alloys in accordance with DIN EN 1706							
ENAC-AlSi9Mg	ENAC-43300	S T6 K T6	–	2 4	230 290	190 210	E = 75 000 MPa; for complicated, thin-walled castings of high strength, good toughness and very good weather resistance, precipitation hardenable, good weldability and solderability, good machinability; engine blocks, gearbox and converter housings

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

[3] DIN EN 485-2: sheet, plate.
DIN EN 754-2: cold drawn rod/bar and tube.
DIN EN 755-2: extruded rod/bar and tube

Magnesium alloys The following table gives the mechanical properties of a number of magnesium alloys as an extract from the corresponding standards.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. MPa	min. MPa	
Wrought magnesium alloys in accordance with DIN 1729 and DIN 9715							
MgAl8Zn	3.5812.08	F27 F29 F31	– ≦10 ≦10	8 10 6	270 290 310	195 205 215	E = 43 000 MPa to 45 000 MPa; very high strength, vibration-resistant, not weldable; components subjected to vibrations and shocks
Cast magnesium alloys in accordance with DIN EN 1753							
EN-MCMgAl8Zn1	EN-MC 21110	S, K F S, K T4 D F	–	2 8 1 ... 7	160 240 200 ... 250	90 90 140 ... 170	E = 41 000 MPa to 45 000 MPa; good castability, weldable, good sliding characteristics, suitable for dynamic loading; components subjected to vibrations and shocks, gearbox and engine housings, oil trays

Source: according to Roloff/Matek Maschinenelemente Band 2, Springer Fachmedien Wiesbaden, 25. Auflage 2021.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 326.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Condition designations and casting methods

Some examples of condition designations and casting methods for non-ferrous metals are given in the following two tables.

Strength	Description
Copper alloys	
R600	Minimum tensile strength $R_m = 600$ MPa

Strength	Description
Wrought magnesium alloys	
F22	Minimum tensile strength $R_m = 10 \cdot 22$ MPa = 220 MPa

Casting methods	Description
Copper alloys	
GS	Sand casting
GM	Gravity diecasting
GZ	Centrifugal casting
GC	Continuous casting
GP	Pressure diecasting
Cast aluminium and cast magnesium alloys	
S	Sand casting
K	Gravity diecasting
D	Pressure diecasting
L	Investment casting

Condition	Description
Wrought aluminium alloys, precipitation hardenable	
T3	Solution annealed, cold formed and cold aged
T351	Solution annealed, stress relieved by controlled stretching and cold aged
T4	Solution annealed and cold aged
T5	Quenched and hot aged
T6	Solution annealed and completely hot aged
Wrought aluminium alloys, not precipitation hardenable	
O	Soft annealed
F	Cast condition
H111	Annealed with subsequent slight work hardening
H12	Work hardened, 1/4 hard
H14	Work hardened, 1/2 hard
H16	Work hardened, 3/4 hard
H18	Work hardened, 4/4 hard
H22	Work hardened and post-annealed, 1/4 hard
H24	Work hardened and post-annealed, 1/2 hard

Plastic

Structure and properties

Plastics are macromolecular, organic materials that are produced artificially from monomers by chemical means, in other words they do not occur in nature.

Depending on the synthesis process, plastics can be subdivided into:
- polycondensation products
- polymerides
- polyaddition compounds

Due to their different chemical structures, polymers can also be subdivided into:
- thermoplastics
- elastomers
- thermosets

Thermoplastics

Thermoplastics occur as amorphous and semi-crystalline polymers. Depending on the type of plastic in question, they consist of linear or ramified macromolecules.
Thermoplastics can soften or melt when heated and solidify when cooled, processes that may be repeated. During the original shaping process, they undergo reversible changes of state. Thermoplastics can be shaped in the free-flowing state by means of various types of processing technology, for example injection moulding, extrusion and calendering, thus arriving at complex components or semi-finished products. Semi-finished products made of (hard) thermoplastics can largely be shaped whilst hot. Thermoplastics are weldable. The mechanical properties are temperature-dependent, with a continuous operating temperature range of 60 °C to 250 °C and a short-term maximum operating temperature of up to 300 °C depending on the thermoplastic used. The type of plastic used determines possible chemical resistance to oils/greases, acids/alkalis and organic solvents.

Elastomer plastics

The structure of elastomers is based on a network of widely meshed macromolecules. In the operating temperature range above the glass temperature, they have a rubber-elastic behaviour. In other words, low stresses result in considerable deformations which recede almost completely once the stress has been removed.
In the event of a temperature increase, they exhibit a rubber-elastic behaviour up to the limit temperature of the irreversible, thermochemical degradation of the (cross-linked) polymer structure.

Elastomers are obtained, for example, through the polymerisation of dienes (NBR) or are the result of polycondensation and polyaddition reactions in starting materials (for example PUR). Elastomers are generally processed in the plastic state, before cross-linking with the addition of vulcanisation agents or cross-linking accelerators.

Thermoplastic elastomers (TPE)

Thermoplastic elastomers are polymers that can be processed thermoplastically with elastomer-like properties.
They are not chemically cross-linked.
TPEs are mostly block copolymers with "hard" and "soft" ranges.
The hard segments form aggregated zones. Owing to secondary compounds, these result in physical cross-linking points in the amorphous matrix, which reversibly dissolve at a temperature that is determined by the chemical structure. These polymers become thermoplastically free-flowing above this temperature.

Thermosetting plastics

In the original shaping process, thermosets come into being by virtue of the fact that free-flowing preproducts of a low molecular weight react with one another, forming chemically narrow cross-linked macromolecules.
Up to the limit temperature of thermochemical degradation, the physical properties of the irreversibly "cured" thermosets are not – unlike thermoplastics or TPE – very dependent on the temperature.
They are not weldable, are not soluble in organic solvents when cured and some of them are capable of swelling.
Thermosetting preproducts can be obtained as "moulding compounds" for processing by means of melting and with subsequent thermal curing.
They are also available as liquid "reaction resins" that can be processed and catalytically cured at room temperature. Moulding compounds consist of the base resin and a number of fillers, such as glass fibres and/or glass beads, mineral fillers and processing aids.

In addition to liquid reaction resins, thermosets can also be present and processed in powder form.

Recycling

Corresponding to their chemical structure, polymer materials are open to diverse recycling possibilities or disposal concepts.
Besides being suitable for more cost-intensive, chemical recycling (for example hydrolysis, hydrogenation or pyrolysis to produce monomers), thermoplastics are also suitable for mostly economical, physical recycling.
Cross-linked polymers can only be recycled chemically, or used as fillers after grinding.

Application

Depending on the application area, thermoplastics are subdivided into:

- so-called mass plastics (for example PE, PS, PVC, PP, ABS, SAN)
- engineering plastics (for example PA, PBT, PET, PC, POM)
- high-performance/high-temperature plastics (for example PPS, PEEK, PEI, PPA, LCP)

Engineering and high-performance plastics are used in automotive engineering, machine construction, the electrical industry and chemicals and plant engineering.
In order to improve the level of the physical and mechanical properties, glass or carbon fibres are frequently added to the base polymers as reinforcements and glass balls or minerals (for example talcum, mica or silica sand, etc.) are added as fillers.
The sliding/friction properties can be improved through modification with graphite or teflon (PTFE).
Thermosets are also often used in the stated areas of engineering.

For further information on plastics for rolling bearings, please see chapter Design elements, section Rolling bearings, Page 617.

Classification of plastics

Plastics can be subdivided into:

- fully synthetic plastics
- modified natural substances (technically low relevance)

The following table lists a number of plastics and the corresponding manufacturing method.

Description	Thermoplastics	Hardenable plastics	Manufacturing method
Fully synthetic plastics			
Polycondensation products	Linear polyester resins, polyamides, mixed polyamides	Phenolic plastics, carbamide polymers, melamine plastics, silicones, polyester resins, alkyd resins	**Polycondensation** During polycondensation, various basic, monomeric building blocks combine with each other, accompanied by the separation of water and other volatile substances of low molecular weight (for example ammonia), to form: ■ linear macromolecules, if monomers with two groups that are capable of reacting are present. ■ spatially cross-linked macromolecules if two or more groups capable of reacting are present. Most thermosets are an important example of this. In such cases, the process often takes place at an increased temperature and under pressure.

Continuation of table, see Page 330.

Continuation of table, Classification of plastics, from Page 329.

Description	Thermoplastics	Hardenable plastics	Manufacturing method
Fully synthetic plastics			
Polymerides	Polyethylenes, polyvinyl chlorides, polystyrenes, polyisobutylenes, polymethacrylates, polyacrylonitriles, polyfluorethylenes	Alkyl resins, unsaturated polyester resins	**Polymerisation** When the double bonds split, the liquid or gaseous initial products, which are mostly equally monomeric, deposit themselves on one another, producing filamentary molecules. No cleavage products are produced. The process is started by initiators (radical or ion generators) and then continues to run exothermally. During this process, the polymeride becomes tenacious and ultimately solid as the molecular weight rises. Most thermoplastics are obtained through polymerisation.
Polyaddition products	Linear polyurethanes	Cross-linked polyurethanes, epoxy resins	**Polyaddition** As a result of the intermolecular repositioning (for example hydrogen), various basic, monomeric building blocks with reactive groups combine to form macromolecules. The process is very similar to that of polycondensation but, in this case, no cleavage products of low molecular weight are produced. Depending on the choice of preproducts, polymers that are either cross-linked or not cross-linked can also be synthesised here. Depending on the starting material, the properties of the products can be intentionally adjusted within wide limits.
Modified natural substances			
Natural substances of high molecular weight	Cellulose ester, cellulose ether	Casein resins	**Chemical transformation** The oldest plastics are modified natural substances such as casein plastics, vulcanised fibre, celluloid, cellophane and artificial silk. High water absorption and associated dimensional changes prevent such substances from being used for technical parts.

Strength characteristics and dimensional stability

The following tables give the strength characteristics and dimensional stability values for thermoplastics and thermosetting plastics.

Unreinforced thermoplastics

The following values are valid for the strength characteristics and dimensional stability of a number of unreinforced thermoplastics:

Plastic	Designation	Tensile strength MPa	Tensile modulus of elasticity (30 s) MPa	Indentation hardness MPa	Flexural strength MPa	Impact strength kJ/m²	Notched impact strength kJ/m²	Dimensional stability (Vicat B) °C
Polyethylene, hard	PE-HD	18 ... 35	700 ... 1 400	40 ... 65	36	■	■	60 ... 70
Polyethylene, soft	PE-LD	8 ... 23	200 ... 500	13 ... 20	–	■	■	<40
Polypropylene	PP	21 ... 37	1 100 ... 1 300	36 ... 70	43	■	3 ... 17	85 ... 100
Polyvinyl chloride, hard	PVC-U	50 ... 75	2 500 ... 3 500	75 ... 155	110	■	2 ... 50	75 ... 110
Polyvinyl chloride, soft	PVC-P	10 ... 25	<100	A90[1]	–	■	■	40
Polystyrene	PS	45 ... 65	3 200 ... 3 250	140 ... 150	90	15 ... 20	2 ... 2,5	78 ... 99
Styrene/acrylonitrile copolymer	SAN	75	3 600	160 ... 170	100	16 ... 20	2 ... 3	100 ... 115
Acrylonitrile-butadiene-styrene copolymer	ABS	32 ... 60	1 900 ... 2 700	80 ... 120	75	70	7 ... 20	95 ... 110
Polymethyl methacrylate	PMMA	50 ... 77	2 700 ... 3 200	180 ... 200	105	18	2	70 ... 100
Polyoxymethylene	POM	62 ... 80	3 200	150 ... 170	110	■	8	160 ... 173
Polytetrafluoroethylene	PTFE	25 ... 36	410	27 ... 35	18	■	13 ... 15	–
Polyamide 6[2]	PA6	70 ... 85	1 400	75	50	■	■	180
Polyamide 66[2]	PA66	77 ... 84	2 000	100	50	■	15 ... 20	200
Polyamide 11[2]	PA11	40	1000	75	–	■	30 ... 40	175
Polyamide 12[2]	PA12	40	1 200	75	–	■	10 ... 20	165
Polycarbonate	PC	56 ... 67	2100 ... 2400	110	100	■	20 ... 30	160 ... 170
Cellulose acetate (432)	CA	40	1 600	50	50	■	15	50 ... 63
Cellulose acetate butyrate (413)	CAB	35	1 600	55	38	■	20	60 ... 75

■ No fracture.

[1] Shore hardness scale A.

[2] Conditioned at +23 °C and 50% relative humidity.

Reinforced thermoplastics The following values are valid for the strength characteristics and dimensional stability of a number of reinforced thermoplastics:

Plastic	Designation	Tensile strength MPa	Tensile modulus of elasticity MPa	Elongation at tear %	Flexural strength MPa	Impact strength kJ/m²	Notched impact strength kJ/m²	Dimensional stability (Vicat B)[1] °C
Polypropylene	PP GF30[2]	50	5 500	5	65	16[5]	6[5]	110
Polybutylene terephthalate	PBT GF30	145	10 500	2,5	210	50[6]	8,5[6]	205
Polyethylene terephthalate	PET GF30	175	11 000	2	228	32[6]	10[6]	260
Polyamide 6[3]	PA6 GF30	180	8 500	3	250	60[6]	12[6]	210
Polyamide 46[3]	PA46 GF30	210	10 000	3,5	300	80[5]	12[5]	290
Polyamide 66[3]	PA66 GF30	190	10 000	3	270	45[6]	8,5[6]	250
Polyoxymethylene	POM GF30	130	10 000	3	170	32[6]	5,5[6]	160
Polyphenylene sulphide	PPS GF40[4]	180	14 000	1,6	240	35[6]	6,5[5]	255
Polysulfone	PSU GF30	125	10 000	1,8	160	20[6]	7[6]	190
Polyethersulfone	PESU GF30	150	10 500	2,1	200	30[6]	8[6]	215
Polyetherimide	PEI GF30	160	9 000	3	220	35[5]	8[5]	220
Polyetherketone	PEEK GF30	190	12 000	2,5	280	55[5]	7,5[5]	› 300
Liquid crystal polymer	LCP GF30	200	23 000	1	–	20[5]	12[5]	170

[1] Tested at 50 °C/h with 50 N.
[2] GF30 = filled with 30% glass fibre.
[3] Values specifically dry.
[4] 30% not commercially available.
[5] Charpy test method.
[6] Izod test method.

Thermosetting plastics The following values are valid for the strength characteristics and dimensional stability of a number of thermosetting plastics:

Type of resin	Group	Type	Filler	Flexural strength min. MPa	Impact strength min. kJ/m²	Notched impact strength min. kJ/m²	Dimensional stability (Martens) °C
Phenol	I	31	Wood flour	70	6	1,5	125
	II	85	Wood flour	70	5	2,5	125
		51	Cellulose	60	5	3,5	125
		83	Cotton fibres	60	5	3,5	125
		71	Cotton fibres	60	6	6	125
		84	Cotton fabric shreds	60	6	6	125
		74	Cotton fabric shreds	60	12	12	125
		75	Artificial silk skeins	60	14	14	125
	III	12	Asbestos fibres	Hardly any asbestos products are now offered in the marketplace.			
		15	Asbestos fibres				
		16	Asbestos rope				
	IV	11,5	Rock flour	50	3,5	1,3	150
		13	Mica	50	3	2	150
		13,5	Mica	50	3	2	150
		30,5	Wood flour	60	5	1,5	100
		31,5	Wood flour	70	6	1,5	125
		51,5	Cellulose	60	5	3,5	125
		HP 2061	Paper webs	150	20	15	–
		Hgw 2081	Cotton fabric, coarse	100	18	15	–
		Hgw 2082	Cotton fabric, fine	130	30	15	–
		Hgw 2083	Cotton fabric, extremely fine	150	35	15	–

Continuation of table, see Page 334.

Group I: types for general use.

Group II: types with increased notched impact strength.

Group III: types with increased dimensional stability under heat.

Group IV: types with increased electrical characteristics.

Continuation of table, Thermosetting plastics, from Page 333.

Type of resin	Group	Type	Filler	Flexural strength min. MPa	Impact strength min. kJ/m^2	Notched impact strength min. kJ/m^2	Dimensional stability (Martens) °C
Aminoplastic and aminoplastic phenol	I	131	Cellulose	80	6,5	1,5	100
		150	Wood flour	70	6	1,5	120
		180	Wood flour	80	6	1,5	120
	II	153	Cotton fibres	60	5	3,5	125
		154	Cotton fabric shreds	60	6	6	125
	III	155	Rock flour	40	2,5	1	130
		156	Asbestos fibres	Hardly any asbestos products are now offered in the marketplace.			
		157	Asbestos fibres + wood flour				
		158	Asbestos fibres				
	IV	131,5	Cellulose	80	6,5	1,5	100
		152	Cellulose	80	7	1,5	120
		181	Cellulose	80	7	1,5	120
		181,5	Cellulose	80	7	1,5	120
		182	Wood and rock flour	70	4	1,2	120
		183	Cellulose + rock flour	70	5	1,5	120
Polyester		801	Glass fibres	60	22	22	125
		802	Glass fibres	55	4,5	3	140
		830	Glass fibre mats	120	50	40	–
		832	Glass fibre mats	160	70	60	–
Epoxy		870	Rock flour	50	5	1,5	110
		871	Glass fibres	80	8	3	120
		872	Glass fibres	90	15	15	125

Group I: types for general use.
Group II: types with increased notched impact strength.
Group III: types with increased dimensional stability under heat.
Group IV: types with increased electrical characteristics.

Processing and use A number of processing procedures and special uses of the most important plastics are presented below.

Plastic	Shaping, reshaping and joining behaviour for workpieces							
	Creating shape by						Shape-changing	
	Melt-on, casting and spraying methods	Low-pressure procedures for reinforced plastics	Injection moulding	Blow moulding	Pressing	Extrusion	(Hot) shaping	Welding
Thermoplastics								
Polyolefins	++	–	+++	+++	(+)	+++	+	+++
Styrene polymers	(+)	–	+++	+	–	++	++	+
Vinyl chloride polymers (hard)	(+)	–	+	++	(+)	+++	++	+++
Polyvinyl chloride (soft)	+	–	+	(+)	+++	++	+	++
Fluorine-containing polymers	+	–	+	–	(+)	(+)	–	+
Poly(meth)acrylic plastics	++	(+)	++	–	–	++	++	–
Heteropolymers	+	–	++	+	–	+	(+)	+
Cellulose ester, cellulose ether	+	–	++	++	–	++	+	–
Cellulose hydrate (viscose sheet, cellophane)	–	–	–	–	–	–	(+)	–
High-temperature plastics								
Polyarylenes, polyarylamides (PARA), polyesters, polyoxides, polyimides	+	+	(+)	–	+	–	–	(+)
Thermosetting plastics								
Phenolic resins, cresylic resins and furan resins	+	(+)	++	–	++	(+)	(+)	–
Urea-formaldehyde resins	–	–	+	–	+	(+)	–	–
Melamine resins	–	–	+	–	++	–	(+)	–
Casein plastics and similar casein products	–	–	–	–	–	(+)	–	–
Unsaturated polyesters	++	+++	+	–	++	(+)	–	–
Epoxy resins	+++	++	++	–	+	(+)	–	–
Special reaction resins	+	(+)	(+)	–	+	–	–	–
Isocyanate resins (PIR)	+++	–	(+)	–	–	–	–	–

Continuation of table, see Page 336.

– = not possible or not common.

+, ++, +++ = corresponds to growing importance.

(+) = special case.

Continuation of table, Processing and use, from Page 335.

Plastic	Shaping, reshaping and joining behaviour for workpieces					
	Special uses					
	Film and fabric synthetic leather	Packaging and insulating films	Foam plastics	Adhesives	Paints and coating materials	Fibres and filaments
Thermoplastics						
Polyolefins	–	+++	+	–	–	+
Styrene polymers	–	++	+++	–	+	(+)
Vinyl chloride polymers (hard)	–	++	+	(+)	(+)	(+)
Polyvinyl chloride (soft)	+++	+	++	–	+	–
Fluorine-containing polymers	–	(+)	(+)	–	–	–
Poly(meth)acrylic plastics	–	–	(+)	+	+	–
Heteropolymers	(+)	++	(+)	(+)	(+)	+++
Cellulose ester, cellulose ether	–	++	–	+	+	++
Cellulose hydrate (viscose sheet, cellophane)	–	+++	–	–	–	++
High-temperature plastics						
Polyarylenes, polyarylamides (PARA), polyesters, polyoxides, polyimides	–	+	+	+	+	(+)
Thermosetting plastics						
Phenolic resins, cresylic resins and furan resins	–	–	+	++	+	–
Urea-formaldehyde resins	–	–	++	+++	+	–
Melamine resins	–	–	(+)	+	+	–
Casein plastics and similar casein products	–	–	–	(+)	(+)	(+)
Unsaturated polyesters	–	–	(+)	++	++	–
Epoxy resins	–	–	–	+++	+	–
Special reaction resins	–	–	–	+	(+)	–
Isocyanate resins (PIR)	++	–	+++	+	+	(+)

– = not possible or not common.

+, ++, +++ = corresponds to growing importance.

(+) = special case.

Material selection

In this chapter, the essential engineering materials, their properties and most important characteristic values have been presented. The task of design engineers and product developers is to select the right materials for the components in a technical system, as there is no one material available which can be used universally.

The following criteria play a decisive role in material selection:

- safety (strength characteristics)
- specific gravity (weight expenditure)
- price
- availability
- workability (castable, hot or cold forming properties, machinable, weldable)
- hardness and hardenability
- elongation and toughness
- damping capability
- corrosion resistance
- surface condition and treatment
- behaviour at high and low temperatures
- resistance to ageing

The optimum material constitutes a compromise between the requirements and possibilities of the various areas:

- Development and Design (use characteristics)
- Production (production characteristics)
- Material and Business Administration (costs)
- Planning and Scheduling (dates)

Product development

Introduction and principles

Developing and manufacturing products that meet customer requirements and prove successful on the market are among the key tasks of manufacturing companies. Today, product developers are faced with a variety of challenges, such as increasing product complexity, an aging society and increasing demands on economic and ecological sustainability, which not only require them to possess an array of individual skills but also to apply extensive specialist knowledge in their work. In addition, the progressive shortening of product life times, the eclectic and complex relationships between the features and properties of mechatronic and cybertronic products, and the large number of players involved require a targeted, orchestrated and systematic approach to product development.

Design phases

According to **Pahl** and **Beitz**, product development is typically divided into the following four design phases:

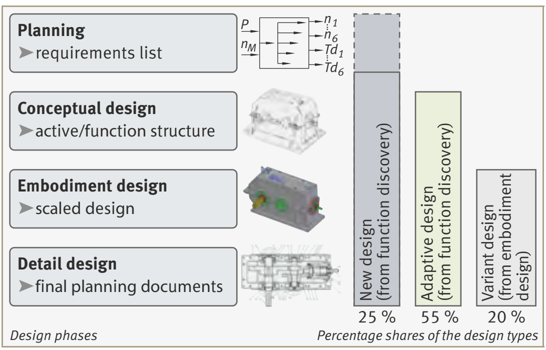
- planning
- conceptual design
- embodiment design
- detail design

A distinction can be made between various design types based on whether all or just some of these phases are completed, examples of which include:

- new design
- adaptive design
- variant design

Figure 1
The four design phases according to Pahl and Beitz and various design types

Source:
FAU Erlangen-Nuremberg, Institute of Engineering Design (KTmfk)



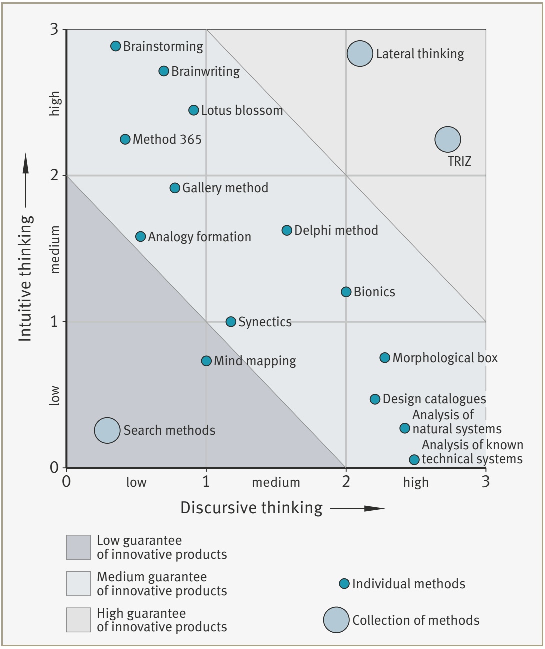
Development and problem-solving methods

The development of technical products requires product developers to overcome and manage various problems and tasks. The distinction typically made between design tasks and problems in this context is that, with design tasks, both the objectives and the means required to achieve them are clear and sufficiently known, whereas with design problems, at least one of these two aspects is unclear or unknown.

To systematically solve these design tasks and problems, product developers have various problem-solving and product discovery methods at their disposal. These can be divided into intuitive and discursive methods and are often grouped together as innovation methods. Patent research, benchmarking and reverse engineering can also serve as inspiration in the development process.

The following representation shows a classification of various product discovery methodologies according to **Gausemeier et. al.** (Produktinnovation, 2001).

Figure 2
Classification of product discovery methodologies according to Gausemeier



Evaluation of technical products

As technical products often have to meet contradictory target criteria and several possible solutions are usually obtained during product development, systematic evaluation methods are required. These allow the product developers to identify the best solutions. Examples of suitable methods include the value analysis in accordance with DIN EN 12973:2020 and/or VDI 2808 and the technical/economic evaluation in accordance with VDI 2225 Sheet 3:1998. These methods enable a systematic comparison of different technical products or product alternatives.
The SWOT analysis also serves to identify strengths, weaknesses, opportunities and threats.

The requirements for these evaluation methods include:

- finally coordinated requirements lists
- clearly defined criteria
- adequate information base
- knowledge of all necessary details of the products or product variants to be evaluated
- implementation of the evaluation in interdisciplinary teams

Design methodology process models

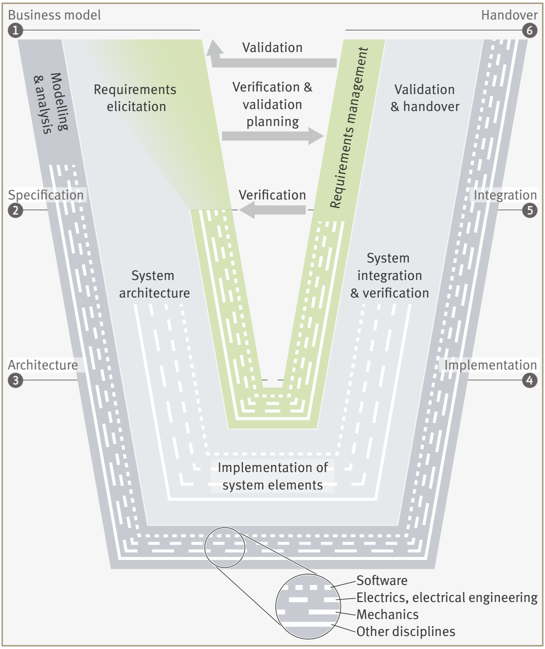
Global competition, with simultaneous development activities spanning multiple personnel and areas, combined with rising levels of product and process complexity increasingly require the planning, design, monitoring and control of development processes. Design methodology process models provide fundamental approaches for solving specific tasks and objectives, thus enabling the structuring, planning and control of development processes. Although an ideal process model for all development processes is not achievable due to the diverse nature of the various factors influencing the development process, design methodology process models can play a significant role in supporting product developers.

In addition to the comparatively generic structuring of the product development process according to **Pahl** and **Beitz** into the planning, conceptual design, embodiment design and detail design phases, the process models according to VDI 2221 and VDI 2206 (“V model”) are also often used.

The following representation shows the V model according to VDI/VDE 2206:2021.

Figure 3
V model

Source:
VDI/VDE Guideline 2206:2021



Rationalisation in product development

The variety of products on offer is increasing in many industries due to customised requirements, which is leading to increased internal complexity. Series, modular systems, platforms and modules are available to reduce this complexity.

Series A series (in this context) is characterised as a technical construct that fulfils the same function with the same solution in several size levels, ideally under identical production conditions, in a wide range of applications. Series are typically derived from a basic design using similarity laws and geometric standard numbers (see table Basic series, Page 359).

Modular system A modular system describes machine parts or assemblies which, as building blocks with often different solutions, fulfil various overall functions when used in combination, and are connected as building blocks to provide detachable or non-detachable solutions. These building blocks are usually divided into basic, auxiliary, special and customisation building blocks.

Platform The platform strategy pursues the goal of a version-neutral product platform with product-specific or variant-specific add-ons. This enables the use of identical parts and structures for different products without making the relationship between them visible to the outside world.

Module The module strategy can be seen as a further development of the platform strategy, as the same modules are again used in different products in order to customise what is perceptible to the customer and standardise what is not perceptible.

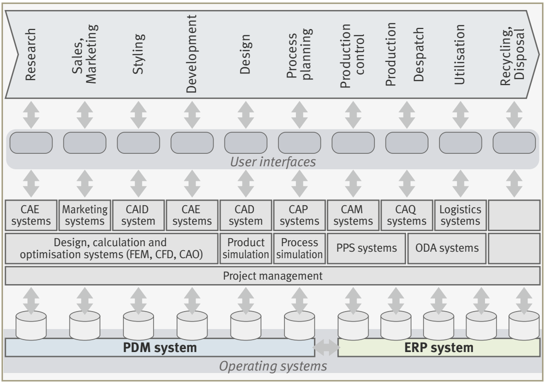
Virtual product development and use of CAx

The efficient implementation of product development processes is now heavily reliant on the use of modern information and communication technology. The collective name for all computer support systems used in a company is “CAx system”. CAx systems consist of hardware (e.g. computers and associated peripheral equipment) and software (e.g. the operating system and application software).

The application tools most commonly encountered in the context of product development and design include computer aided design (CAD) and various design, calculation and optimisation tools, which are grouped together under the term computer aided engineering (CAE). CAE includes calculation tools used in the finite element method (FED) and computational fluid dynamics (CFD) as well as mathematical optimisation methods (computer aided optimisation, CAO).
Modern product development processes also call for computer aided manufacturing (CAM) or computer aided quality (CAQ) tools.

According to **Vanja et al.** (CAx für Ingenieure, 2018) the current CAx landscape can be represented as follows:

Figure 4
Current CAx landscape according to Vanja et al.

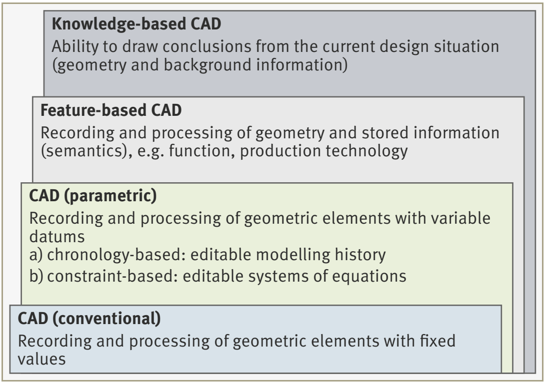


CAD modelling

A particularly important technology in this context is 3D product modelling using CAD tools, the principles and systematic application of which are set out in VDI 2209, for example. This technology is typically used at the beginning of the design phase in product development. In the use of CAD tools, an additional distinction is made between conventional, parametric, feature-based or knowledge-based CAD modelling.

Figure 5
3D modelling principles – overview

Source: VDI Guideline 2209



Data exchange and management

In practice, the exchange of data between various CAD and CAE tools can take place either via direct or neutral interfaces. While direct interfaces provide a direct link between the various software tools, neutral interfaces are based on neutral data formats, such as the STEP format in accordance with ISO 10303 (Standard for the Exchange of Product Model Data) or the JT format in accordance with ISO 14306 (Jupiter Tesselation).

Nowadays, the holistic and computer-aided design and management of the product life and exchange of data across the life cycle under the terms of product lifecycle management is largely carried out by interlinking product data management systems (PDM), enterprise resource planning systems (ERP) and manufacturing execution systems (MES).

Knowledge-based engineering and knowledge processing

As previously described, the development of technical products requires extensive design knowledge which not only encompasses theoretical specialist knowledge (knowledge of facts and methods) and heuristic empirical knowledge but also explicit and implicit knowledge.

Knowledge-based engineering (KBE)

In this context, knowledge-based engineering (KBE) and knowledge-based systems help to preserve existing knowledge and improve the exchange of knowledge within a company. These knowledge-based systems are intelligent information systems, which map and render knowledge usable using knowledge representation and knowledge modelling methods. The essential steps and elements involved in the development and operation of KBE applications are described, for example, in VDI 5610 Sheet 2. These applications use inference and various search and problem-solving strategies in order to process knowledge.

The following table shows sample solutions for KBE solutions in accordance with VDI 5610-2.

	Required employees	Required skills and knowledge	Required licences and software	Required expenditure	Potential users
CAD systems and integrated program functions					
Application to derive gearbox concept designs	A designer	Basic programming experience	CAD system, SQL database	Minimal maintenance and testing outlay	Students, training participants
Automated design of window lift kinematics using intelligent features	Application developers, component specialists, designers	Internal CAD programming language	CAD system and KWA licence	Moderate maintenance, training and testing outlay	Global designers
Development of a KBE application using CAD programming interface	Product developers, designers, production planners, knowledge engineers or programmers	Good programming knowledge, interview techniques	ERP/PLM and CAD system, UDFs and programming interface as on-board tools	Gathering of expert knowledge, moderate maintenance outlay	Designers and internal sales force (configuration)
KBE modules and systems					
Design of a cutter head based on customer-specific diagrams	Designers, sales employees, knowledge engineers or programmers, external consultants	Good knowledge of the programming language, particularly of the commercial KBE module	Commercial KBE module of a CAD system, possibly licence to design the user interface	Moderate maintenance, training and testing outlay	Designers and sales employees
Training example for learning basic KBE functions in CAD	Designers, knowledge engineers or programmers, external consultants where appropriate	Good programming knowledge	KBE module with on-board CAD system tools, UDFs, design table, spreadsheet software	Moderate maintenance, training and testing outlay	Designers
Product configurators and design automation					
Knowledge-based configuration of elevator systems	Designers, knowledge engineers or programmers, product managers	Good programming knowledge (e.g. for XML interface)	ERP/PLM system, CAD system with integrated constraint-based configurator	Moderate maintenance, training and testing outlay	Designers and sales employees
Web portal with configurator for stairs, platforms and crossovers	Designers, production planning, production, knowledge engineers or programmers, marketing	Good programming knowledge, knowledge of rules and relationships, knowledge of business processes	CAD, PDM, CRM and ERP system, integrated 3D viewing system, commercial authoring tool	Minimal maintenance, training and testing outlay	Sales employees, customers, technical departments

Knowledge discovery in databases and data mining

In addition to the introduction and application of human knowledge, product development activities increasingly require product developers to view, evaluate and interpret large quantities of data. The methods used to generate such data include simulations or other validation measures. The area of knowledge discovery in databases (KDD) provides methods and processes that use computer-based support to extract knowledge from data and prepare it for product developers.

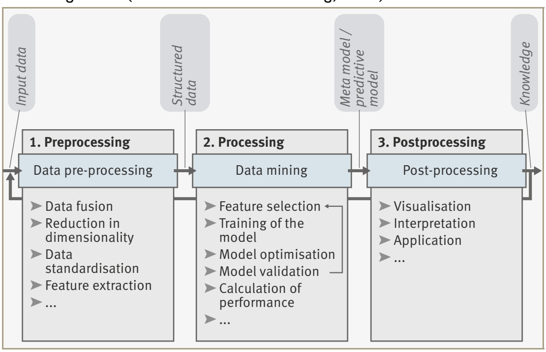
The three main steps involved in knowledge discovery in databases are:

- data pre-processing
- data mining using machine learning methods
- data post-processing

The KDD process according to **Fayyad** and the Cross-Industry Standard Process for Data Mining (CRISP-DM) also propose more detailed approaches to knowledge discovery in databases.

The following representation shows the main steps of a KDD process according to **Tan** (Introduction to Data Mining, 2006):

Figure 6
Main steps of a KDD process according to Tan

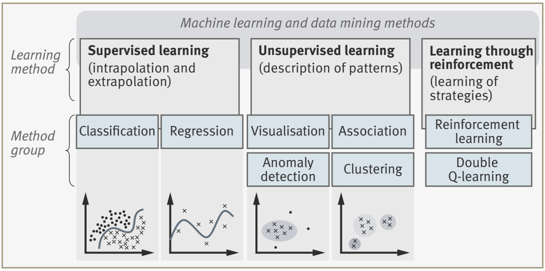


The actual data mining itself is carried out using various machine learning methods, which can be divided into the following three areas:

- supervised learning
- unsupervised learning
- reinforced learning

The choice of machine learning method is essentially determined by the specific goal and task set and by the available data.

Figure 7
Machine learning and data mining – structuring of methods



Supervised learning methods are often used in the context of product development to predict target values based on known input values. This requires the data set under analysis to contain a sufficiently large number of both the input values and the resulting target value. Overfitting and underfitting of the machine learning model to the available data during the process known as training should be avoided.

The predictive quality of the model can also be determined and evaluated using various validation parameters, examples of which include:

- coefficient of determination
- accuracy
- precision
- recall
- root mean square error

Technical drawing guideline

Representation of technical objects

Representation options

There are various possibilities for the clear representation of technical objects, from a freehand drawing through a technical drawing to a digital data set in a 3-D CAD system.

Technical product documentation (TPD)

In the past, technical drawing encompassed the technical drawings used in various engineering disciplines such as civil engineering, ship building, mechanical engineering and electrical engineering. In particular, the increasing importance of 3-D dimensioning, as applied in computer aided design (CAD), has resulted in Technical Product Documentation (TPD) being increasingly influenced by the Geometrical Product Specification (ISO GPS). The specification is defined by the function of the component and is intended to be unambiguous and consistent with the verification process and the manufactured component, Thus enabling the specification to provide usable data in production and quality assurance along the entire value chain. For further information, see the chapter Tolerances and fits.

The technical drawing principles for two-dimensional specification in TPD are expanded for three-dimensional specification and model-based definition (MBD) to include the practices necessary for digital product definition data, as defined in ISO 16792.

Representation of technical objects

The following descriptions are focussed exclusively on the area of technical drawing. The technical drawing as an information carrier serves in the case of a production order as a binding means of communication and legal document between Design and Production and between supplier and customer. In addition to the pictorial representation of the object and its dimensioning, it also contains technological and organisational information (cf. ISO 10209:2012).

Projection methods

Orthogonal projection

Orthogonal projection is normally used in technical drawings. The body is depicted and dimensioned in several two-dimensional views (2-D views). These are always perpendicular parallel projections.

As a result of different projection methods, there are different arrangements of views on the drawing. The standards ISO 128-3:2022 and ISO 5456-2:1996 give more detailed explanations of the basic rules governing views and orthogonal projection in technical drawings.

Descriptions are given here of three projection methods:

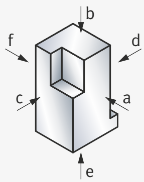
- arrow method
- first angle projection
- third angle projection

Basic rules

The basic rules of orthogonal projection are as follows:

- The most informative view is always selected as the front view or main view.
- The body is to be depicted in the main view as far as possible in its operating position, manufacturing position or mounting position.
- The number of additional views and sections should be restricted to what is necessary.

The following table shows the standardised descriptions of the views:

Views of body	Direction of observation		Description of views	
	View from	View in direction	Letter	Name
	front	a	A	Front view, main view
	above	b	B	Top view
	left	c	C	Lateral view from left
	right	d	D	Lateral view from right
	below	e	E	Bottom view
	rear	f	F	Rear view

Arrow method

In the arrow method according to ISO 5456-2:1996, the views on the technical drawing in relation to the main view can be in any arrangement but are individually marked.

Rules for the arrow method:

- Arrows with any letters, predominantly in the main view, indicate the direction of observation of the other views.
- Each view except the main view must be marked at the top by means of the corresponding upper case letter.
- In the drawing, no graphical symbol is necessary for this method (in contrast to first and third angle projection).

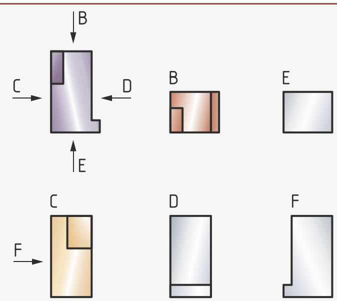


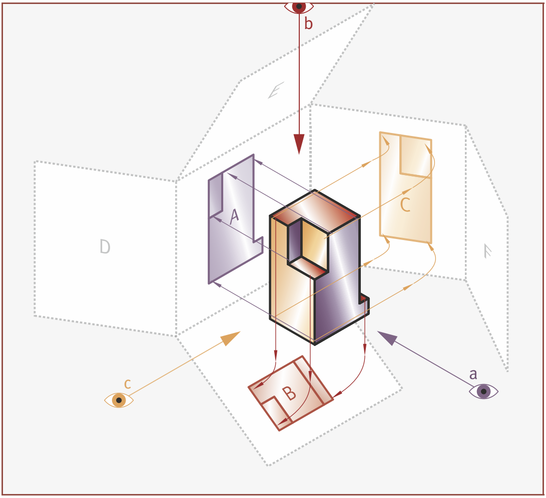
Figure 1
Arrow method, arrangement of views

First angle projection method

The first angle projection method is used in preference in the German-speaking world and substantial parts of Europe and is applied in ISO GPS.

The body is positioned between the observer and the surface that represents the plane of projection. Each view is projected in a perpendicular manner through the body onto the plane at the rear. The planes of projection of all 6 views form a cube, the inner faces of which bear the projected views.

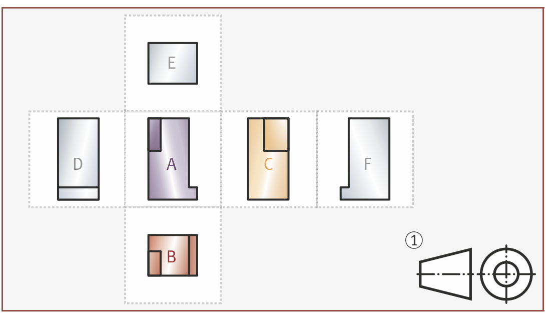
Figure 2
Folding rule in first angle projection method



The technical drawing corresponds to the internal surface of this cube arrangement, which means that the views can be placed on the drawing area in this arrangement without any further description. The title block on the drawing contains a symbol indicating the first angle projection method.

Figure 3
First angle projection method, arrangement of views

① Graphical symbol (doubled in size)

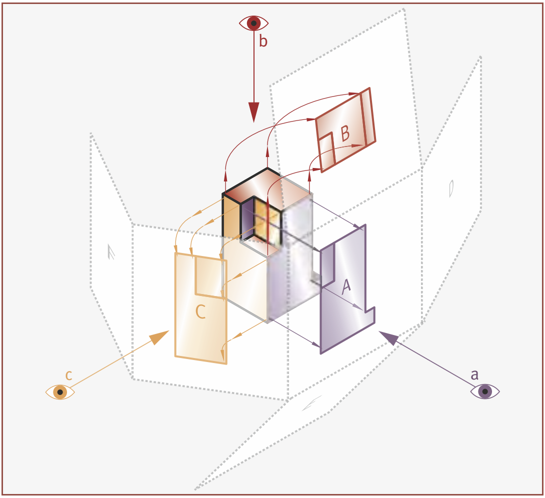


Third angle projection method

The third angle projection method is used in the ASME (The American Society of Mechanical Engineers).

The body is positioned, from the observer's perspective, behind the plane of projection. Each of the 6 views is projected perpendicular to the plane in front, which is closer to the observer. All the planes of projection form a cube on which the projected views are shown.

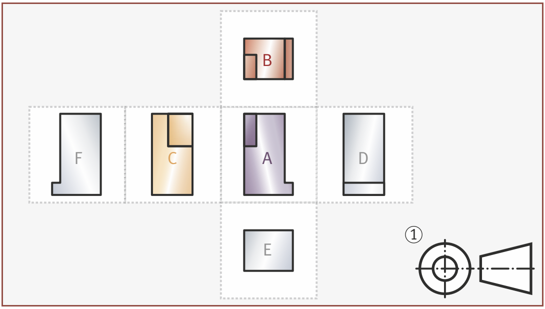
Figure 4
Folding rule in third angle projection method



The technical drawing corresponds to the external surface of the cube arrangement. The views are arranged on the drawing area in this way without any further description. The title block on the drawing contains a symbol indicating the third angle projection method.

Figure 5
Third angle projection method, arrangement of views

① Graphical symbol (doubled in size)



Pictorial representation

The pictorial representation of an object gives a three-dimensional view (3-D view). It conveys the impression of a three-dimensional object, is easier to understand than 2-D views and is therefore used in a supporting capacity in technical drawings.

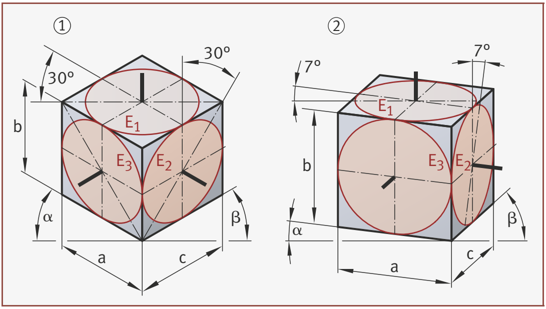
Axonometric representations

The standard ISO 5456-3:1996 shows axonometries for technical drawings. Axonometries are parallel projections that create a three-dimensional view of a three-dimensional object. Lines that run parallel in the 3-D object remain parallel in axonometric representations.

Two common methods of axonometric representation, isometric projection and dimetric projection, are explained here.

Figure 6
Axonometric representations

① Isometric projection
② Dimetric projection



Isometric projection

In **isometric projection**, see Figure 6 Part ①, the following apply:

- representation of a cube and the circles in three views
- ratio of sides a:b:c = 1:1:1
- $\alpha = \beta = 30°$
- ellipse E_1: large axis horizontal
- ellipses E_2 and E_3: large axes at 30° to perpendicular
- ratio of the axes of the three ellipses 1:1,7

Dimetric projection

In **dimetric projection**, see Figure 6 Part ②, the following apply:

- representation of a cube and the circles in three views
- ratio of sides a:b:c = 1:1:0,5
- $\alpha = 7°$ and $\beta = 42°$
- ellipse E_1: large axis horizontal
- ellipses E_1 and E_2: large axes at 7° to perpendicular
- ellipses E_1 and E_2 have ratio of axes of 1:3.
- ellipse E_3 becomes a circle for the purposes of simplicity.

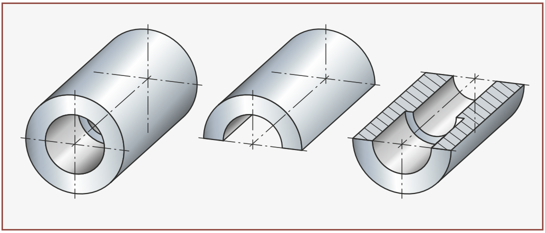
Indication of cuts and sections

Representation in accordance with DIN

Cuts and sections allow insights into the interior of components or hollow bodies such as housings, workpieces with holes or openings. A virtual cut through the relevant body provides clarity. Its representation is implemented in accordance with ISO 128-3:2022.

Areas of cuts and sections are indicated by hatching: parallel narrow continuous lines at an angle of 45°.

Figure 7
Areas of cuts and sections



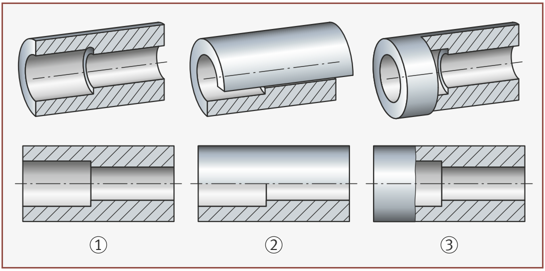
Types of sections

The types of sections can be classified as follows, see Figure 8:

- Complete cut or section:
  Complete cut or section of the relevant component.
- Cut/section of symmetrical parts:
  Showing both a cut or section and a view.
- Local cut or section:
  Internal contours exposed only in certain selected areas.
  – Break: A local cut or section is limited to the area to be indicated. Hatching is limited by a continuous narrow line with zigzags or freehand curve, see Figure 9, Page 354.
  – Cut: representation of a detail and associated with a magnification. The hatching lines end at a straight virtual edge, see Figure 9, Page 354.

Figure 8
Types of sections

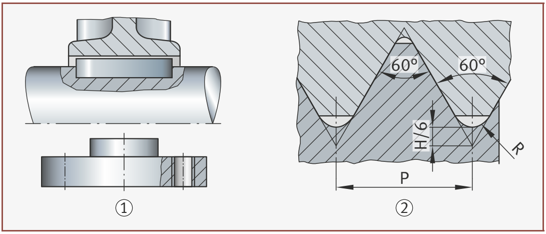
① Complete cut or section
② Cut/section of symmetrical parts
③ Local cut or section



A local cut/section and cut can be represented as follows:

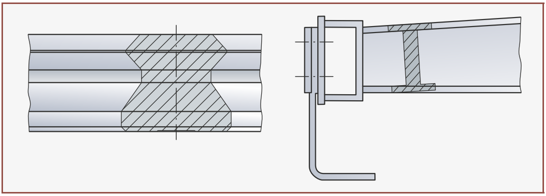
Figure 9
Local cut/section and cut

① Local cut/section
② Cut



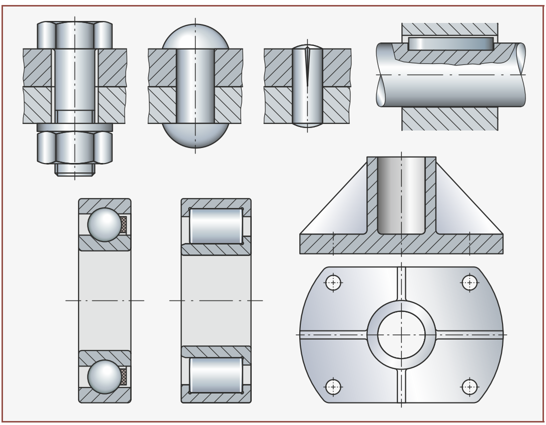
If unambiguity is given, a section can be revolved in a simplified form in the relevant view, see Figure 10. In this case, the outline of the section shall be drawn with continuous narrow lines. Further identification is not necessary.

Figure 10
Revolved section in relevant view



Solid workpieces are not drawn in longitudinal section, see Figure 11. These include, for example, shafts, studs, rivets, pins, screws, nuts, washers, feather keys, splines, rolling bearing bodies as well as ribs on castings and knobs on handwheels.

Figure 11
Longitudinal sections



Identification of the cutting plane

In the case of more complex components, the cutting line can be represented more specifically by means of:

- representation of cutting planes
- several cutting planes in order to represent all features
- identification within one cutting plane by means of capital letters
- arrangement of sections or cuts on the projection axis, possibly below the projection axis, see Figure 12
- representation of the cutting line, especially where the cutting plane ends or changes direction, see Figure 13
- change of cutting plane direction, when applied in several cutting planes
- change of cutting plane direction away from the 90° angle, see Figure 14, Page 356
- outlines and edges behind the cutting plane can be omitted if they do not contribute to the clarification of the drawing.

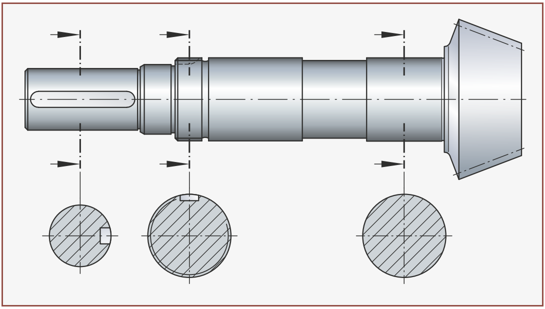


Figure 12
Arrangement of section or cut below the projection axis

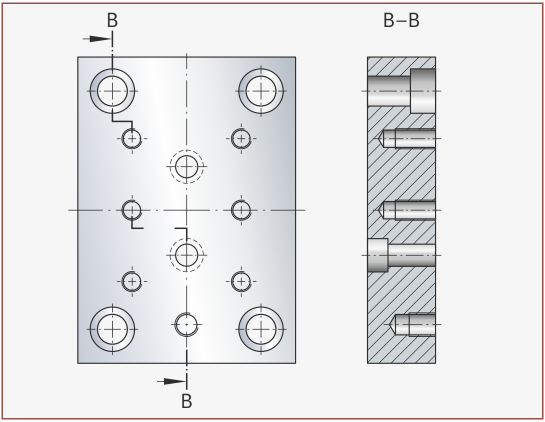
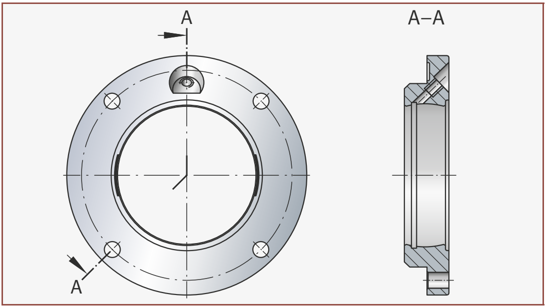


Figure 13
Change of cutting plane direction

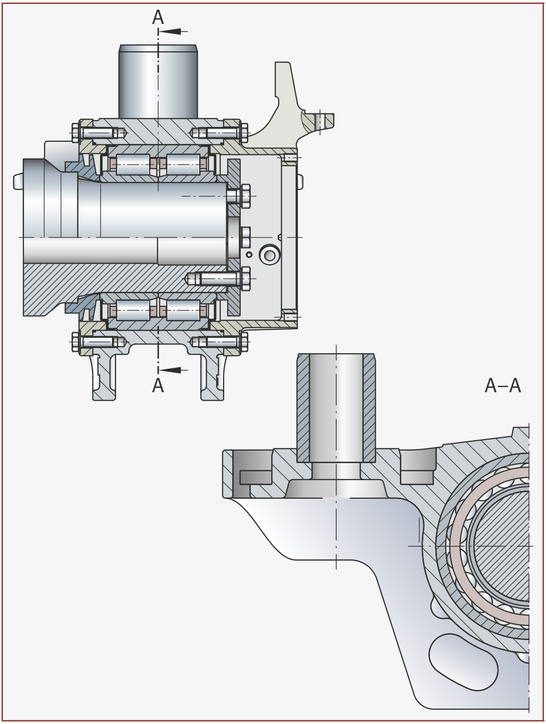
Figure 14
Change of cutting plane direction away from the 90° angle



Example of a section and cut representation

The following example shows a more complex cross-section.

Figure 15
Example of a section and cut representation



Basic elements in drawings

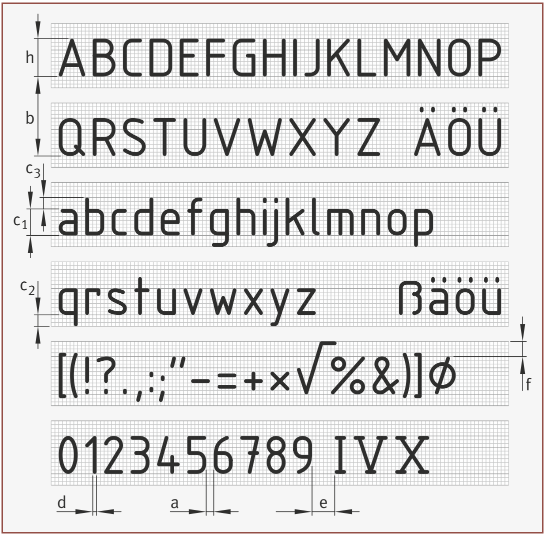
Standard font

In technical drawings, standard fonts are used for labelling. The standard ISO 3098:2015 defines the fonts A, B, CA and CB (CA and CB only for CAD applications) in either a vertical (V) or inclined (S) form as appropriate.

The Roman alphabet (L), which also contains numbers and characters, the Greek alphabet (G) and the Cyrillic alphabet (K) are defined.

The following diagram shows the preferred font for manual labelling. In the case of the font size of 5 mm shown here, this font is designated as follows: font ISO 3098 – BVL – 5.

Figure 16
Font B, vertical (V), Roman alphabet (L), with German characters



Font B For font B (d = h/10) in the standard font, the dimensions are as follows:

Labelling feature		Ratio	Dimension mm						
Font size, height of upper case letters	h	(10/10) h	2,5	3,5	5	7	10	14	20
Height of lower case letters (x height)	c_1	(7/10) h	1,75	2,5[1]	3,5	5[1]	7	10[1]	14
Descenders of lower case letters	c_2	(3/10) h	0,75	1,05	1,5	2,1	3	4,2	6
Ascenders of lower case letters	c_3								
Range of diacritical characters	f	(4/10) h	1	1,4	2	2,8	4	5,6	8
Line thickness	d	(1/10) h	0,25	0,35	0,5	0,7	1	1,4	2
Minimum spacing between:									
Characters	a	(2/10) h	0,5	0,7	1	1,4	2	2,8	4
Base lines 1[2]	b_1	(19/10) h	4,75	6,65	9,5	13,3	19	26,6	38
Base lines 2[3]	b_2	(15/10) h	3,75	5,25	7,5	10,5	15	21	30
Base lines 3[4]	b_3	(13/10) h	3,25	4,55	6,5	9,1	13	18,2	26
Words	e	(6/10) h	1,5	2,1	3	4,2	6	8,4	12

1) Rounded off value.
2) In the use of upper case and lower case letters with diacritical characters (Ä, Ö, Ü, ...).
3) In the use of upper case and lower case letters without diacritical characters.
4) Only upper case letters or numerals are used, diacritical characters are not used.

Preferred numbers and preferred number series

In accordance with the standard DIN 323:1974, the structure of the preferred number series is based on geometric number sequences with 5, 10, 20 or 40 elements in each group of ten.

Starting from the basic series

Equation 1

$$a \cdot q^0,\ a \cdot q^1,\ a \cdot q^2,\ \ldots,\ a \cdot q^{n-1}$$

the individual preferred number series are determined as having the increment steps q:

Equation 2

$$q = \sqrt[5]{10} = 1{,}6 \quad \text{for R 5} \qquad q = \sqrt[20]{10} = 1{,}12 \quad \text{for R 20}$$

$$q = \sqrt[10]{10} = 1{,}25 \quad \text{for R 10} \qquad q = \sqrt[40]{10} = 1{,}06 \quad \text{for R 40}$$

Basic series The following table gives the basic series (main values and precise values).

Main value Basic series				Sequence number	Mantissa	Precise value	Deviation of main value from precise value
R 5	R 10	R 20	R 40	N			%
1,00	1,00	1,00	1,00	0	000	1,0000	0
			1,06	1	025	1,0593	+0,07
		1,12	1,12	2	050	1,1220	−0,18
			1,18	3	075	1,1885	−0,71
	1,25	1,25	1,25	4	100	1,2589	−0,71
			1,32	5	125	1,3353	−1,01
		1,40	1,40	6	150	1,4125	−0,88
			1,50	7	175	1,4962	+0,25
1,60	1,60	1,60	1,60	8	200	1,5849	+0,95
			1,70	9	225	1,6788	+1,26
		1,80	1,80	10	250	1,7783	+1,22
			1,90	11	275	1,8836	+0,87
	2,00	2,00	2,00	12	300	1,9953	+0,24
			2,12	13	325	2,1135	+0,31
		2,24	2,24	14	350	2,2387	+0,06
			2,36	15	375	2,3714	−0,48
2,50	2,50	2,50	2,50	16	400	2,5119	−0,47
			2,65	17	425	2,6607	−0,40
		2,80	2,80	18	450	2,8184	−0,65
			3,00	19	475	2,9854	+0,49
	3,15	3,15	3,15	20	500	3,1623	−0,39
			3,35	21	525	3,3497	+0,01
		3,55	3,55	22	550	3,5481	+0,05
			3,75	23	575	3,7584	−0,22

Continuation of table, see Page 360.

The method of writing preferred numbers without trailing zeroes is also used internationally.

Continuation of table, Basic series, from Page 359.

Main value Basic series				Sequence number	Mantissa	Precise value	Deviation of main value from precise value
R 5	R 10	R 20	R 40	N			%
4,00	4,00	4,00	4,00	24	600	3,9811	+0,47
			4,25	25	625	4,2170	+0,78
		4,50	4,50	26	650	4,4668	+0,74
			4,75	27	675	4,7315	+0,39
	5,00	5,00	5,00	28	700	5,0119	−0,24
			5,30	29	725	5,3088	−0,17
		5,60	5,60	30	750	5,6234	−0,42
			6,00	31	775	5,9566	+0,73
6,30	6,30	6,30	6,30	32	800	6,3096	−0,15
			6,70	33	825	6,6834	+0,25
		7,10	7,10	34	850	7,0795	+0,29
			7,50	35	875	7,4989	+0,01
	8,00	8,00	8,00	36	900	7,9433	+0,71
			8,50	37	925	8,4140	+1,02
		9,00	9,00	38	950	8,9125	+0,98
			9,50	39	975	9,4406	+0,63
10,00	10,00	10,00	10,00	40	000	10,0000	0

The method of writing preferred numbers without trailing zeroes is also used internationally.

Line types and line groups The standard ISO 128-2:2022 defines line types and line groups.

Line types The following table shows some line types from the standard.

Name[1]	Line type	Application in accordance with ISO 128-2:2022
01.2	Continuous wide line	■ Visible edges ■ Visible outlines ■ Crests of screw threads ■ Limit of length of usable full thread depth ■ Main representations in diagrams, charts, flow charts ■ System lines (steel construction) ■ Parting lines of moulds in views ■ Lines of cuts and section arrows
01.1 (subtypes 01.1.1 to 01.1.18)	Continuous narrow line	■ Imaginary lines of intersection ■ Dimension lines ■ Extension lines ■ Leader lines and reference lines ■ Hatching ■ Outlines of revolved sections ■ Short centre lines ■ Root of screw threads ■ Bending lines ■ Origin and terminations of dimension lines ■ Diagonals for the indication of flat surfaces ■ Bending lines on blanks and machined parts ■ Framing of details ■ Indication of repetitive details, e.g. root circles of gears ■ Dimensioning and tolerance lines of tapered features ■ Location of laminations ■ Projection lines ■ Grid lines ■ Axis of the coordinate system
Subtype 01.1.19	Continuous narrow freehand line	■ Preferably manually represented termination of partial or interrupted views, cuts and sections, if the limit is not a line of symmetry or a centre line[2].
Subtype 01.1.20	Continuous narrow line with zigzags	■ Termination represented in CAD of partial or interrupted views, cuts and sections, if the limit is not a line of symmetry or a centre line[2].
02.2	Dashed wide line	■ Possible indication of permissible areas of surface treatment or coating
02.1	Dashed narrow line	■ Hidden edges ■ Hidden outlines
04.2	Long-dashed dotted wide line (long dash)	■ Indication of cutting planes ■ Indication of a restricted area (examples: for a toleranced feature, for heat treatment, for coating)

Continuation of table, see Page 362.

[1] The first part of the number of the lines indicates the basic type of lines in accordance with ISO 128-2.
[2] It is recommended to use only one type of line on one drawing.

Continuation of table, Line types, from Page 361.

Designation[1)]	Line type	Application in accordance with ISO 128-2:2022	
04.1	Long-dashed dotted narrow line (long dash)	■ Centre lines ■ Lines of symmetry, planes of symmetry ■ Pitch circles of gears ■ Pitch circle of holes	■ Indication of expected or wished spread of surface-hardened area (example: heat treatment)
05.1	Long-dashed double-dotted narrow line (long dash)	■ Outlines (adjacent parts, alternative designs, finished parts into blank parts) ■ Initial outlines prior to forming ■ Extreme positions of movable parts ■ Centroidal lines ■ Parts situated in front of a cross-sectional plane ■ Outlines of alternative executions	■ Outlines of the finished part within blanks ■ Framing of particular fields/areas ■ Projected toleranced feature ■ Optical axes ■ Indication of structural outlines used in mechanical processes ■ Outlines of datum target areas
07.2	Dotted wide line	■ Indication of areas where heat treatment is not permissible	
09.2	Long-dashed double-dotted wide line	■ Situation elements (as non-median features)	

1) The first part of the number of the lines indicates the basic type of lines in accordance with ISO 128-2.

Line thicknesses and line groups

The following table lists some line groups and the associated line thicknesses.

Line group	Associated line thickness (nominal dimensions in mm) for		
	Line type		Dimension and text notation; graphical symbol ISO 21920-1:2021
	01.2, 02.2, 04.2, 07.2, 09.2	01.1, 02.1, 04.1, 05.1	
0,25	0,25	0,13	0,18
0,35	0,35	0,18	0,25
0,5[1)]	0,5	0,25	0,35
0,7[1)]	0,7	0,35	0,5
1	1	0,5	0,7
1,4	1,4	0,7	1
2	2	1	1,4

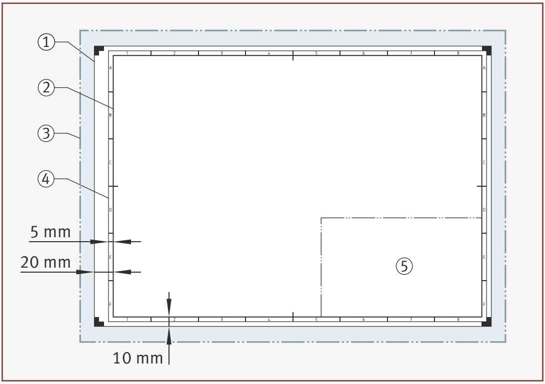
1) These line groups should be used in preference.

Sheet sizes

The standard ISO 5457:1999 + Amd. 1:2010 defines sheet sizes. For details about the title block, see ISO 7200:2004.

Figure 17
Sheet sizes based on the example of DIN A3

1 Trimmed drawing
2 Drawing area
3 Untrimmed sheet
4 Field division
5 Title block



The preferred formats are defined in accordance with ISO 216 in ISO main series A. All formats are used in landscape format, A4 is also used in portrait format.

Format of ISO-A series in accordance with ISO 5457:1999 + Amd. 1:2010				Usable favourable roller width			Base sheet[3]
Sheet size, format	Trimmed drawing, trimmed sheet (T)[1] mm	Drawing area mm	Untrimmed sheet (U)[2] mm	mm			mm
A0	841×1189	821×1159	880×1230	–	900	–	–
A1	594×841	574×811	625×880	–	900	660	660×900
A2	420×594	400×564	450×625	(2×450)	900	660	450×660
A3	297×420	277×390	330×450	(2×330) (2×450)	660	900	330×450
A4	210×297	180×277	240×330	250	660	–	225×330

[1] Finished sheet.
[2] Base sheet for single printing.
[3] From 660 mm×900 mm.

Scales

The standard ISO 5455:1979 defines notation for scales.

The title block contains the principal scale of the drawing and the other scales at different locations. The latter are presented in each case against the associated representations. As far as possible, all objects (except in standards) must be drawn true to scale.

If parts are depicted in an enlarged representation, it is advisable to add a representation on a 1:1 scale in order to show the natural size. In this case, it is not necessary to reproduce the details.

Enlargement scale	50:1 5:1	20:1 2:1	10:1
Natural scale	1:1		
Reduction scales	1:2 1:20 1:200 1:2 000	1:5 1:50 1:500 1:5 000	1:10 1:100 1:1 000 1:10 000

Dimensioning in accordance with standards

Dimension indications in accordance with standards

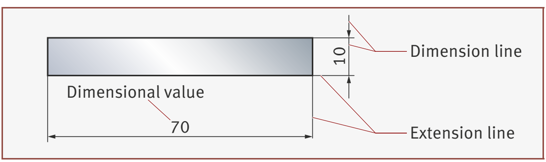
Dimension indications in technical drawings apply to the final condition of a part (raw, premachined or finished). Their representation is implemented in accordance with ISO 129-1:2018 + Amd 1:2020. Standards DIN 406-10 and DIN 406-11 have been withdrawn.

Elements of dimension indication

The following elements and specifications must be observed when indicating dimensions:

- Dimension lines (continuous narrow lines):
  - are at a distance of approx. 10 mm from the body outlines.
  - other parallel dimension lines have a spacing of at least 7 mm.
  - centre lines and edges must not be used as dimension lines.
- Extension lines (continuous narrow lines):
  - project approx. 2 mm beyond the dimension line.
- Dimension lines and extension lines should not intersect any other lines.
- Dimensional values:
  - are positioned at the approximate centre of the dimension line (although exceptions occur).
  - are positioned approx. 0,5 mm to 1,5 mm above the dimension line.

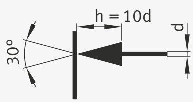
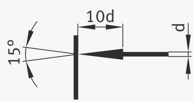
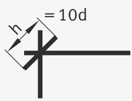
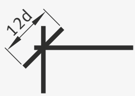
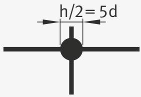
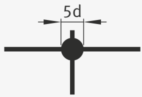
Figure 18
Elements of dimension indication



Termination of dimension lines

For the termination of dimension lines, the following applies:

- Within a drawing, only one of the possible terminations of dimension lines may be used, see table Terminations of dimension lines
- In general, a black arrow is used as the dimension arrow, while an open arrow is used in CAD drawings
- In subject-related drawings (for example in the case of construction), obliques can be used instead of dimension arrows
- If there is a shortage of space, solid dots can also be used in conjunction with dimension arrows (in the case of chain dimensions)
- Arrows are always used as terminations of dimension lines on arcs, radii and diameters
- In order to indicate an origin, an open circle is used.

Terminations of dimension lines	To ISO 129-1	To DIN 406-10 (obsolete, standard withdrawn)
Dimension arrows		
Obliques		
Solid dots		

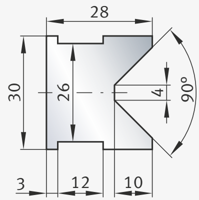
Methods of dimension indication

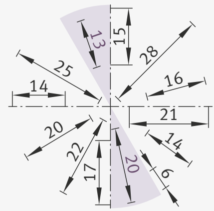
Dimensions can be indicated in accordance with the following methods:

- in two main reading directions (standard):
  Dimensional values should where possible be indicated such that in the reading position of the drawing, they can be read in the main reading directions from the bottom and from the right.
- on dimension reference surfaces:
  Suitable reference planes are selected for the dimensioning.

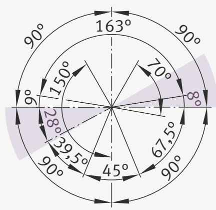
Dimension indication in two main reading directions

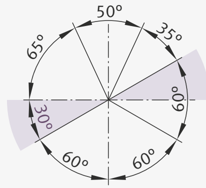
For length dimensions



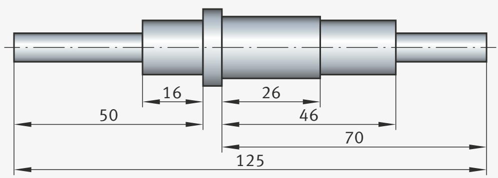


For angle dimensions



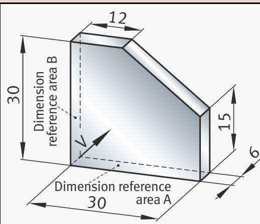


Example

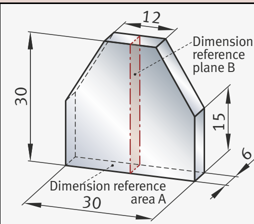


Dimension indication on dimension reference areas

As a function of the symmetry of components



For asymmetrical components



For symmetrical components

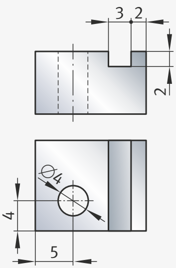
Arrangement of dimensions, dimensioning rules

Each dimension of a component shall only be indicated once within a drawing.

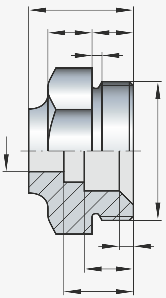
Indications of dimensions are arranged as follows:

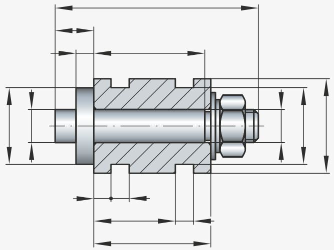
- according to views
- according to internal and external dimensions
- according to individual components
- for two-point sizes without dimension chains in order to avoid, for example, summation of individual tolerances, but possibly with an auxiliary dimension (in brackets)
- with dimension chain if the specifications are constructed using theoretical exact dimensions (TEDs), e.g. for distances or hole patterns.

Arrangement of dimensions

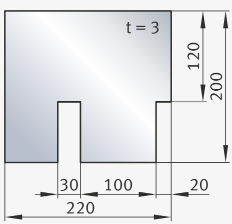


According to views

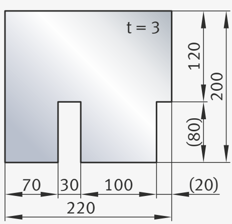

According to internal and external dimensions


According to individual components

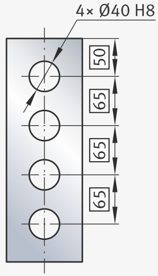
Dimension chains



Without dimension chains



Without dimension chains, with an auxiliary dimension (in brackets)

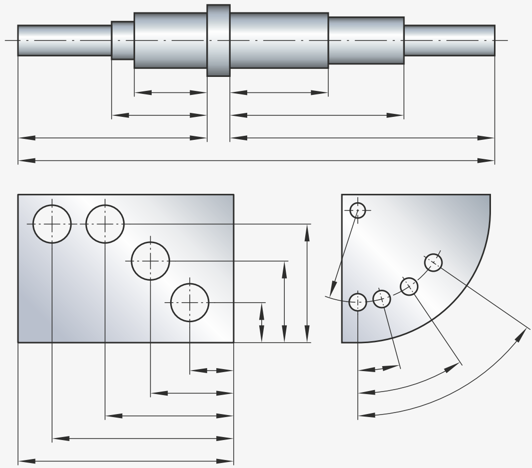


With dimension chain, when using TEDs

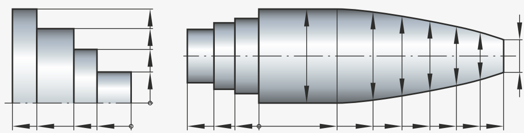
Types of dimension indication

Indication of dimensions can be carried out as follows:

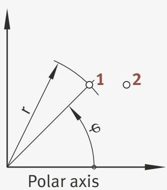
Parallel dimensioning



Running dimensioning

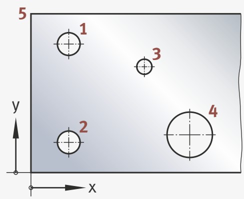


Coordinate dimensioning



Pos.	r	φ
1	15	45°
2	19	35°

With polar coordinates

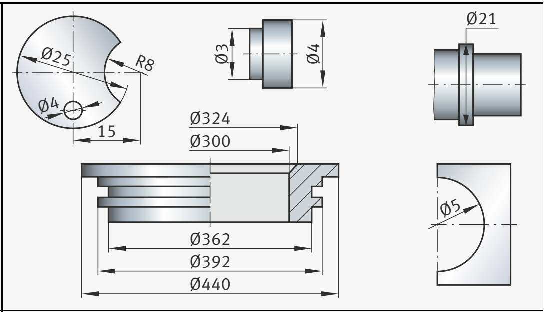
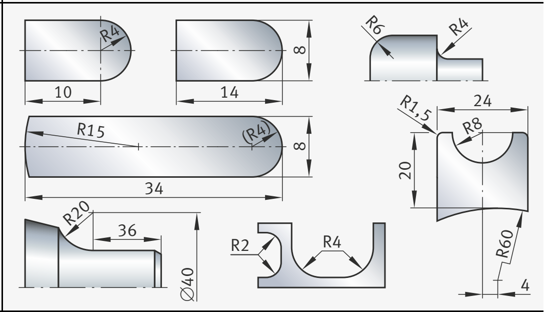
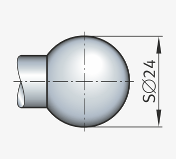
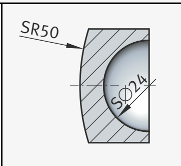
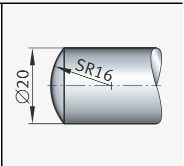
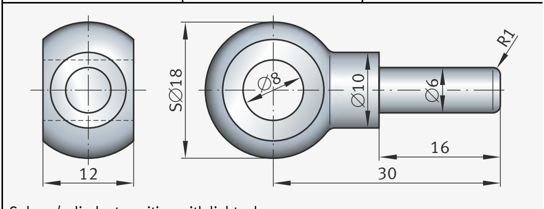


Pos.	x	y	d
1	5	17	∅3
2	5	4	∅3
3	15	14	∅2
4	21	5	∅6
5	0	21	–

With Cartesian coordinates

Dimensioning of features

Examples of the dimensioning of geometrical parts are given below:

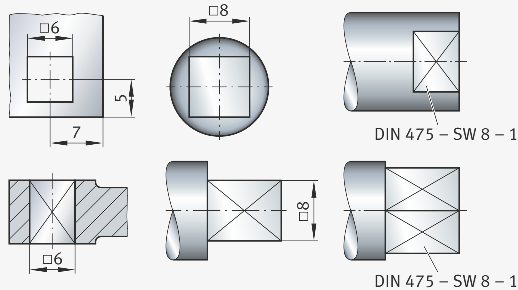
Diameter	
Radius	
Sphere	Sphere diameter Sphere radius Oval point Sphere/cylinder transition with light edge

Continuation of table, see Page 370.

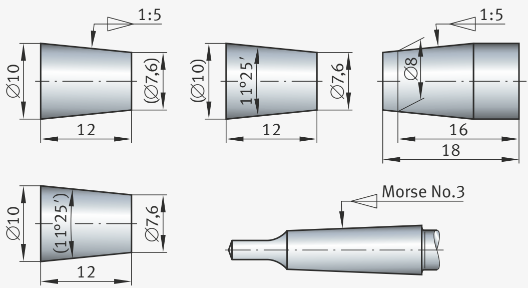
Continuation of table, Dimensioning of features, from Page 369.

Square form and width across flats

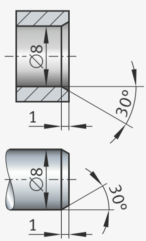
The width across flats is selected in accordance with DIN 475-1:2016



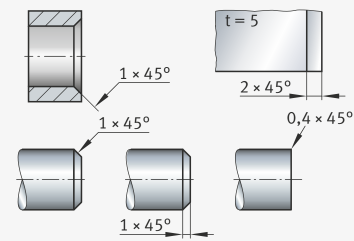
Taper (in accordance with ISO 3040)



Chamfer (obsolete: bevel)

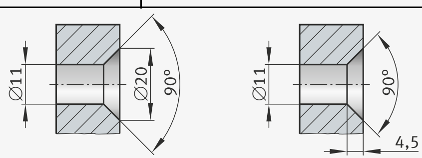


Chamfer other than 45°



Chamfer at 45°

Countersink



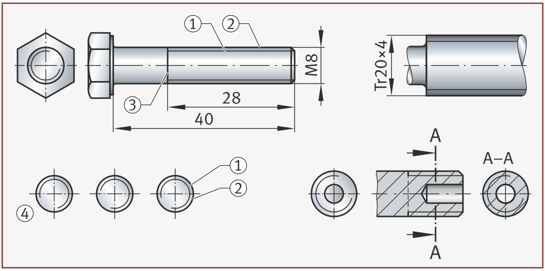
Dimensioning of threads

The representation of threads is defined in ISO 6410-1:1993, the simplified representation in ISO 6410-3:2021.

In the case of external threads, the thread root must be drawn as a continuous narrow line while the outside diameter and the thread limit must be drawn as a continuous wide line.

Figure 19
Bolt thread/ external thread

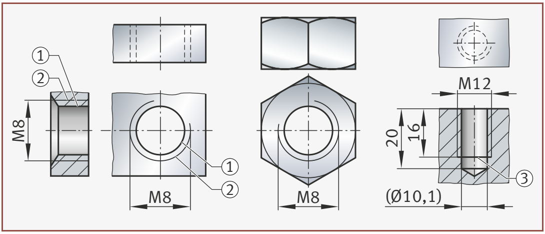
① Thread root
② Outside diameter
③ Thread limit
④ 3/4 circle, variable position and opening



In the case of internal threads, the outside diameter must be drawn as a continuous narrow line while the core diameter must be drawn as a continuous wide line.

Figure 20
Nut thread/ internal thread

① Core diameter
② Outside diameter
③ Thread limit



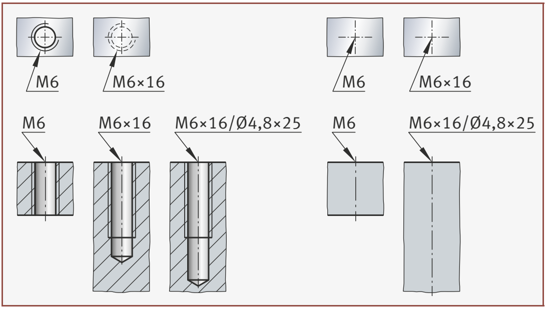
Short designations for threads are defined in DIN 202. These comprise:

- designation for thread type: M, R, Tr
- nominal diameter (thread size)
- lead or pitch as appropriate
- number of turns
- any additional indications necessary: tolerance, direction of turns

The simplified representation is permitted for a thread diameter ≤ 6 mm or if holes or threads of the same type and size are arranged in a regular pattern. The necessary features that are also present in the conventional representation are indicated in the simplified representation.

According to ISO 129-1, the symbol for the quantity “×” can be replaced, as an alternative, by “↧” in the representation.

Figure 21
Simplified representation in accordance with ISO 6410-3:2021



Dimensions for threads, thread relief grooves and thread runouts, see chapter Design elements, section Thread runout and thread undercut, Page 508, and section Metric ISO threads, Page 500.

Dimensioning of relief grooves

The representation of relief grooves produced with indexable inserts is defined in ISO 18388:2016 and is supplemented with the tolerances in DIN 509:2022. Relief grooves are used at the surface transitions of stepped shafts or flat surfaces with steps that are to be ground. As a result, the edge of the grinding wheel can run out cleanly.

A distinction is made between the following types of relief grooves:
- type E for workpieces with one machining surface
- type F, G and H for workpieces with two machining surfaces perpendicular to each other featuring different geometries

The representation of relief grooves in drawings is indicated as follows:
- simplified as a designation with extension line, see Figure 22, Page 373, with the indications:
  - “DIN 509” or “ISO 18388”
  - type, radius × relief groove depth
  - optional indications (DIN 509 only) on surface texture in accordance with ISO 1302: designation for production, e.g. MMR for “material removal required” and roughness parameter (recommendation: Ptmax 25 or as agreed with the customer)
- complete with all dimensions:
  - conventional representation: large representation of the workpiece in a break or small representation of a detail represented in magnification, see Figure 22, Page 373. This representation is ambiguous according to ISO GPS.
  - drawing compliant with ISO GPS, see Figure 23, Page 373

Dimensions z_1 and z_2 for the machining allowance are only defined in DIN 509.

Figure 22
Dimensions and representation of relief grooves, type E and F

① Conventional representation
② Countersink on counterpart
③ Simplified representations of relief groove type F, radius r = 4 mm, depth t_1 = 0,5 mm

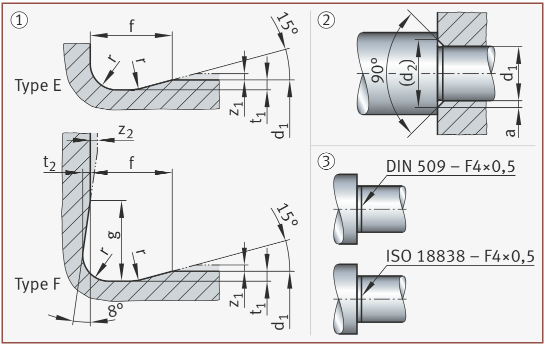
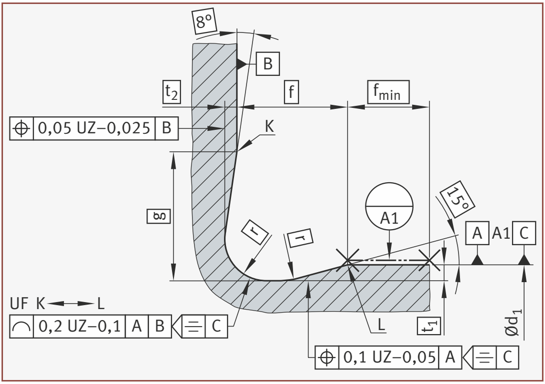


Figure 23
Relief groove type F, dimensioning compliant with ISO GPS



The necessary countersink depth in the counterpart is dependent on the relief groove type.

The following table shows the dimensions of relief grooves of type E and F and the countersink in the counterpart in accordance with ISO 18388 and DIN 509, see also Figure 22 and Figure 23, Page 373. The tolerances are only defined in DIN 509.

Dimensions mm					Recommended allocation[2] to diameter d_1 for workpieces mm		Minimum dimension a for countersink mm	
r	t_1	t_2	f	g	under normal load	under higher alternating load	Type E	Type F
±0,1	+0,1 0	+0,05 0	+0,2 0					
R0,2	0,1	0,1	1	(0.9)	> ∅1,6 to ∅3	–	0,2	0
R0,4	0,2	0,1	2	(1.1)	> ∅3 to ∅18	–	0,3	0
R0,6	0,2	0,1	2	(1.4)	> ∅10 to ∅18	–	0,5	0,15
R0,6	0,3	0,2	2,5	(2.1)	> ∅18 to ∅80	–	0,4	0
R0,8	0,3	0,2	2,5	(2.3)	> ∅18 to ∅80	–	0,6	0,05
R1	0,2	0,1	2,5	(1.8)	–	> ∅18 to ∅50	0,9	0,45
R1	0,4	0,3	4	(3.2)	> ∅80	–	0,7	0
R1,2	0,2	0,1	2,5	(2)	–	> ∅18 to ∅50	1,1	0,6
R1,2	0,4	0,3	4	(3.4)	> ∅80	–	0,9	0,1
R1,6	0,3	0,2	4	(3.1)	–	> ∅50 to ∅80	1,4	0,6
R2,5	0,4	0,3	5	(4.8)	–	> ∅80 to ∅125	2,2	1
R4	0,5	0,3	7	(6.4)	–	> ∅125	3,6	2,1

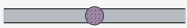
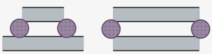
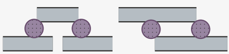
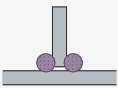
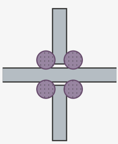
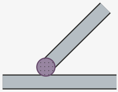
[1] Numerals printed **bold** correspond to relief grooves of series 1. Series 1 should be used in preference.

[2] The allocation to the diameter range is a guideline only; it does not apply in the case of short steps and thin-walled parts.

Symbols for weld connections

Weld connections Weld connections are material contact connections between materials of the same type (for example steel, aluminium, certain plastics). Depending on the intended purpose and type of loading, use is made of various joint types, weld seam types and seam configurations.

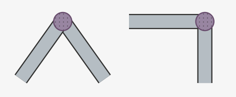
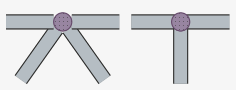
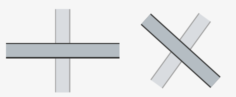
Joint types The following table shows a selection of joint types in accordance with ISO 17659:2002.

Joint type	Arrangement of parts	Explanation of joint type	Suitable seam configurations (symbols), guidelines
Butt joint		The parts lie in a single plane and are abutted against each other	 Favourable in relation to flow of forces and material usage
Parallel joint		The parts lie parallel on top of each other	 Frequently used in top flange plates of bending beams
Overlap joint		The parts lie parallel on top of and overlap each other	 Frequently used as member connections in steelwork
T joint		The parts are butted perpendicular against each other	 Measures are necessary in the case of transverse tensile loading[1)]
Double T joint (cross joint)		(derived from T joint) Two parts lying in a single plane are butted perpendicular against an interjacent third part – unfavourable!	 Measures are necessary in the case of transverse tensile loading[1)]
Angular joint		(derived from T joint) One part is butted obliquely against another	 Fillet angle ≧ 60° Measures are necessary in the case of transverse tensile loading[1)]

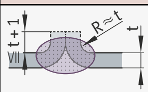
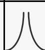
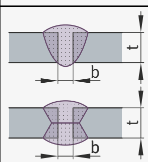
Continuation of table, see Page 376.

1) Due to the risk of fractures (lamellar fractures): examples include ultrasonic inspection, increase in weld connection area.

Continuation of table, Joint types, from Page 375.

Joint type	Arrangement of parts	Explanation of joint type	Suitable seam configurations (symbols), guidelines
Corner joint		Two parts are butted against each other at a corner of any angle	 Lower load capacity than T joint
Multiple joint		Three or more parts are butted against each other at any angle	Difficult to assess all parts. Unsuitable for higher loading.
Crossing joint		Two parts lie crosswise on top of each other	 Isolated cases in steelwork

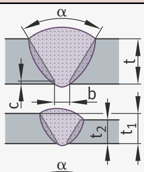
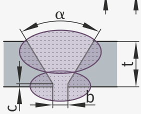
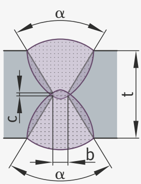
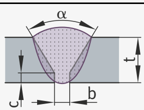
Seam types and seam configurations The following table shows a selection of butt seam configurations and their preparation in accordance with ISO 9692-1.

Seam type	Seam configuration (joining configuration)	Work-piece thick-ness t mm	Ex-ecution	Sym-bol	Dimensions Gap b mm	Angle α, β °	Welding process[1]	Production costs[2]	Comments Application
Flanged seam		up to 2	One side		–	–	G, E, WIG, MIG, MAG	–	Welding of thin sheet metal without filler material
I seam		up to 4	One side		≈ t	–	G, E, WIG	0,5	No seam preparation, little filler material. With welding on one side, the possibility of root defects and fusion defects cannot be ruled out.
		up to 8	Both sides		≈ t/2	–	E, WIG (MIG, MAG)		

Continuation of table, see Page 377.

[1] Recommended welding process.
[2] Relative production costs.

Continuation of table, Seam types and seam configurations, from Page 376.

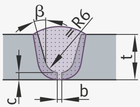
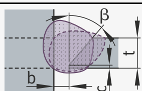
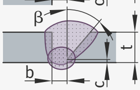
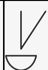
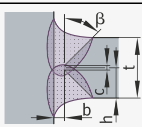
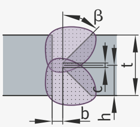
Seam type	Seam configuration (joining configuration)	Work-piece thick-ness t mm	Ex-ecution	Sym-bol	Dimensions Gap b mm	 Angle α, β °	Welding process[1]	Production costs[2]	Comments Application
V seam		3 to 10	One side	V	≦ 4	40 to 60	G	1	In case of dynamic loading: ■ Work out and back weld the root. ■ For $t_1 - t_2 > 3$ mm, bevel the thicker part to an inclination of 1:4 (flow of forces!).
		3 to 40	Both sides		≦ 3	≈ 60	E, WIG		
						40 to 60	MIG, MAG		
DV seam or X seam		over 10	Both sides	X	1 to 4	≈ 60	E, WIG	2	More favourable for larger sheet thicknesses than V seam since, for an identical angle α, only half the weld deposit quantity is required. Almost no angle shrinkage with welding on alternate sides. If necessary, work out root before welding of the opposing position.
						40 to 60	MIG, MAG		
Y seam		5 to 40	One side	Y	1 to 4	≈ 60	E, WIG, MIG, MAG	1,5	Web height c = 2 ... 4 mm

Continuation of table, see Page 378.

1) Recommended welding process.

2) Relative production costs.

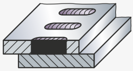
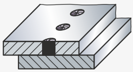
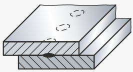
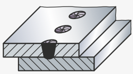
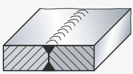
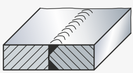
Continuation of table, Seam types and seam configurations, from Page 377.

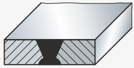
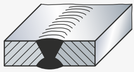
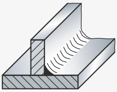
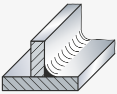
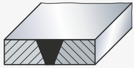
Seam type	Seam configuration (joining configuration)	Workpiece thickness t mm	Execution	Symbol	Dimensions Gap b mm	Dimensions Angle α, β °	Welding process[1]	Production costs[2]	Comments Application
U seam		over 12	One side		1 to 4	8 to 12	E, WIG, MIG, MAG	4	Web height c = 3 mm advantageous with inaccessible opposing side. Expensive preparation (planing).
HV seam		3 to 10	One side		2 to 4	35 to 60	E, WIG, MIG, MAG	0,7	Frequently used in conjunction with a fillet seam in a T joint. Execution with unwelded web (HV web seam) reduces production costs. Web height c ≦ 2 mm
		3 to 10	Both sides		1 to 4				
DHV seam (double HV seam, K seam)	 	over 10	Both sides		1 to 4	35 to 60	E, WIG, MIG, MAG	1	Frequently used in conjunction with fillet seams in a T joint. Execution with unwelded centre web (K web seam) reduces production costs. Flank height h = t/2 or t/3

1) Recommended welding process.

2) Relative production costs.

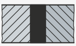
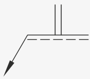
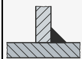
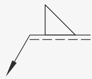
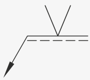
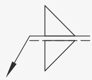
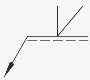
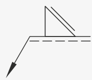
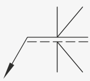
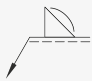
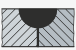
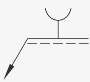
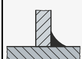
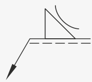
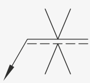
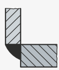
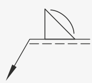
The following table shows further examples of seam types taken from ISO 2553:2019.

Seam type	Seam configuration (joining configuration)	Symbol
Plug seam	 	
Spot seam	 	
Composite symbols		
DY seam (double Y seam)		
DHY seam (double HY seam, K web seam)		

Seam type	Seam configuration (joining configuration)	Symbol
Application examples for additional symbols		
Flat V seam with flat opposing seam		
Y seam with worked out root and opposing seam		
Fillet seam with concave surface		
Fillet seam with notch-free seam transition (machined as necessary)		
Flat V seam levelled on upper workpiece surface by additional machining		

Seam symbols for butt and corner seams

The following table shows a selection of weld seam symbols for butt seams and corner seams in accordance with ISO 2553:2019, in the form in which they can be used in drawings.

Symbol for seam type	Seam configuration (cross-section)	Symbolic representation in drawings	Name	Symbol for seam type	Seam configuration (cross-section)	Symbolic representation in drawings	Name
Butt seams				**Corner seams and fillet seams**			
			I seam				Fillet seam
			V seam				Double fillet seam
			HV seam				Flat seam
			DHV seam or K seam				Camber seam
			U seam				Concave seam
			DV seam or X seam				Corner seam (outer fillet seam, executed here as a camber seam)

Indication of surface texture and roughness parameters in drawings

Surface texture

The indication of profile surface texture in technical product documentation (TPD) is carried out with the aid of graphical symbols and is defined in standard series ISO 21920-1, 2 and 3:2021.

Surfaces on workpieces that are to remain unfinished (unmachined), i.e. surfaces which are the result of the manufacturing process, such as rolling, forging, casting or flame cutting etc., are not assigned a surface symbol.

Indications of surface texture are necessary where there are higher requirements for the quality of the surface. The required surface texture can be achieved by cutting or non-cutting production processes.

Various profile surface texture parameters are defined for identification of the surface texture, see section Roughness profile parameters, Page 396.

The values for the surface texture parameters Ra that can be achieved using various production processes are compiled in the section Achievable mean roughness values, Page 402. The values are based on the previously valid default case (application of the 16% rule) and the previously applied measurement practice (surface imperfections were not taken into account during measurement).

Graphical symbols without indications

Requirements for profile surface texture are represented in TPD by means of the following graphical symbols:

Graphical symbol	Explanation
	**Basic symbol** Any manufacturing process permitted
	**Expanded symbol** Surface must be machined with removal of material
	**Expanded symbol** Surface must not be machined by removal of material or must remain in its delivered condition

Tolerance acceptance rules

The application of tolerance limits to the measured values of the parameters is stipulated in the tolerance acceptance rules:

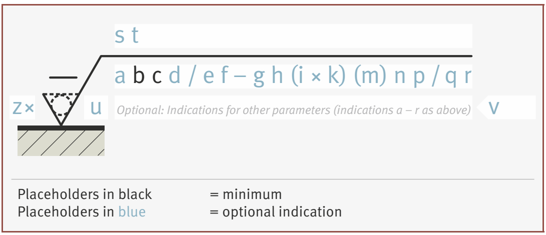
Tolerance acceptance rule	Explanation
Tmax	No measured value may exceed the maximum value; the maximum tolerance acceptance rule is the default case
T16%	A maximum of 16 % of all measured values may violate the tolerance limit
Tmed	The median value of all measured values must be met

Indications with the symbol

If the texture of the surface is specified or further indications relating to the coating, production process or surface structure (direction of grooves) are necessary, the horizontal line top right of the graphical symbol is expanded.

The possible additional requirements a-z are positioned in the symbol as shown in Figure 24 and are explained in the following table. The letters a to z symbolise placeholders for optional information. These can be specified if they differ from the default case or from an additional requirement. Placeholders b and c represent mandatory information.

Figure 24
Position of indications with the symbol



Indication	Type of indication
a	Tolerance type
b	Symbol for R-parameter, P-parameter or W-parameter
c	Tolerance limit value for profile surface texture parameter
d	Tolerance acceptance rule
e	Profile S-filter type
f	Profile S-filter nesting index
g	Profile L-filter type (indication only appropriate for R-parameters)

Continuation of table, see Page 383.

Continuation of table, Indications with the symbol, from Page 382.

Indication	Type of indication
h	Profile L-filter nesting index (indication only appropriate for R-parameters)
i	Section length (only appropriate when indicating section length-based R-/P-/W-parameters)
k	Number of sections (only appropriate when indicating section length-based R-/P-/W-parameters)
m	Evaluation length (only appropriate when indicating evaluation length-based R-/P-/W-parameters)
n	Association method and element of profile F-operator
p	Profile F-operator nesting index
q	Method of profile extraction
r	Placeholder for the OR(n) symbol for other requirements
s	Manufacturing process
t	Surface lay and direction of machining marks
u	Profile direction in relation to surface lay
v	Intersection plane indicator for specified profile direction
z	Number of identical specifications

The parameter type and length type determine whether information is provided for the individual placeholders in the middle part g to m of the surface symbol:

- evaluation lengths for R-/P-/W-parameters
- section lengths for R-/P-/W-parameters

Values and settings that are not explicitly specified with the symbol are defined by the default cases in ISO 21920-3:2021. They are taken from the tables based on the specified parameter with tolerance type and tolerance limit.

Preferred parameters

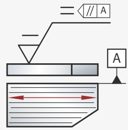
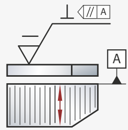
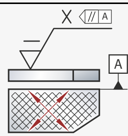
When indicating roughness parameters in drawings, it must be ensured that only the preferred parameters are used for mean roughness values.

The preferred parameters for Ra and Rz are as follows:

Ra	0,025	0,05	0,1	0,2	0,4	0,8	1,6	3,2	6,3	12,5	25	50
Rz	0,2	0,4	0,8	1,6	3,2	6,3	12,5	25	50	100	200	200

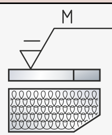
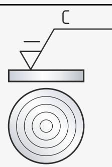
Indication of direction of grooves with datum

In order to determine the direction of grooves relative to a feature, the following symbols are added at position "t" of the surface symbol, see Figure 24, Page 382:

Symbol	Example	Explanation
=		■ Parallel to the direction specified by the intersection plane indicator and the associated reference to a feature
⊥		■ Perpendicular to the direction specified by the intersection plane indicator and the associated reference to a feature
X		■ Crossed in two directions, inclined at 45° to the direction specified by the intersection plane indicator and the associated reference to a feature

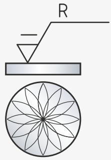
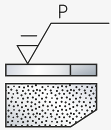
Direction of grooves without datum

In order to determine the direction of grooves without a datum, the following symbols are added at position "t" of the surface symbol, see Figure 24, Page 382:

Symbol	Example	Explanation
M		■ Multi-directional
C		■ Approximately concentric to the centre of the surface to which the symbol applies

Continuation of table, see Page 385.

Continuation of table, Indications with the symbol, from Page 384.

Symbol	Example	Explanation
R		■ Approximately radial to the centre of the surface to which the symbol applies
P		■ Surface without grooves, non-directional or with troughs

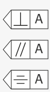
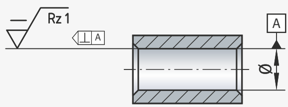
Indication of the profile direction

In order to determine the direction of the profile, the following symbols are added at position “u” of the surface symbol, see Figure 24, Page 382:

Symbol	Explanation
	Profile direction perpendicular to the prevailing direction
	Profile direction parallel to the prevailing direction
	Profile direction at an angle to the prevailing direction ($0° < \alpha < 90°$). The angle is given after the symbol.
	Profile direction circular to the centre of the surface to which the symbol applies

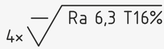
Indication of the profile direction relative to a feature

In order to determine the profile direction using an intersection plane indicator on the surface symbol with reference to a feature, the following symbols are added at position “v” of the surface symbol, see Figure 24, Page 382:

Symbol	Example, explanation
	 Specified profile direction determined by intersection plane indicator on the surface symbol. Reference to a feature is also required in the drawing.

Surface symbols The following tables show some examples of surface symbols and their meaning or application.

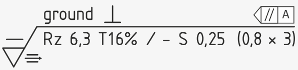
Example 1:



Indication[1]	Type of indication	Indication, explanation
Indications with the symbol		
	Graphical symbol	Profile surface texture, any production method
b	Ra	R-parameter: Ra
c	6,3	Tolerance limit value: 6,3 μm, leads to the relevant default settings in accordance with ISO 21920-3:2021, table 3
d	Tolerance acceptance rule	16% rule
z	4×	Number of identical specifications: 4
Applied default settings in accordance with ISO 21920-3:2021, table 1 and table 3		
a	Tolerance type	Upper tolerance limit
e	Profile S-filter type	Gaussian filter in accordance with ISO 16610-21
f	Profile S-filter nesting index N_{is}	8 μm with maximum sampling distance of 1,5 μm
g	Profile L-filter type	Gaussian filter in accordance with ISO 16610-21
h	Profile L-filter nesting index N_{ic}	2,5 mm
i	Section length l_{sc}	2,5 mm
k	Number of sections n_{sc}	5
m	Evaluation length l_e	12,5 mm
n	Profile F-operator association method and element	Association and removal of the specified form element with total least square
p	Profile F-operator nesting index	Not required
q	Method of profile extraction	Mechanical profile
r	Other requirements (OR(n))	No other requirements
s	Manufacturing process	No requirement
t	Surface lay and direction of machining marks	No requirement
u	Profile direction	Perpendicular to surface lay
v	Intersection plane indicator for specified profile direction	No requirement

[1] Position of indications with symbol, see Figure 24, Page 382.

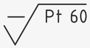
Example 2:



Indication[1]	Type of indication	Indication, explanation
Indications with the symbol		
	Graphical symbol	Profile surface texture, material must be removed
b	Rz	R-parameter: Rz
c	6,3	Tolerance limit value: 6,3 µm, leads to the relevant default settings in accordance with ISO 21920-3:2021, table 3
d	Tolerance acceptance rule	16% rule
g	Profile L-filter type	Spline filter
h	Profile L-filter nesting index N_{ic}	0,25 mm
i	Section length l_{sc}	0,8 mm
k	Number of sections n_{sc}	3
s	Ground	Grinding production method
t	⊥ ◁// A	Surface lay and direction of machining marks perpendicular to the direction specified by the intersection plane indicator and datum
u	⥱	Profile direction parallel to surface lay
Applied default settings in accordance with ISO 21920-3:2021, table 1 and table 3		
a	Tolerance type	Upper tolerance limit
e	Profile S-filter type	Gaussian filter in accordance with ISO 16610-21
f	Profile S-filter nesting index N_{is}	2,5 µm with maximum sampling distance of 0,5 µm
n	Profile F-operator association method and element	Association and removal of the specified form element with total least square
p	Profile F-operator nesting index	Not required
q	Method of profile extraction	Mechanical profile
r	Other requirements (OR(n))	No other requirements
v	Intersection plane indicator for specified profile direction	No requirement
z	Number of identical specifications	Not required

[1] Position of indications with symbol, see Figure 24, Page 382.

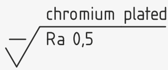
Example 3:



Indication[1)]	Type of indication	Indication, explanation
Indications with the symbol		
	Graphical symbol	Profile surface texture, any production method
b	Pt	P-parameter: Pt
c	60	Tolerance limit value: 60 µm, leads to the relevant default settings in accordance with ISO 21920-3:2021, table 6
Applied default settings in accordance with ISO 21920-3:2021, table 1 and table 6		
a	Tolerance type	Upper tolerance limit
d	Tolerance acceptance rule	Maximum tolerance acceptance rule
e	Profile S-filter type	Gaussian filter in accordance with ISO 16610-21
f	Profile S-filter nesting index N_{is}	8 µm with maximum sampling distance of 1,5 µm
m	Evaluation length l_e	Length of feature to be evaluated
n	Profile F-operator association method and element	Association and removal of the specified form element with total least square
q	Method of profile extraction	Mechanical profile
r	Other requirements (OR(n))	No other requirements
s	Manufacturing process	No requirement
t	Surface lay and direction of machining marks	No requirement
u	Profile direction	Perpendicular to surface lay
v	Intersection plane indicator for specified profile direction	No requirement
z	Number of identical specifications	Not required

1) Position of indications with symbol, see Figure 24, Page 382.

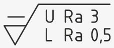
Example 4:



Indication[1]	Type of indication	Indication, explanation
Indications with the symbol		
	Graphical symbol	Profile surface texture, any production method
b	Ra	R-parameter: Ra
c	0,5	Tolerance limit value: 0,5 µm, leads to the relevant default settings in accordance with ISO 21920-3:2021, table 3
s	Manufacturing process	Chromium plated surface
Applied default settings in accordance with ISO 21920-3:2021, table 1 and table 3		
a	Tolerance type	Upper tolerance limit
d	Tolerance acceptance rule	Maximum tolerance acceptance rule
e	Profile S-filter type	Gaussian filter in accordance with ISO 16610-21
f	Profile S-filter nesting index N_{is}	2,5 µm with maximum sampling distance of 0,5 µm
g	Profile L-filter type	Gaussian filter in accordance with ISO 16610-21
h	Profile L-filter nesting index N_{ic}	0,8 mm
i	Section length l_{sc}	0,8 mm
k	Number of sections n_{sc}	5
m	Evaluation length l_e	4 mm
n	Profile F-operator association method and element	Association and removal of the specified form element with total least square
p	Profile F-operator nesting index	Not required
q	Method of profile extraction	Mechanical profile
r	Other requirements (OR(n))	No other requirements
t	Surface lay and direction of machining marks	No requirement
u	Profile direction	Perpendicular to surface lay
v	Intersection plane indicator for specified profile direction	No requirement
z	Number of identical specifications	Not required

[1] Position of indications with symbol, see Figure 24, Page 382.

Example 5:

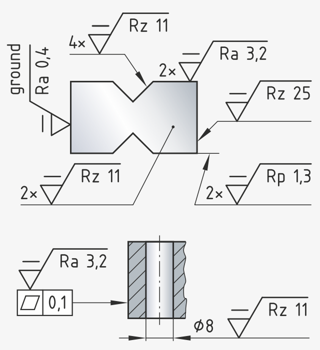
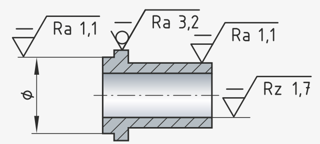
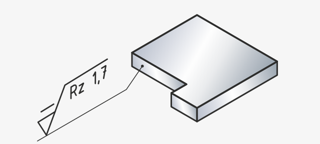
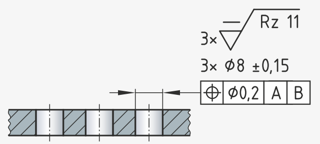
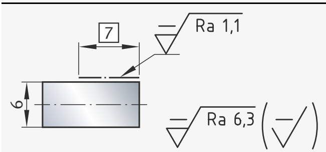


Indication[1]	Type of indication	Indication, explanation
Indications with the symbol		
	Graphical symbol	Profile surface texture, material must be removed
a	Tolerance type	Bilateral tolerance limits
b	Ra	R-parameter: Ra
c Top line	3; U	Upper tolerance limit value: 3 µm
c Bottom line	0,5; L	Lower tolerance limit value: 0,5 µm
2 lines	Indications U and L	Bilateral tolerance limits, tolerance centre C = (U + L)/2 = (3 + 0,5) µm/2 = 1,75 µm, leads to the relevant default settings in accordance with ISO 21920-3:2021, table 4
Applied default settings in accordance with ISO 21920-3:2021, table 1 and table 4		
d	Tolerance acceptance rule	Maximum tolerance acceptance rule
e	Profile S-filter type	Gaussian filter in accordance with ISO 16610-21
f	Profile S-filter nesting index N_{is}	8 µm with maximum sampling distance of 1,5 µm
g	Profile L-filter type	Gaussian filter in accordance with ISO 16610-21
h	Profile L-filter nesting index N_{ic}	2,5 mm
i	Section length l_{sc}	2,5 mm
k	Number of sections n_{sc}	5
m	Evaluation length l_e	12,5 mm
n	Profile F-operator association method and element	Association and removal of the specified form element with total least square
p	Profile F-operator nesting index	Not required
q	Method of profile extraction	Mechanical profile
r	Other requirements (OR(n))	No other requirements
s	Manufacturing process	No requirement
t	Surface lay and direction of machining marks	No requirement
u	Profile direction	Perpendicular to surface lay
v	Intersection plane indicator for specified profile direction	No requirement
z	Number of identical specifications	Not required

[1] Position of indications with symbol, see Figure 24, Page 382.

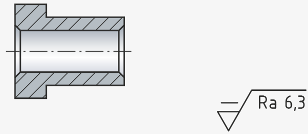
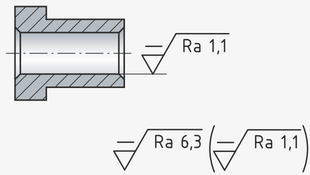
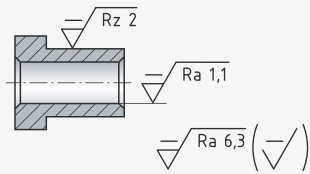
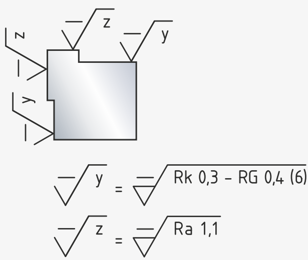
Arrangement of symbols

In the drawing, the symbols are arranged as follows:

<table>
<tr><th>Drawing</th><th>Applications</th></tr>
<tr><td>
</td><td>Arrangement, general rules:
■ Symbols must be readable from the right or from the bottom.
■ Symbols are located directly on the surface or connected with it by means of reference or extension lines and a dimension arrow.
■ Symbols may also be placed on the tolerance frame for geometrical tolerances.
■ The surface indication can be given together with a dimension indication if there is no possibility of misinterpretation.</td></tr>
<tr><td>
</td><td>■ Cylindrical and prismatic surfaces</td></tr>
<tr><td>
</td><td>■ On a three-dimensional model</td></tr>
<tr><td>
</td><td>■ (Indication) of identical specifications</td></tr>
<tr><td>
</td><td>■ Restricted area of a feature</td></tr>
</table>

Continuation of table, see Page 392.

Continuation of table, Arrangement of symbols, from Page 391.

Drawing	Applications
	■ Simplified drawing indication for identical surface texture
	■ Simplified drawing indication for predominantly identical surface texture
	
	■ Simplified drawing indication for identical surface quality on several individual texture

Profile surface texture parameters

Geometrical parameters are defined for the description of surface texture (ISO 21920-2:2021):

- R-parameters (relating to the roughness profile)
- W-parameters (relating to the waviness profile)
- P-parameters (relating to the primary profile)

The quality of a surface is measured in accordance with the standardised profile method (ISO 3274:1996). No distinction is made between periodic and non-periodic profiles.

The terms used in conjunction with measurement and recording of profile surface texture parameters are explained in the following table.

Term	Definition in accordance with ISO 21920 and other standards
Number of sections n_{sc}	Number of section lengths for specifying the section length parameters (e.g. Rz, Rp, Rv). Previous designation: number of individual measurement distances
Section length l_{sc}	Length in the direction of the x axis that is used to obtain the section length parameters. Section length parameters such as Rz, Rp, Rv are evaluated over several consecutive sections of length l_{sc}. Previous designation: individual measurement distance lr, lw, lp
Evaluation length l_e	Length in the direction of the x axis used to determine the geometric structures that describe the scale-limited profile. The travel path is longer than the evaluation length. The majority of parameters are defined using the evaluation length l_e, exceptions include Rz, Rp, Rv. Previous designation: measurement distance In
Setting class Scn	Identifier for determining default settings. Previously undetermined.
F-operation	Operation which removes form components. Previous definition: profile filter λf
L-filter	Filter that removes large-scale lateral components (high pass). For roughness parameters, the “N_{ic} filter” removes large-scale lateral components that do not belong to the R-profile. Previous definition: profile filter λc: Gaussian filter in accordance with ISO 11562 with exceptions
Mechanical surface	Limitation of mathematical erosion of the geometric location of the centre of an ideal stylus ball with radius r, which is rolled over the actual workpiece surface by a sphere which also has the radius r.
Mechanical profile	Limitation of mathematical erosion of the geometric location of the centre of an ideal stylus ball with radius r, which is rolled along a track over the actual workpiece surface using a circular disk which also has the radius r.

Continuation of table, see Page 394.

Continuation of table, Profile surface texture parameters, from Page 393.

Term	Definition in accordance with ISO 21920 and other standards
Nesting index N_{is}, N_{ic}, N_{if}	Numerical value of the filter that separates a profile into large-scale and small-scale lateral components. Previous designation: cut-off wavelength λs, λc, λf
Nesting index N_{is}	For roughness parameters, N_{is} removes very small-scale lateral components that do not belong to the R-profile. Previous definition: cut-off wavelength λs
Nesting index N_{ic}	Filter that determines the transition from roughness to waviness. For roughness parameters, N_{ic} removes large-scale lateral components that do not belong to the R-profile. For waviness parameters, N_{ic} removes small-scale lateral components that belong to the R-profile and not to the W-profile. Previous definition: cut-off wavelength λc
Nesting index N_{if}	Filter relating to a nominal feature or a filter.
Surface imperfections in accordance with ISO 8785	Verification must be carried out at the location of the surface where the highest values are expected. By default, surface imperfections such as scratches, pores and cavities must be included in the verification. Previous practice: Surface imperfections were excluded from the surface determination, ISO 4288:1989.
Ordinate value z(x)	Height of the measured profile at any specified position x. The heights are considered as negative if ordinates lie below the x axis (centre line).
Primary surface profile	Surface profile trace produced with defined nesting index N_{is} (the “morphological filter”) from the mechanical surface (default), the electromagnetic surface or the auxiliary surface.
Primary profile P-profile	Scale-limited profile derived from the primary surface profile using a profile F-operation with nesting index N_{if}. The primary profile is the basis for evaluation of the P-parameters.
Profile element	Peak followed by a trough, or trough followed by a peak with the dimensions: ■ Z_{ph} peak height ■ Z_{vd} pit depth ■ Z_t profile element height ■ X_s profile element spacings
Profile position	Location where maximum values are expected, with surface imperfections included. Previous practice: Imperfections (e.g. scratches, pores) were excluded from the verification.

Continuation of table, see Page 395.

Continuation of table, Profile surface texture parameters, from Page 394.

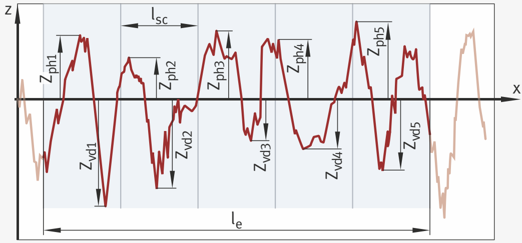
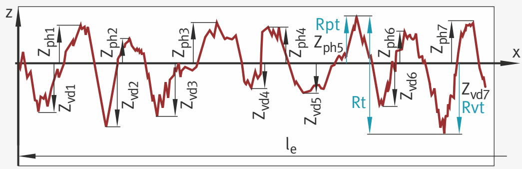
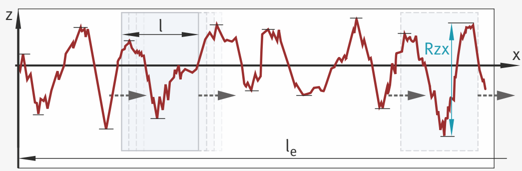
Term	Definition in accordance with ISO 21920 and other standards
Roughness profile R-profile	Scale-limited profile derived from the primary profile by using a specific profile L-filter with a nesting index N_{ic} The roughness profile is the basis for evaluation of the R-parameters.
S-filter	Filter that removes small-scale lateral components (low pass). For roughness parameters, the "N_{is} filter" removes very small-scale lateral components that do not belong to the R-profile. For waviness parameters, the "N_{ic} filter" removes small-scale lateral components that belong to the R-profile and not to the W-profile. Previous definition: profile filter λs: Gaussian filter in accordance with ISO 11562
Scale-limited profile	Profile structure scale components between specified nesting indices. A profile is scale-limited once a profile filter with a specified nesting index has been applied.
Specification of parameters	Control values are based on the drawing indication. Previous practice: Default values were selected on the basis of the workpiece.
Traced section	Determination of the surface texture in accordance with the profile method
Traced length l_t	Total length covered by the tracing system in order to record the traced profile. This is the sum of the pre-travel length (for stabilising the filter), the evaluation length le and the post-travel length (for the excursion of the filter).
Tolerance acceptance rule	Determines how the tolerance limits are applied to the measured values of the parameters. Default case is the maximum tolerance acceptance rule. Previous practice: 16% rule
16% tolerance acceptance rule T16%	Application of the 16% tolerance acceptance rule must be explicitly specified if it is to be applied. Previous practice: The 16% rule was the default case without actually being specified.
Waviness profile W-profile	Scale-limited profile derived from the primary profile by using a specific profile S-filter with a nesting index N_{ic}. The waviness profile is the basis for evaluation of the W-parameters.

Roughness profile parameters Roughness profile parameters (R-parameters) are calculated from the roughness profile, W-parameters from the waviness profile and P-parameters from the primary profile.

R-parameters The following table shows a selection of R-parameters in accordance with ISO 21920-2:2021.

Parameter		Definition	Mathematical definition
Rp	Mean peak height	Arithmetic mean of the largest peak heights Z_{phi} of each profile section in the roughness profile	$Rp = \frac{1}{n_{sc}} \cdot \sum_{i=1}^{n_{sc}} \max_{j \in N_i}\left(Z_{ph,j}\right)$
Rv	Mean pit depth	Arithmetic mean of the largest pit depths Z_{vdi} of each profile section in the roughness profile	$Rv = \frac{1}{n_{sc}} \cdot \sum_{i=1}^{n_{sc}} \max_{j \in N_i}\left(Z_{vd,j}\right)$
Rz	Maximum height of the profile	Arithmetic mean of all sums of the largest peak heights and the largest pit depths from all sections	$Rz = \frac{1}{n_{sc}} \cdot \sum_{i=1}^{n_{sc}} \left(\max_{j \in N_{p,i}}\left(Z_{ph,j}\right) + \max_{k \in N_{v,i}}\left(Z_{vd,k}\right) \right)$
Rt	Total height of the profile	Sum of the height of the largest profile peak Z_{phi} and the largest depth of the profile pit Z_{vdi} within the evaluation length l_e	$Rt = \max_{x \in X}\left(z(x)\right) - \min_{x \in X}\left(z(x)\right)$ where $X = \left(x \in R \mid 0 \leqq x \leqq l_e\right)$
Rpt	Maximum peak height	Largest peak height Z_{phi} of the roughness profile within the evaluation length l_e	
Rvt	Maximum pit depth	Largest pit depth Z_{vdi} of the roughness profile within the evaluation length l_e	
Rzx	Maximum height per section	Greatest difference between the highest and the lowest profile value within a section of length l moved along the evaluation length l_e. ISO 21920 defines the new parameter Rzx as the successor to Rmax. Meaning of Rmax: maximum roughness depth is only defined in VDA 2006 and ASME B46.1 as the largest individual roughness depth within the evaluation length l_e.	–

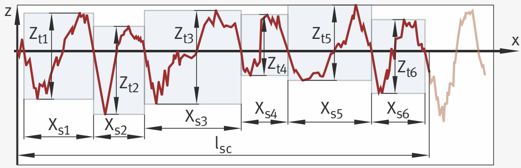
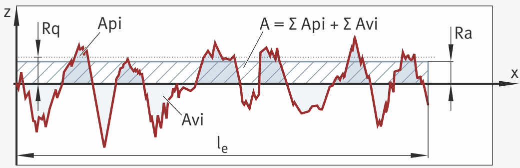
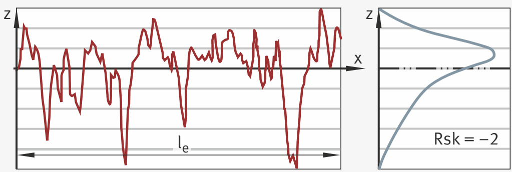
Continuation of table, see Page 398.

Statement	Geometrical representation
■ Defined by outliers ■ No statement on profile shape	
■ Rt is defined by means of the total evaluation length l_e, which is larger than the individual measurement distance. As a result, Rt ≥ Rzx	
–	

Continuation of table, R-parameters, from Page 396.

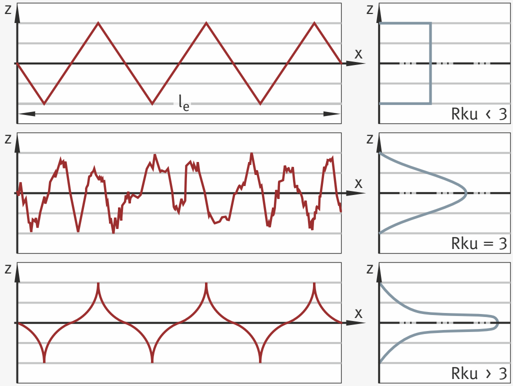
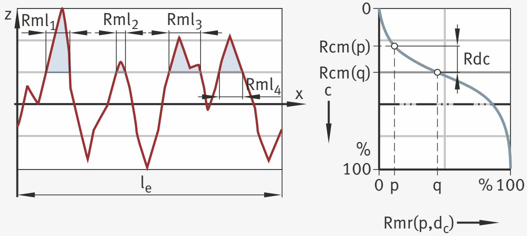
Parameter		Definition	Mathematical definition
Rc	Mean height of the profile elements	Mean of the height of the profile elements Z_t within an individual measurement distance	$Rc = \frac{1}{n_{pe}} \cdot \sum_{i=1}^{n_{pe}} Z_{t,i}$ where n_{pe} = total number of profile elements
Ra	Arithmetic mean height	Arithmetic mean of the absolute values of the ordinate values z(x)	$Ra = \frac{1}{l_e} \cdot \int_0^{l_e} \lvert z(x) \rvert dx$
Rq	Root mean square height	Root mean square of the ordinate values z(x)	$Rq = \sqrt{\frac{1}{l_e} \cdot \int_0^{l_e} z^2(x) dx}$
Rsk	Skewness	Quotient of the mean cube value of the ordinate values z(x) and the cube of Rq within the evaluation length	$Rsk = \frac{1}{Rq^3} \cdot \frac{1}{l_e} \cdot \int_0^{l_e} z^3(x) dx$

Continuation of table, see Page 400.

Statement	Geometrical representation
–	
■ Surfaces of the same character can be compared ■ “Good-natured” response ■ No statement on profile shape	
■ Parameter with higher statistical security than Ra ■ No statement on profile shape	
■ Good description of the profile shape ■ A negative Rsk value indicates a plateau-like surface with good load behaviour	

Continuation of table R-parameters from Page 398.

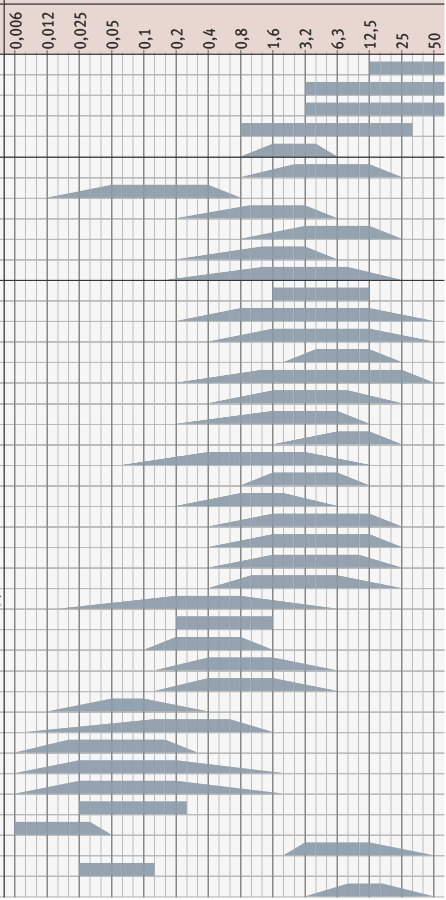
Parameter		Definition	Mathematical definition
Rku	Kurtosis	Quotient of the mean fourth power of the ordinate values z(x) and the fourth power of Rq within the evaluation length	$Rku = \frac{1}{Rq^4} \cdot \frac{1}{l_e} \cdot \int_0^{l_e} z^4(x)dx$
Rmc(c)	Material ratio	Quotient of the sum of the material lengths of the profile elements Rml(c) at the specified section height c and the evaluation length l_e. This gives the Abbott-Firestone curve, which represents the material ratio of the profile as a function of the section height.	$Rmc(c) = \frac{Rml(c)}{l_e}$
Rmr(p,d_c)	Relative material ratio	Material ratio with a section height $c_p + d_c$, where c_p is the inverse material ratio with a material ratio p and d_c is a relative section height.	$Rmr(p,d_c) = Rmc(c_p + d_c)$ where $c_p = Rcm(p)$, usually $d_c < 0$
Rdc(p,q)	Material ratio at the section line height	Difference in height between two section planes of a given material ratio p and q	$Rdc(p,q) = Rcm(q) - Rcm(p)$ where $p \leqq q$

Statement	Geometrical representation
–	
■ Integral value, so a proper definition of the profile is possible	$Rml(c) = Rml_1 + Rml_2 + Rml_3 + Rml_4$ 

Achievable mean roughness values

The standard DIN 4766-2:1981 (which is now invalid but nevertheless of considerable practical relevance) shows the achievable mean roughness values Ra for various production processes.

Production processes		Achievable mean roughness values Ra µm
Main group	Description	
Forming[1]	Sand casting[2]	
	Lost mould casting[2]	
	Gravity diecasting	
	Pressure diecasting	
	Investment casting	
Reforming	Drop forging	
	Burnishing	
	Sheet metal deep drawing	
	Extrusion	
	Stamping	
	Rolling of shaped parts	
Cutting	Slicing	
	Longitudinal turning	
	Face turning	
	Plunge turning	
	Planing	
	Ramming	
	Shaving	
	Drilling	
	Drilling out	
	Countersinking	
	Reaming	
	Side milling	
	Face milling	
	Broaching	
	Filing	
	Circular longitudinal grinding	
	Circular face grinding	
	Circular plunge grinding	
	Plane peripheral grinding	
	Plane end grinding	
	Polish grinding	
	Long stroke honing	
	Short stroke honing	
	Cylindrical lapping	
	Plane lapping	
	Ultrasonic lapping	
	Polish lapping	
	Blasting	
	Barrel finishing	
	Flame cutting	



[1] For further details, see VDG Instruction Sheet K 100, obtainable from Verein Deutscher Giessereifachleute (VDG), Sohnstrasse 70, 40237 Düsseldorf.

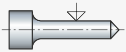
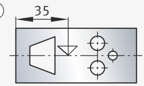
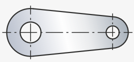
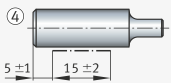
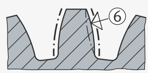
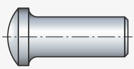
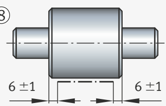
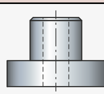
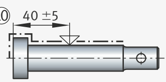
[2] In this casting method, Ra values of up to 125 µm must be expected in the case of castings up to 250 kg.

Hardness indications in drawings in accordance with ISO 15787:2016

The drawing must, in addition to the indications of the material, define the required final condition such as “hardened”, “surface-hardened and tempered”, “case-hardened” or “nitrided” and include the necessary indications of the surface hardness[1], core hardness[1] and the hardening depth (SHD, CHD, NHD).

Examples of hardness indications

The following table shows some examples of hardness indications in drawings:

Explanation	Example
Hardening, tempering, quenching and tempering	
Indications of heat treatment of the entire part by hardening, hardening and tempering, and quenching and tempering are shown in Figure ①, ② and ③: In addition to the text indication such as “hardened”, the hardness indication must always be provided together with an upper limit deviation and lower limit deviation. The indication of Brinell hardness in Figure ③ must, following the abbreviation HBW, state the ball diameter and the associated testing force in accordance with ISO 6506.	①  quench-hardened $(60\ ^{+4}_{0})$ HRC ②  quench-hardened and tempered (60 +4/0) HRC ③  quench-hardened and tempered (350 ±25) HBW 2,5/187,5
Surface hardening	
The surface-hardened areas are indicated by long-dashed dotted wide lines outside the body outlines of the part, see Figure ④ and ⑤: Figure ⑤ shows the pattern of the surface hardening in the tooth by means of long-dashed dotted narrow lines and a measurement point ⑥.	④  –·– surface-hardened (700 ±80) HV30 SHD 500 = 1,2 ±0,4 ⑤  –·– surface-hardened and entire part tempered ⑥ (58 ±3) HRC SHD 475 = 1,5 ±0,5
Case hardening	
In case hardening, a distinction is made as follows: ■ case hardening on all sides, see Figure ⑦ ■ case hardening on all sides with differing surface hardness or case hardening depth ■ case hardening at specific points, see Figure ⑧ When case hardening is carried out at specific points, the case-hardened area is indicated by long-dashed dotted wide lines outside the body outlines of the part.	⑦  case-hardened and tempered (62 ±2) HRC CHD = 1 ±0,2 ⑧  –·– case-hardened and entire part tempered (750 ±50) HV10 CHD = 0,4 ±0,1
Nitriding	
In nitriding, a distinction is made as follows: ■ nitriding of the entire part, see Figure ⑨ ■ localised nitriding, see Figure ⑩ In the case of localised nitriding, the areas of the workpiece to be nitrided are indicated by long-dashed dotted wide lines outside the body outlines of the part.	⑨  nitrided NHD HV0,3 = 0,12 ±0,2 ≧800 HV3 ⑩  –·– nitrided NHD400 = 0,4 ±0,1 ≧900 HV10

Source: ISO 15787:2016

[1] See ISO 6508-1 Rockwell hardness; ISO 6507-1 Vickers hardness; ISO 6506-1 Brinell hardness.

Tolerances and fits

Tolerances – general definition

Allocation of tolerances

The absolutely precise production of components to the size indicated in the technical drawing, the nominal size, is not possible for manufacturing reasons.

It is possible to determine, however, the maximum permissible variations from these nominal sizes. These variations are described as deviations or limit deviations. In addition, the deviation from the represented geometrical form or their position relative to each other can be permitted or restricted in such a way that the function of the component is still ensured.

In order to give an adequate definition of all the required sizes and geometrical characteristics of a workpiece in relation to its production, tolerances are allocated that must then be indicated in the technical drawing.

Tolerances in drawings

There are different types of geometrical deviations: dimensional, geometrical, locational, orientation, run-out and surface deviations. These must be restricted by means of tolerance indications.

Tolerances can be indicated in a technical drawing by means of:

- Dimensional tolerancing:
  Indication of deviations for linear and angular sizes in the form of values after the nominal size or by specifying the permitted maximum and minimum size
- Symbols for tolerance classes:
  The use of fit systems in accordance with the ISO code system for tolerances on linear sizes, starting Page 425
- Geometrical tolerancing:
  Identification of the permissible variation of geometry, orientation, location or run-out by means of symbols, starting Page 451
- Tolerancing principle:
  Global and individual indication on the drawing, starting Page 464
- General tolerances:
  Global indication of tolerances for the simplification of drawings, starting Page 468

Dimensional tolerances in drawings

The following image shows the nominal size, limit deviations and limits of size.

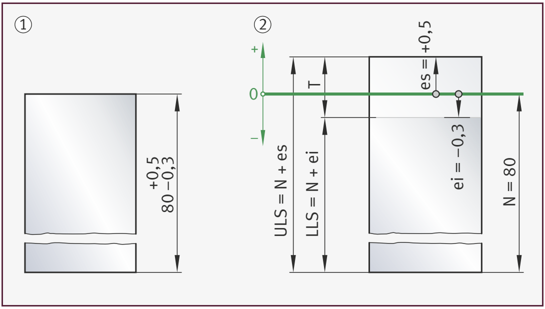


Figure 1
Nominal size, limit deviations and limits of size

(1) Drawing with linear size and tolerance
(2) Appropriate representation of nominal size, limits of size and limit deviations

Definition of terms

For the indication of sizes, deviations and tolerances, the standard ISO 286-1:2010 defines the following fixed terms:

Name		Symbol	Comments
Nominal size		N[1)]	Dimension of a feature of perfect form, as defined by the specification on the drawing. This dimension is used, with the aid of the upper and lower limit deviation, to derive the limit dimensions
Actual size		I[1)]	Dimension of the associated complete dimensional feature, determined by measurement
Local actual size		–	Any measured dimension in any cross-section of a form element
Limit of size		–	Extreme permitted sizes of a dimensional feature. The actual size of a workpiece may lie between the two limits of size “upper limit of size” and “lower limit of size” (including the limits of size themselves)
Upper limit of size		ULS	The larger permitted size of the two limits of size
Lower limit of size		LLS	The smaller permitted size of the two limits of size
Limit deviation	upper	ES, es	Upper limit of size minus nominal size (designation ES for holes, es for shafts)
	lower	EI, ei	Lower limit of size minus nominal size (designation EI for holes, ei for shafts)
Tolerance		T[1)]	Upper limit of size minus lower limit of size
Standard tolerance		IT	International tolerance; any tolerance in the ISO system for tolerances of linear sizes

1) Still included in the standard DIN 7182-1; no longer included in the standard ISO 286-1:2010 as a symbol.

Geometrical Product Specification (ISO GPS)

Concept

The Geometrical Product Specification (ISO GPS) is an internationally recognised concept that defines all the different requirements relating to geometry (especially the sizes, geometry, position and surface of a workpiece or a component) in their specification and applies to all the associated principles in relation to inspection, the measuring equipment used and its calibration. The result ensures optimum function.

The Masterplan from 1995 (ISO 14638) gave the first specifications for ISO GPS, while the GPS symbols for dimensional tolerancing have been defined in ISO 14405 since 2010. Together with the symbols for geometrical tolerancing in accordance with ISO 1101, the definitions for datums and datum systems in accordance with ISO 5459, and other GPS standards, the GPS symbols now serve worldwide as a common "language". They facilitate precise, clear and detailed indication of the technical requirements in technical drawings or in a 3D model.
The standard ISO 8015 revised in 2011 defines the fundamental concepts, principles and rules that apply in the creation, interpretation and application of all other international standards, technical specifications and technical reports (where these concern the geometrical product specification and verification). In industry, technical drawings worldwide have since been successively created in accordance with the GPS standard. Particularly in America, reference is often made instead to ASME standards and, most notably, to ASME Y14.5.

Aspects of ISO GPS

Significant aspects of ISO GPS are:

- Objective: functionally appropriate specification – specification has a considerable influence on all phases of the product life and the quality of products.
- The term "drawing" encompasses the complete package of documentation for specification of the workpiece, for example the technical drawing, 3D model, product definition data set, standards, etc.
- Specification and verification: The specification describes the product while the verification checks the implementation of the specification on the workpiece with the aid of measurements.
- Specification in accordance with the GPS principles can be clearly recognised from the indication "Linear size ISO 14405" or "ISO 8015" in or above the title block of a technical drawing. This means that all standards from the ISO GPS standard system generally apply.

- In order to exactly describe linear sizes, specification modifiers in accordance with ISO 14405-1 can be used. They define features of size such as the type “cylinder”, “sphere” or “two parallel opposite planes” (see also section Sizes, Page 417, and tables Specification modifiers for linear sizes, Page 420, as well as Complementary specification modifiers for linear sizes, Page 421).
- In order to exactly describe angular sizes, specification modifiers in accordance with ISO 14405-3 can be used.
  They define:
  - revolute angular features of size:
    cone or frustum
  - prismatic angular features of size:
    wedge (truncated or not)
- The principle of general geometrical and size specifications is defined in ISO 22081; in addition, tolerance values can be found in DIN 2769.
- Important standards in the GPS Masterplan that specify products in dimensional and geometrical terms and regulate the verification of component characteristics are:
  - ISO 8015: Fundamentals – Concepts, principles and rules
  - ISO 14405-1: Dimensional tolerancing – Linear sizes
  - ISO 14405-2: Dimensional tolerancing – Dimensions other than linear sizes or angular sizes
  - ISO 14405-3: Dimensional tolerancing – Angular sizes
  - ISO 1101: Geometrical tolerancing – Tolerances of form, orientation, location and run-out
  - ISO 5459: Geometrical tolerancing – Datums and datum systems
  - ISO 2692: Geometrical tolerancing – Maximum material requirement (MMR), least material requirement (LMR) and reciprocity requirement (RPR)
  - ISO 5458: Geometrical tolerancing – Pattern and combined geometrical specification
  - ISO 14638: GPS matrix model
  - ISO 22081: Geometrical tolerancing – General geometrical specifications and general size specifications
  - ISO 21920-1 to 3: Surface texture: Profile
  - ISO 25178-1 to 3: Surface texture: Areal
  - ISO 21204: Transition specification

Advantages of ISO GPS

The advantages of ISO GPS compared to the previous standards are:

- Clarity of workpieces and measurement methods:
  By means of the GPS standard, the designer can clearly specify the nominal geometry, sizes with tolerances and characteristics of the surface of a workpiece.
- Specification is free from contradictions or additions and can therefore no longer be interpreted in different ways, thus achieving the prerequisite for legal compliance.
- Uniform standards apply worldwide for mechanical engineering, automotive and the rolling bearing industry (contradictions and redundancies are no longer present).
- The uniform technical "language" increases quality and facilitates more efficient cooperation within the company and globally.
- ISO GPS is the foundation for machine-readable specifications and model-based definition (MBD), and thus creates the preconditions for Industry 4.0.
- The specification and verification are contained in a common specification, for example in the technical drawing, 3D model or product definition data set. The GPS specification is independent of any measuring device or measurement method. It does not prescribe how measurement or inspection is to be carried out (measurement strategy).

Unambiguity with ISO GPS

The greatest advantage of the GPS symbolic language is the clarity of workpieces and measurement methods. The finished workpiece in the mounting and functional respect is clearly represented in the specification. This was not always the case in the past.
A complex production drawing could be understood in different ways by different parties, see Figure 2.
The use of the GPS symbolic language reduces errors and simplifies not only internal communication but also regional and international exchange with suppliers and customers.
Companies now speak a uniform technical language globally that defines the requirements for the workpiece so clearly that misunderstandings and misinterpretations are eliminated. With ISO GPS, a specification is obtained that is unambiguous worldwide.

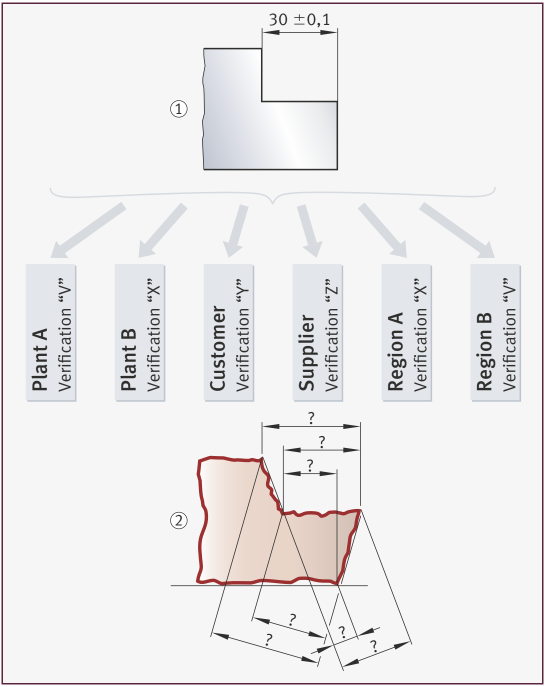
Linear distance as an example of ISO GPS

Figure 2 shows an example of ISO 14405-2 for the use of ± tolerancing in the case of a distance, Part ①. This type of indication was previously general practice. This specification indication can result in significant ambiguity, see Figure 2 Part ②.

Figure 2
Without ISO GPS, ambiguous solution

Source: ISO 14405-2

① Distance with symmetrical tolerance indication
② Differing verifications possible



Starting from the functional requirements of the product, Figure 3, Part ① to ③ describes datums in accordance with ISO 5459, geometrical tolerances and a modifier for the combination of tolerance zones in accordance with ISO 1101. The datums represent the mounting situation at the customer.

In Part ①, the position of the integral toleranced feature, in this case a straight line or flat surface, is specified as a function of datum A at the theoretically exact distance 30.

In Part ②, the surface profile of the flat surface is specified as a function of datum A at the theoretically exact distance 30.
The extracted surface must be contained between two planes parallel to the datum plane A at a distance of 0,2.

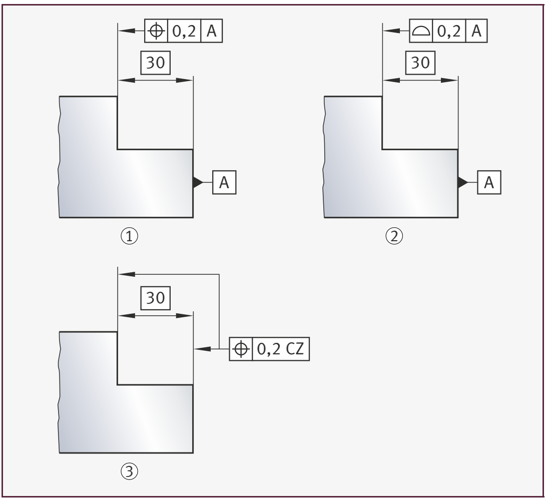
In the case of Part ③, and in contrast to Part ①, the two tolerance zones of the position tolerance must be checked in relation to each other in location and orientation at the theoretically exact distance 30.
The modifier CZ joins the two individual tolerance zones to create a combined zone.

Each of these indications conforms to GPS. It represents an unambiguous specification that can be clearly verified.

Figure 3
Conformity with GPS, unambiguous dimensioning concept

Source: ISO 14405-2

① With position tolerance
② With surface profile tolerance
③ With position tolerance and combined zone



ISO GPS in tolerance specifications for rolling bearings – examples

The special tolerance specifications for rolling bearings in accordance with ISO GPS are defined, starting from the basis of ISO 14405-1 and ISO 8015, in the rolling bearing standards ISO 492 and ISO 199. Based on examples from the field of rolling bearings, the following section shows changes to and the advantages of tolerance specification using ISO GPS in comparison with previous practice and its significance is explained.

Bearing outer ring, deviation of diameter

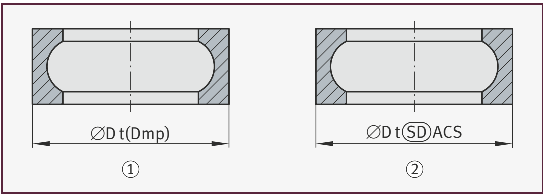
Figure 4 shows, taking the example of the deviation in diameter of a bearing outer ring, the differences in representation with and without ISO GPS.

Part 1 shows an outer ring that was specified using the function symbolic language in accordance with the rolling bearing standard ISO 1132-1. The mean outside diameter Dmp in a plane is stated. Evaluation is carried out on the arithmetic mean derived from the maximum and minimum outside surface diameter ascertainable in a radial plane, thus (Dmax + Dmin)/2.

Part 2 shows the same ring using the GPS symbolic language in accordance with ISO 492. The modifier SD applied to the outside diameter D means that this is a statistically determined value. The middle value of the range is determined, which is the arithmetic mean value derived from the maximum and minimum measured outside diameter, (Dmax + Dmin)/2. This applies in ACS (Any Cross Section), in other words in any plane of the cylinder. In this representation, there is no explicit indication of the default modifier LP for the two-point size in accordance with ISO 14405-1.

Figure 4
Bearing outer ring, tolerancing of diameter

1 Without ISO GPS (prior to 2011)
2 With ISO GPS



While the measurement method and evaluation strategy are identical, the GPS indications facilitate understanding of the measurement method and evaluation strategy without requiring special knowledge of rolling bearing standards.

Bearing inner ring, straightness of end face

Figure 5, Page 412, shows, taking the example of the straightness of the end face of a bearing inner ring, the differences in representation with and without ISO GPS.

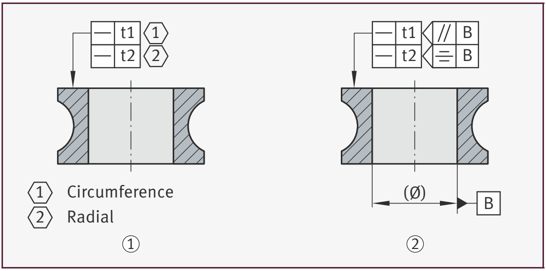
Part 1 shows an inner ring whose end face is specified by the indication of two straightness values. One value is to be determined in a circumferential direction, while the other is to be determined in a radial direction. For precise interpretation of the measurement direction, indications in the form of words are necessary, which are stated with the support of flag note.

Part ② shows the same inner ring with straightness indications on the end face, where the specification indications can be given in a non-verbal form. The straightness t1 is supplemented by an intersection plane indicator, which states: “Intersection plane parallel to axis B”, which forms the datum as the axis of the bore diameter. Measurement is thus carried out in a circumferential direction on the end face.

The straightness t2 is measured taking account of axis B, symbolised by the intersection plane indicator “Axis B included”.

Figure 5
Bearing inner ring, straightness of end face

① Without ISO GPS (prior to 2011)
② With ISO GPS



Due to the additional indications of the two intersection plane indicators, the measurement direction can be specified exactly and no indications in the form of words are used. In this way, the scope for interpretation in measurement (including towards the customer) is reduced.

Tolerances for linear and angular sizes

Dimensional tolerances in technical drawings

In ISO 129-1:2018 (as well as ISO 14405), the methods are defined for indicating sizes in technical drawings and specifically the tolerances for linear and angular sizes.

Dimensional tolerances are indicated in the form of deviations after the nominal size. In each case, the sum of the nominal size and the specific deviation gives the two permissible limits of size between which the actual size of the finished component may lie.

Rules for the indication of dimensional tolerances in drawings

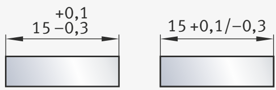
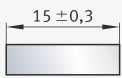
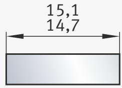
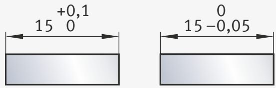
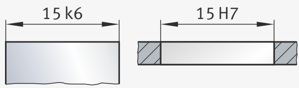
The values for the deviations are always indicated in the same unit as the nominal size after which they are indicated. Exception: If an individual nominal size is indicated in a unit different from the rest of the drawing, the units are added for the nominal size and the associated deviations.

Both deviations must have the same number of decimal places. Exception: In the case of the deviation 0, no decimal places are given. In the case of linear sizes, the symbol for the tolerance class can be placed instead of the deviations after the nominal size.

The deviations and symbols for the tolerance class are indicated in the same font size as the nominal size.

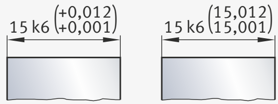
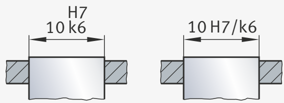
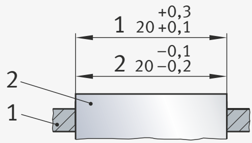
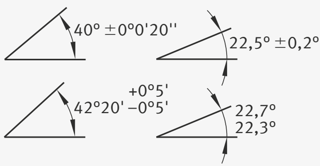
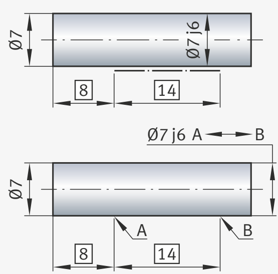
Indication in drawings of deviations, limits of size and tolerance classes

The following table shows further rules for the indication of tolerances for linear and angular sizes.

Drawing indication	Rule
	The upper deviation is placed above or before the lower deviation.
	If the upper and lower deviation are of the same magnitude, they are combined.
	By agreement, limits of size calculated from the nominal size and deviations may be indicated instead of the nominal size.
	The deviation zero is also given as a limit deviation in accordance with ISO 14405-1 and must not be omitted.
	Indication of symbol for tolerance class

Continuation of table, see Page 414.

Continuation of table, Indication in drawings of deviations, limits of size and tolerance classes, from Page 413.

Drawing indication	Rule
	Deviations or limits of size may be placed in parentheses after the symbol for the tolerance class.
	Where parts are mated, the symbol for the tolerance class of the internal size is placed before or above that of the external size.
	Where parts are mated, the internal size and its deviations is placed above the external size and the components are additionally allocated by means of numbers.
	In the case of angular sizes, the units must always be indicated for the nominal size and deviations. In other respects, the rules are identical to those for linear sizes.
	If tolerances apply to a specific fixed restricted portion, this is shown by: ■ a long-dashed dotted wide line, see image above or ■ two letters, which use theoretically exact dimensions (TEDs) to determine the start and end of the area, and an auxiliary indication after the size tolerance, see image below

Tolerance zones dimensional and position tolerances

The following example shows that dimensional tolerances alone are often not sufficient. Drawings that contain dimensional tolerances only may be ambiguous. The task is to define the position of a hole axis in a plate in relation to the lateral surfaces and restrict possible variations of the axis by means of tolerancing.

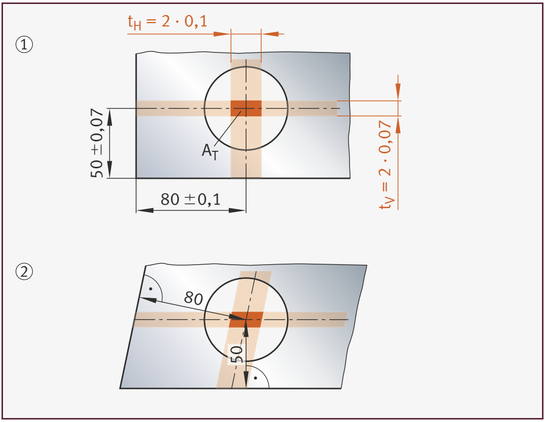
1. Approach: Dimensional tolerances

The axis of the hole is toleranced by means of deviations. This generates a rectangular tolerance range, see Figure 6. The ideal situation is shown in section ①. Section ② shows the actual situation, namely a workpiece with variations.

Figure 6
Dimensional tolerances

t_H = horizontal tolerance range
t_V = vertical tolerance range
A_T = tolerance range for hole axis

① Dimensional tolerancing
② Workpiece subject to variations (exaggerated representation)



It is clear that the specification of dimensional tolerances alone is not sufficient, since this may lead to ambiguity. Datums are important for production and inspection. A complete datum system will therefore be necessary in the future for each workpiece.

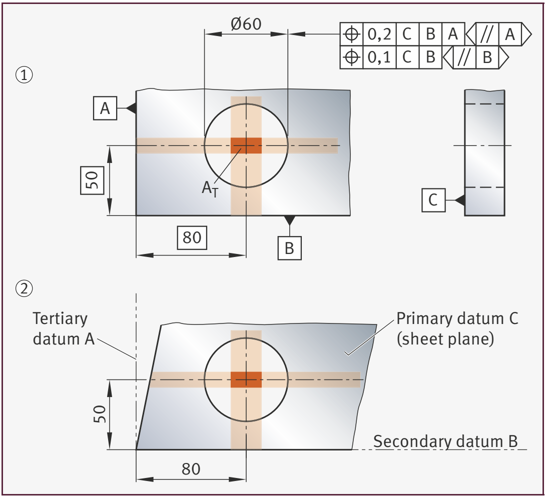
2. Approach: Position tolerances with datum system

The allocation of position tolerances with three datum planes gives the following situation, see Figure 7. The ideal situation is shown in section ①. Section ② shows the actual situation, namely a workpiece with variations.

Figure 7
Position tolerances

A_T = tolerance zone for hole axis

① Position tolerancing
② Workpiece subject to variations (exaggerated representation)

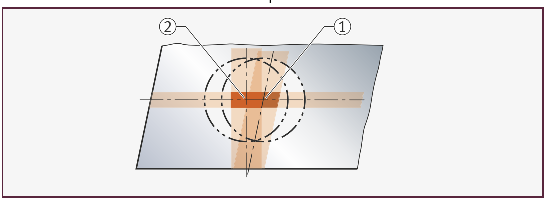


3. Comparison

The following representation shows a comparison of the position of the hole axis for dimensional and position tolerances:

Figure 8
Comparison: Dimensional and position tolerances

① Dimensional tolerancing
② Position tolerancing



Conclusion

The different tolerancing systems lead to different hole axis positions for the various tolerances. The position tolerance is clearer than the dimensional tolerance, since the datums and thus the clear alignment are defined.
The formation of a suitable datum system is the most important task in the position tolerancing of a component, providing clear information, even in cases where the design language and production language differ, for example.
It is therefore advisable to indicate a functionally appropriate datum system. The datums are important for production and inspection, representing the installation situation.

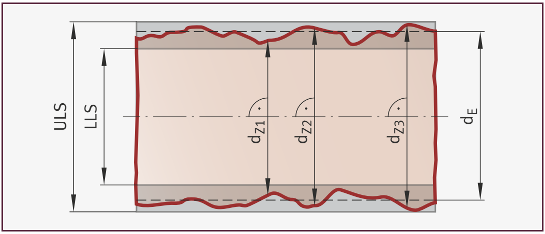
Measurement of components

During measurement of the finished component, the actual size, namely the size of the feature, is determined. The actual size is determined from various local actual sizes.
If further specification modification symbols (see ISO 14405-1 for example for spherical sizes, section sizes or portion sizes) are not indicated, local actual sizes are two-point sizes. They are formed by the associated median plane or straight line (in the case of cylinders) of the feature of size and are obtained by means of two-point measurement, see Figure 9.

The actual size and the local actual sizes may not be more or less than the upper and lower limit of size (maximum and minimum size) respectively.

Figure 9
Local actual sizes

d_E = size of the substitute element
d_Z = local actual size, two-point size
ULS = upper limit of size
LLS = lower limit of size



Sizes

Manufactured workpieces always deviate from the ideal geometrical form. The actual value of the sizes of a feature of size depends on the geometrical deviations and the specific type of the size used. The function of the workpiece determines the type of size that is used for a feature of size.

In ISO 14405-1, a differentiation is made between the following types of linear sizes:

- local size
- global size
- calculated size
- rank-order size (statistical value)

Local sizes are explained in further detail below.

Local sizes

The local size is defined in accordance with ISO 14405-1 as:

- two-point size (default or marked with the specification modifier (LP))
- spherical size (with specification modifier (LS))
- cross-section size (global size for a given cross-section of the measured feature)
- portion size (global size for a given portion of the measured feature)

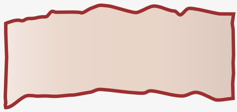
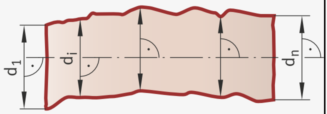
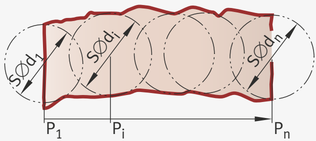
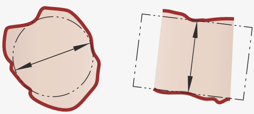
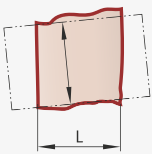
The local size of a measured cylinder and the local diameter of a measured cylinder are defined in accordance with ISO 17450-3:2016 as the distance between two opposing points on the feature, where:

- The connecting line between the points includes the associated circle centre.
- The cross-sections are perpendicular to the axis of the associated cylinder resulting from the measured face.

The local size of two parallel, measured faces is defined, in accordance with ISO 17450-3:2016, as the distance between two points on opposing, measured faces, where:

- The connecting lines of pairs of opposing points are perpendicular to the associated median plane.
- The associated median plane is the median plane of two associated parallel planes that are derived from the measured faces.

The table describes the types of local sizes in accordance with ISO 14405-1.

Representation	Definition in accordance with ISO 14405-1
	Measured feature under consideration that corresponds to the inner or outer feature and a cylinder or two parallel opposing planes
	Two-point size
	Spherical size determined from the diameter of the maximum inscribed sphere
	Cross-section size obtained from a direct global size with the criterion "maximum inscribed" (other criteria are possible)
	Portion size obtained from a direct global size with the criterion "maximum inscribed" Only a portion of the measured feature of length L is under consideration, other criteria are possible.

Legend

d Size	mm	P Position	
L Length of portion under consideration of cylinder	mm	S∅d Diameter of the maximum inscribed sphere.	mm

Specification modifiers for linear sizes
The type of linear size is indicated on the drawing by a specification modifier in accordance with ISO 14405-1, see tables Specification modifiers for linear sizes, Page 420, and Complementary specification modifiers for linear sizes, Page 421.

The specification modifiers comprise the type of size and the required type of evaluation of the size, see Figure 10, Page 422.

Specification modifiers for linear sizes	
Description	**Symbol**
Two-point size	(LP)
Local size defined by a sphere	(LS)
Least-squares association criterion	(GG)
Maximum inscribed feature association criterion	(GX)
Minimum circumscribed feature association criterion	(GN)
Minimax (Chebyshev) association criterion	(GC)
Circumference diameter (calculated size)	(CC)
Area diameter (calculated size)	(CA)
Volume diameter (calculated size)	(CV)
Maximum size[1)]	(SX)
Minimum size[1)]	(SN)
Average size[1)]	(SA)
Median size[1)]	(SM)
Mid-range size[1)]	(SD)
Range of sizes[1)]	(SR)
Standard deviation of sizes[1)]	(SQ)

Source: ISO 14405-1:2016.

1) Rank order sizes can be used to complement calculated or global portion sizes or local sizes.

The following table shows complementary specification modifiers for linear sizes.

Complementary specification modifiers for linear sizes		
Description	**Symbol**	**Example of drawing indication**
United feature of size	UF	UF 2× Ø30 ±0,2 (GN)
Envelope requirement	Ⓔ	30 ±0,2 Ⓔ
Any restricted portion of feature	/length	Ø30 ±0,2 (GG) /5
Any cross-section	ACS	Ø30 ±0,2 (GX) ACS
Specific fixed cross-section	SCS	30 ±0,2 (GX) SCS
Any longitudinal section	ALS	30 ±0,2 (GX) ALS
More than one feature	Quantity ×	4× 30 ±0,2 Ⓔ
Common toleranced feature of size	CT	4× 30 ±0,2 Ⓔ CT
Free-state condition	Ⓕ	Ø30 ±0,2 (LP) (SA) Ⓕ
Between	◄─►	Ø30 ±0,2 C◄─►D
Intersection plane[1]	◁ // | B	8 ±0,01 ALS ◁ ⊥ | A
Direction feature[1]	◄─ // | B	8 ±0,01 ALS ◄─ ⊥ | A
Flag note	⟨1⟩	30 ±0,2 ⟨1⟩

Source: ISO 14405-1:2016.

[1] Further information, see ISO 1101.

The following diagram shows the types of sizes and the system of specification modifiers.

Figure 10
Types of sizes – system of specification modifiers

① For linear sizes
② For angular sizes

Type of Size	(X	Y)	Type for Evaluation of Size
① local ② two-line	(L	P) S) G) C)	point spherical least-squares (Gauß) minimax (Chebyshev)
global	(G	G) X) N) C)	least-squares (Gauß) maximum inscribed minimum circumscribed minimax (Chebyshev)
calculated	(C	C) A) V)	circumference diameter area diameter volume diameter
statistical (rank-order)	(S	X) N) A) M) D) R) Q)	maximum minimum average median mid-range range standard deviation (quadratic sum)

Specification modifiers for angular sizes

The type of angular size is indicated on the drawing by a specification modifier in accordance with ISO 14405-3, see tables Specification modifiers for angular sizes, Page 423, and Complementary specification modifiers for angular sizes, Page 424.

The specification modifiers comprise the type of size and the required type of evaluation of the size, see Figure 10, Page 422.

Specification modifiers for angular sizes	
Description	**Symbol**
Two-line angular size with minimax association criterion	(LC)
Two-line angular size with least-squares association criterion	(LG)
Global angular size with least-squares association criterion	(GG)
Global angular size with minimax association criterion	(GC)
Maximum angular size[1)]	(SX)
Minimum angular size[1)]	(SN)
Average angular size[1)]	(SA)
Median angular size[1)]	(SM)
Mid-range angular size[1)]	(SD)
Range of angular sizes[1)]	(SR)
Standard deviation of angular sizes[1)] [2)]	(SQ)

Source: ISO 14405-3:2016.

[1)] The rank order angular size can be used in addition to the portion angular size, the global portion angular size or the local portion angular size.

[2)] Standard deviation from the root mean square.

The following table shows complementary specification modifiers for angular sizes.

Complementary specification modifiers for angular sizes			
Description	Symbol	Example of drawing indication	
		Prismatic angular feature of size	Revolute angular feature of size
Any restricted portion of angular feature of size	/linear distance	35° ±1°/15[1]	35° ±1°/15[1]
Any restricted portion of angular feature of size	/angular distance	Not applicable	35° ±1°/15°[1]
Specific fixed cross-section	SCS	45° ±2° SCS	Not applicable
More than one angular feature of size	Quantity ×	2× 45° ±2°	2× 45° ±2°
Common toleranced feature of size	CT	2× 45° ±2° CT	2× 45° ±2° CT
Free-state condition	Ⓕ[2]	35° ±1° Ⓕ	35° ±1° Ⓕ
Between	◄—►	35° ±1° A ◄—► B	35° ±1° A ◄—► B

Source: ISO 14405-3:2016.

[1] The specification modifier "/linear distance" applies to prismatic features of size and revolute features of size along the axis of the revolute feature.
The specification modifier "/angular distance" applies to a revolute feature of size.

[2] Further information: see ISO 10579.

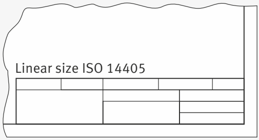
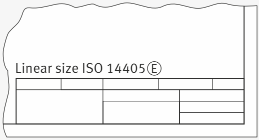
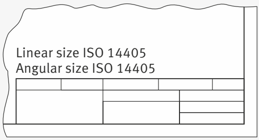
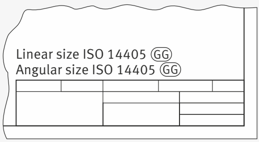
ISO GPS in technical drawings

In technical drawings, the new possibilities for tolerancing of sizes in accordance with ISO GPS unites the requirements in relation to function, production and inspection of a component.

Drawing-specific specification operator

The indication of an ISO GPS specification or the explicit indications "Linear size ISO 14405" or "Angular size ISO 14405" in or near the drawing title block shows the application of the GPS concept to the technical drawing. If no further specification modifier is indicated after the explicit indication, the drawing is based by default on the two-point size and the principle of independence.

The indication of a specification modifier changes the basis of the entire drawing.

Specification operator above the drawing title block	Principle for the entire drawing
	Default: Two-point size and principle of independence. There is no angular size on the drawing
	Example of a change: Change of default specification operator to envelope requirement. There is no angular size on the drawing
	Default specification operator for the angular size is the “two-line angular size” with minimax association criterion. The default for the linear size is the two-point size and the principle of independence
	Example of a change: Change of default specification operator to global linear sizes and global angular sizes with least-squares association criterion

ISO tolerances and fits

Functions and areas of application

Precise manufacturing and **precise measurement** of machine parts are the basis for interchangeable manufacture and the prerequisite for economical series and mass production in the entire field of technology. Systematic production and the easy repair of technical devices is only possible if the sizes of mutually interchangeable machine parts lie within certain limits and these parts can be assembled or replaced (without special fitting or adaptation work).

In order to achieve a particular joining characteristic or a particular fit tolerance zone of two machine parts (interference fit, transition fit or clearance fit; formerly: press seat, transition seat or clearance seat), the sizes and deviations of the components at the joint must conform to a particular tolerance and it must be possible to check these by means of appropriate measuring equipment.

ISO code system for tolerances on linear sizes

In order to fulfil all technical requirements to a substantial degree, the standard ISO 286-1:2010 was developed to include the "ISO code system for tolerances on linear sizes – Part 1: Basis of tolerances, deviations and fits" in accordance with ISO GPS. It is valid for nominal sizes up to 3150 mm and is graduated as far as appropriate in technical terms. ISO 286-2:2010 contains the associated tables of standard tolerance grades and limit deviations for holes and shafts.

The ISO system contains the following:

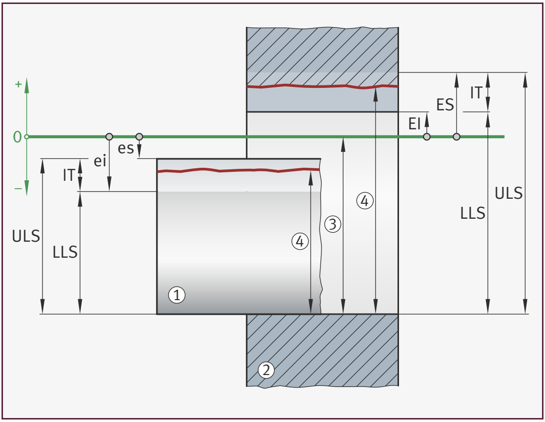
- ISO code system for tolerances on linear sizes:
  It contains principles and the associated terminology for the system and also provides a standardised selection of tolerance classes for general usage from the extensive possibilities of the system; see also ISO 286-1:2010.
- ISO fundamental deviation system:
  The letter defines the position of the tolerance zone in relation to the zero line; it is identified as a fundamental deviation identifier (table ISO fundamental deviations for external sizes and ISO fundamental deviations for internal sizes, Page 430 and Page 432; see also ISO 286-1:2010.
- ISO standard tolerance system:
  Definition of the dimensional tolerances that are indicated by means of ISO tolerance grades (table ISO standard tolerances, Page 436); see also ISO 286-2:2010.
- System of ISO tolerance classes and ISO tolerance zones:
  The short designation of the tolerance class comprises the letters for the fundamental deviation (fundamental deviation identifier: lower case letter for external sizes, upper case letter for internal sizes) and the tolerance grade (the number of the standard tolerance grade). When the magnitude of the tolerance class using the appropriate limit deviations is represented in graphical form, this is described as a tolerance zone (formerly: "tolerance field" or "tolerance interval") (graphical representation and calculation starting Page 427).
  The values for limit deviations for generally applied tolerance classes for holes and shafts are part of ISO 286-2 (table ISO tolerances for shafts, Page 438, and table ISO tolerances for holes, Page 442).

The terms in the ISO code system for tolerances on linear sizes (see also section Definition of terms, Page 405) are shown below by means of a clearance fit.

Figure 11
Dimensions, limit deviations and tolerances for ISO fits

es, ES = upper limit deviations
ei, EI = lower limit deviations
IT = standard tolerance
LLS = lower limit of size
ULS = upper limit of size

① Inner part (outer fit surface)
② Outer part (inner fit surface)
③ Nominal size
④ Local actual size

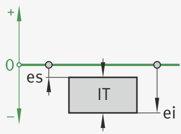
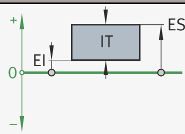
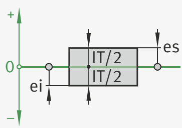
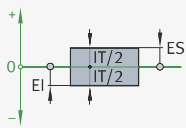


The terms "hole" and "shaft" refer in the ISO tolerance system of linear sizes not only to cylindrical fit surfaces but also to parallel fit surfaces of workpieces, for example the width of a slot or the thickness of a feather key.

Derivation of ISO tolerance classes or ISO tolerance zones

The ISO tolerance classes or the ISO tolerance zones (in diagrammatic form) are derived from ISO fundamental deviations and ISO standard tolerances. For this purpose, the ISO fundamental deviations for external sizes are taken from the table ISO fundamental deviations for external sizes, Page 430. In the case of internal sizes, the table ISO fundamental deviations for internal sizes, Page 432, is used.
The ISO fundamental deviations are the limit deviations closest to the zero line (smallest distances) taking account of the mathematical signs. The other limit deviation is produced by adding or subtracting the ISO standard tolerance IT, see table ISO standard tolerances, Page 436.

The following table shows the calculations of limit deviations for external and internal sizes in comparison with each other.

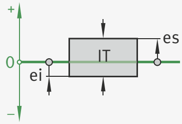
Limit deviations for external sizes (shafts)	Limit deviations for internal sizes (holes)
Position of tolerance zones a to h Beneath the zero line  Lower limit deviation ei = fundamental deviation – standard tolerance IT	**Position of tolerance zones A to H** Above the zero line  Upper limit deviation ES = fundamental deviation + standard tolerance IT
Example for fit size **25 d13**:	Example for fit size **420 C10**:
Upper limit deviation es Table Page 430 = Fundamental deviation = −65 μm	Lower limit deviation EI Table Page 432 = Fundamental deviation = +440 μm
Standard tolerance for tolerance grade 13 Table Page 436 = 330 μm	Standard tolerance for tolerance grade 10 Table Page 436 = 250 μm
Lower limit deviation ei = −65 μm − 330 μm = −395 μm	Upper limit deviation ES = +440 μm + 250 μm = +690 μm
Therefore: 25 d13 = $25^{-0,065}_{-0,395}$	Therefore: 420 C10 = $420^{+0,690}_{+0,440}$
Position of tolerance zone js Symmetrical on both sides of the zero line 	**Position of tolerance zone JS** Symmetrical on both sides of the zero line 
Example for fit size **25 js8**:	Example for fit size **200 JS9**:
Standard tolerance for tolerance grade 8 Table Page 436 = 33 μm	Standard tolerance for tolerance grade 9 Table Page 436 = 115 μm
Upper limit deviation es = +IT/2 = +33 μm/2 = +16,5 μm	Upper limit deviation ES = +IT/2 = +115 μm/2 = +57,5 μm
Lower limit deviation ei = −IT/2 = −33 μm/2 = −16,5 μm	Lower limit deviation EI = −IT/2 = −115 μm/2 = −57,5 μm
Therefore: 25 js8 = $25\pm0{,}0165$	Therefore: 200 JS9 = $200\pm0{,}0572$
From the position of the tolerance zone j, the fundamental deviations can also change with the tolerance grade. When determining the fundamental deviation, it is therefore necessary to observe not only the position of the tolerance zone but also the tolerance grade (table Page 430).	From the position of the tolerance zone J, the fundamental deviations can also change with the tolerance grade. When determining the fundamental deviation, it is therefore necessary to observe not only the position of the tolerance zone but also the tolerance grade (table Page 432).

Continuation of table, see Page 429.

Continuation of table, Calculations of limit deviations for external sizes and internal sizes, from Page 428.

Limit deviations for external sizes (shafts)

Position of tolerance zone j
Approximately symmetrical on both sides of the zero line



Upper limit deviation es
= fundamental deviation + standard tolerance IT

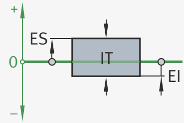
Example for fit size **25 j6**:

Lower limit deviation ei Table Page 430	= Fundamental deviation = −4 μm
Standard tolerance for tolerance grade 6 Table Page 436	= 13 μm
Upper limit deviation es = −4 μm + 13 μm	= +9 μm
Therefore:	25 j6 = $25^{+0{,}009}_{-0{,}004}$

In the position of tolerance zone j, the fundamental deviation table always indicates the lower limit deviation ei.

Limit deviations for internal sizes (holes)

Position of tolerance zone J
Approximately symmetrical on both sides of the zero line



Lower limit deviation EI
= fundamental deviation – standard tolerance IT

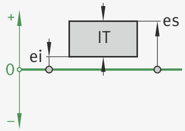
Example for fit size **125 J7**:

Upper limit deviation ES Table Page 432	= Fundamental deviation = +26 μm
Standard tolerance for tolerance grade 7 Table Page 436	= 40 μm
Lower limit deviation EI = +26 μm – 40 μm	= −14 μm
Therefore:	125 J7 = $125^{+0{,}026}_{-0{,}014}$

In the position of tolerance zone J, the fundamental deviation table always indicates the upper limit deviation ES.

Limit deviations for external sizes (shafts)

Position of tolerance zones k to zc
Above the zero line



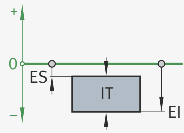
Upper limit deviation es
= fundamental deviation + standard tolerance IT

Example for fit size **25 p6**:

Lower limit deviation ei Table Page 430	= Fundamental deviation = +22 μm
Standard tolerance for tolerance grade 6 Table Page 436	= 13 μm
Upper limit deviation es = 22 μm + 13 μm	= +35 μm
Therefore:	25 p6 = $25^{+0{,}035}_{+0{,}022}$

Limit deviations for internal sizes (holes)

Position of tolerance zones K to ZC
Predominantly beneath the zero line



Lower limit deviation EI
= fundamental deviation – standard tolerance IT

Example for fit size **125 T10**:

Upper limit deviation ES Table Page 432	= Fundamental deviation = −122 μm
Standard tolerance for tolerance grade 10 Table Page 436	= 160 μm
Lower limit deviation EI = −122 μm −160 μm	= −282 μm
Therefore:	125 T10 = $125^{-0{,}122}_{-0{,}282}$

ISO fundamental deviations for external sizes

The following table shows the values for ISO fundamental deviations (minimum distances) for external sizes (shafts):

Tolerance classes[2)]		Nominal size range											
		mm											
Fundamental deviation identifier	Tolerance grade IT	over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to ISO 286-1 µm										
a	All tolerance grades	–	270	270	280	290	290	300	300	310	320	340	360
b		–	140	140	150	150	150	160	160	170	180	190	200
c		–	60	70	80	95	95	110	110	120	130	140	150
d		–	20	30	40	50	50	65	65	80	80	100	100
e		–	14	20	25	32	32	40	40	50	50	60	60
f		–	6	10	13	16	16	20	20	25	25	30	30
g		–	2	4	5	6	6	7	7	9	9	10	10
h		–	0	0	0	0	0	0	0	0	0	0	0
j[1)]	5 + 6	–	2	2	2	3	3	4	4	5	5	7	7
j[1)]	7	–	4	4	5	6	6	8	8	10	10	12	12
js	All grades	The limit deviations amount to ±1/2 IT in the relevant tolerance grade											
k	4 – 7	+	0	1	1	1	1	2	2	2	2	2	2
k	up to 3, from 8	+	0	0	0	0	0	0	0	0	0	0	0
m	All tolerance grades	+	2	4	6	7	7	8	8	9	9	11	11
n		+	4	8	10	12	12	15	15	17	17	20	20
p		+	6	12	15	18	18	22	22	26	26	32	32
r		+	10	15	19	23	23	28	28	34	34	41	43
s		+	14	19	23	28	28	35	35	43	43	53	59
t		+	–	–	–	–	–	–	41	48	54	66	75
u		+	18	23	28	33	33	41	48	60	70	87	102
v		+	–	–	–	–	39	47	55	68	81	102	120
x		+	20	28	34	40	45	54	64	80	97	122	146
y		+	–	–	–	–	–	63	75	94	114	144	174
z		+	26	35	42	50	60	73	88	112	136	172	210
za		+	32	42	52	64	77	98	118	148	180	226	274
zb		+	40	50	67	90	108	136	160	200	242	300	360
zc		+	60	80	97	130	150	188	218	274	325	405	480

1) In the case of the fundamental deviation identifier j, the table always indicates the lower limit deviation as the fundamental deviation.

2) The special tolerance classes with the fundamental deviation identifiers cd, ef and fg for clockmaking and precision engineering are not given here.

80	100	120	140	160	180	200	225	250	280	315	355	400	450
100	120	140	160	180	200	225	250	280	315	355	400	450	500
380	410	460	520	580	660	740	820	920	1050	1200	1350	1500	1650
220	240	260	280	310	340	380	420	480	540	600	680	760	840
170	180	200	210	230	240	260	280	300	330	360	400	440	460
120	120	145	145	145	170	170	170	190	190	210	210	230	230
72	72	85	85	85	100	100	100	110	110	125	125	135	135
36	36	43	43	43	50	50	50	56	56	62	62	68	68
12	12	14	14	14	15	15	15	17	17	18	18	20	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	9	11	11	11	13	13	13	16	16	18	18	20	20
15	15	18	18	18	21	21	21	26	26	28	28	32	32
3	3	3	3	3	4	4	4	4	4	4	4	5	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	13	15	15	15	17	17	17	20	20	21	21	23	23
23	23	27	27	27	31	31	31	34	34	37	37	40	40
37	37	43	43	43	50	50	50	56	56	62	62	68	68
51	54	63	65	68	77	80	84	94	98	108	114	126	132
71	79	92	100	108	122	130	140	158	170	190	208	232	252
91	104	122	134	146	166	180	196	218	240	268	294	330	360
124	144	170	190	210	236	258	284	315	350	390	435	490	540
146	172	202	228	252	284	310	340	385	425	475	530	595	660
178	210	248	280	310	350	385	425	475	525	590	660	740	820
214	254	300	340	380	425	470	520	580	650	730	820	920	1000
258	310	365	415	465	520	575	640	710	790	900	1000	1100	1250
335	400	470	535	600	670	740	820	920	1000	1150	1300	1450	1600
445	525	620	700	780	880	960	1050	1200	1300	1500	1650	1850	2100
585	690	800	900	1000	1150	1250	1350	1550	1700	1900	2100	2400	2600

ISO fundamental deviations for internal sizes

The following table shows the values for ISO fundamental deviations (minimum distances) for internal sizes (holes):

Tolerance classes[2)] Fundamental deviation identifier	Tolerance grade IT	Nominal size range mm											
		over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to ISO 286-1 µm										
A	All tolerance grades	+	270	270	280	290	290	300	300	310	320	340	360
B		+	140	140	150	150	150	160	160	170	180	190	200
C		+	60	70	80	95	95	110	110	120	130	140	150
D		+	20	30	40	50	50	65	65	80	80	100	100
E		+	14	20	25	32	32	40	40	50	50	60	60
F		+	6	10	13	16	16	20	20	25	25	30	30
G		+	2	4	5	6	6	7	7	9	9	10	10
H			0	0	0	0	0	0	0	0	0	0	0
J[1)]	6	+	2	5	5	6	6	8	8	10	10	13	13
J[1)]	7	+	4	6	8	10	10	12	12	14	14	18	18
J[1)]	8	+	6	10	12	15	15	20	20	24	24	28	28
JS	All grades	The limit deviations amount to ±1/2 IT in the relevant tolerance grade											
K	5	+	0	0	1	2	2	1	1	2	2	3	3
K	6	+	0	2	2	2	2	2	2	3	3	4	4
K	7	+	0	3	5	6	6	6	6	7	7	9	9
K	8	+	0	5	6	8	8	10	10	12	12	14	14
M	6	–	2	1	3	4	4	4	4	4	4	5	5
M	7	–	2	0	0	0	0	0	0	0	0	0	0
M	8		–2	+2	+1	+2	+2	+4	+4	+5	+5	+5	+5
M	9	–	2	4	6	7	7	8	8	9	9	11	11
N	6	–	4	5	7	9	9	11	11	12	12	14	14
N	7	–	4	4	4	5	5	7	7	8	8	9	9
N	8	–	4	2	3	3	3	3	3	3	3	4	4
N	9	–	4	0	0	0	0	0	0	0	0	0	0
P	6	–	6	9	12	15	15	18	18	21	21	26	26
R		–	10	12	16	20	20	24	24	29	29	35	37
S		–	14	16	20	25	25	31	31	38	38	47	53
T		–	–	–	–	–	–	–	37	43	49	60	69

Continuation of table, see Page 434.

1) In the case of the fundamental deviation identifier J, the table always indicates the upper limit deviation as the fundamental deviation.

2) The special tolerance classes with the fundamental deviation identifiers CD, EF and FG for clockmaking and precision engineering are not given here.

80	100	120	140	160	180	200	225	250	280	315	355	400	450
100	120	140	160	180	200	225	250	280	315	355	400	450	500
380	410	460	520	580	660	740	820	920	1050	1200	1350	1500	1650
220	240	260	280	310	340	380	420	480	540	600	680	760	840
170	180	200	210	230	240	260	280	300	330	360	400	440	480
120	120	145	145	145	170	170	170	190	190	210	210	230	230
72	72	85	85	85	100	100	100	110	110	125	125	135	135
36	36	43	43	43	50	50	50	56	56	62	62	68	68
12	12	14	14	14	15	15	15	17	17	18	18	20	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	16	18	18	18	22	22	22	25	25	29	29	33	33
22	22	26	26	26	30	30	30	36	36	39	39	43	43
34	34	41	41	41	47	47	47	55	55	60	60	66	66
2	2	3	3	3	2	2	2	3	3	3	3	2	2
4	4	4	4	4	5	5	5	5	5	7	7	8	8
10	10	12	12	12	13	13	13	16	16	17	17	18	18
16	16	20	20	20	22	22	22	25	25	28	28	29	29
6	6	8	8	8	8	8	8	9	9	10	10	10	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+6	+6	+8	+8	+8	+9	+9	+9	+9	+9	+11	+11	+11	+11
13	13	15	15	15	17	17	17	20	20	21	21	23	23
16	16	20	20	20	22	22	22	25	25	26	26	27	27
10	10	12	12	12	14	14	14	14	14	16	16	17	17
4	4	4	4	4	5	5	5	5	5	5	5	6	6
0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	30	36	36	36	41	41	41	47	47	51	51	55	55
44	47	56	58	61	68	71	75	85	89	97	103	113	119
64	72	85	93	101	113	121	131	149	161	179	197	219	239
84	97	115	127	139	157	171	187	209	231	257	283	317	347

Continuation of table, ISO fundamental deviations for internal sizes, from Page 432.

Tolerance classes: Fundamental deviation identifier	Tolerance grade IT	Nominal size range mm over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to ISO 286-1 μm										
U	6	–	18	20	25	30	30	37	44	55	65	81	96
V		–	–	–	–	–	36	43	51	63	76	96	114
X		–	20	25	31	37	42	50	60	75	92	116	140
Y		–	–	–	–	–	–	59	71	89	109	138	168
Z		–	26	32	39	47	57	69	84	107	131	166	204
ZA		–	32	39	49	61	74	94	114	143	175	220	268
ZB		–	40	47	64	87	105	132	156	195	237	294	354
ZC		–	60	77	94	127	147	184	214	269	320	399	474
P	7	–	6	8	9	11	11	14	14	17	17	21	21
R		–	10	11	13	16	16	20	20	25	25	30	32
S		–	14	15	17	21	21	27	27	34	34	42	48
T		–	–	–	–	–	–	–	33	39	45	55	64
U		–	18	19	22	26	26	33	40	51	61	76	91
V		–	–	–	–	–	32	39	47	59	72	91	109
X		–	20	24	28	33	38	46	56	71	88	111	135
Y		–	–	–	–	–	–	55	67	85	105	133	163
Z		–	26	31	36	43	53	65	80	103	127	161	199
ZA		–	32	38	46	57	70	90	110	139	171	215	263
ZB		–	40	46	61	83	101	128	152	191	233	289	349
ZC		–	60	76	91	123	143	180	210	265	316	394	469
P	from 8	–	6	12	15	18	18	22	22	26	26	32	32
R		–	10	15	19	23	23	28	28	34	34	41	43
S		–	14	19	23	28	28	35	35	43	43	53	59
T		–	–	–	–	–	–	–	41	48	54	66	75
U		–	18	23	28	33	33	41	48	60	70	87	102
V		–	–	–	–	–	39	47	55	68	81	102	120
X		–	20	28	34	40	45	54	64	80	97	122	146
Y		–	–	–	–	–	–	63	75	94	114	144	174
Z		–	26	35	42	50	60	73	88	112	136	172	210
ZA		–	32	42	52	64	77	98	118	148	180	226	274
ZB		–	40	50	67	90	108	136	160	200	242	300	360
ZC		–	60	80	97	130	150	188	218	274	325	405	480

80	100	120	140	160	180	200	225	250	280	315	355	400	450
100	120	140	160	180	200	225	250	280	315	355	400	450	500
117	137	163	183	203	227	249	275	306	341	379	424	477	527
139	165	195	221	245	275	301	331	376	416	464	519	582	647
171	203	241	273	303	341	376	416	466	516	579	649	727	807
207	247	293	333	373	416	461	511	571	641	719	809	907	987
251	303	358	406	458	511	566	631	701	781	889	989	1087	1237
328	393	463	528	593	661	731	811	911	991	1139	1289	1437	1587
438	518	613	693	773	871	951	1041	1191	1291	1489	1639	1837	2087
578	683	793	893	993	1141	1241	1341	1541	1691	1889	2089	2387	2587
24	24	28	28	28	33	33	33	36	36	41	41	45	45
38	41	48	50	53	60	63	67	74	78	87	93	103	109
58	66	77	85	93	105	113	123	138	150	169	187	209	229
78	91	107	119	131	149	163	179	198	220	247	273	307	337
111	131	155	175	195	219	241	267	295	330	369	414	467	517
133	159	187	213	237	267	293	323	365	405	454	509	572	637
165	197	233	265	295	333	368	408	455	505	569	639	717	797
201	241	285	325	365	408	453	503	560	630	709	799	897	977
245	297	350	400	450	503	558	623	690	770	879	979	1077	1227
322	387	455	520	585	653	723	803	900	980	1129	1279	1427	1577
432	512	605	685	765	863	943	1033	1180	1280	1479	1629	1827	2077
572	677	785	885	985	1133	1233	1333	1530	1680	1879	2079	2377	2577
37	37	43	43	43	50	50	50	56	56	62	62	68	68
51	54	63	65	68	77	80	84	94	98	108	114	126	132
71	79	92	100	108	122	130	140	158	170	190	208	232	252
91	104	122	134	146	166	180	196	218	240	268	294	330	360
124	144	170	190	210	236	258	284	315	350	390	435	490	540
146	172	202	228	252	284	310	340	385	425	475	530	595	660
178	210	248	280	310	350	385	425	475	525	590	660	740	820
214	254	300	340	380	425	470	520	580	650	730	820	920	1000
258	310	365	415	465	520	575	640	710	790	900	1000	1100	1250
335	400	470	535	600	670	740	820	920	1000	1150	1300	1450	1600
445	525	620	700	780	880	960	1050	1200	1300	1500	1650	1850	2100
585	690	800	900	1000	1150	1250	1350	1550	1700	1900	2100	2400	2600

ISO standard tolerances

The following table shows the values for ISO standard tolerances:

Standard tolerance grade	Nominal size range mm						
	over –	3	6	10	18	30	50
	incl. 3	6	10	18	30	50	80
	ISO standard tolerances IT to ISO 286-1 µm						
IT01	0,3	0,4	0,4	0,5	0,6	0,6	0,8
IT0	0,5	0,6	0,6	0,8	1	1	1,2
IT1	0,8	1	1	1,2	1,5	1,5	2
IT2	1,2	1,5	1,5	2	2,5	2,5	3
IT3	2	2,5	2,5	3	4	4	5
IT4	3	4	4	5	6	7	8
IT5	4	5	6	8	9	11	13
IT6	6	8	9	11	13	16	19
IT7	10	12	15	18	21	25	30
IT8	14	18	22	27	33	39	46
IT9	25	30	36	43	52	62	74
IT10	40	48	58	70	84	100	120
IT11	60	75	90	110	130	160	190
IT12	100	120	150	180	210	250	300
IT13	140	180	220	270	330	390	460
IT14	250	300	360	430	520	620	740
IT15	400	480	580	700	840	1000	1200
IT16	600	750	900	1100	1300	1600	1900
IT17	1000	1200	1500	1800	2100	2500	3000
IT18	1400	1800	2200	2700	3300	3900	4600

Continuation of table, see Page 437.

Continuation of table, ISO standard tolerances, from Page 436.

Standard tolerance grade	Nominal size range mm					
	over 80 incl. 120	120 180	180 250	250 315	315 400	400 500
	ISO standard tolerances IT to ISO 286-1 µm					
IT01	1	1,2	2	2,5	3	4
IT0	1,5	2	3	4	5	6
IT1	2,5	3,5	4,5	6	7	8
IT2	4	5	7	8	9	10
IT3	6	8	10	12	13	15
IT4	10	12	14	16	18	20
IT5	15	18	20	23	25	27
IT6	22	25	29	32	36	40
IT7	35	40	46	52	57	63
IT8	54	63	72	81	89	97
IT9	87	100	115	130	140	155
IT10	140	160	185	210	230	250
IT11	220	250	290	320	360	400
IT12	350	400	460	520	570	630
IT13	540	630	720	810	890	970
IT14	870	1 000	1 150	1 300	1 400	1 550
IT15	1 400	1 600	1 850	2 100	2 300	2 500
IT16	2 200	2 500	2 900	3 200	3 600	4 000
IT17	3 500	4 000	4 600	5 200	5 700	6 300
IT18	5 400	6 300	7 200	8 100	8 900	9 700

ISO tolerances for shafts The following table shows a selection of ISO tolerances for shafts and their corresponding limit deviations:

Tolerance classes	Nominal size range mm											
	over 1 incl. 3	3 6	6 10	10 18	18 30	30 40	40 50	50 65	65 80	80 100	100 120	120 140
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = es Lower limit deviation = ei					
a12	−270	−270	−280	−290	−300	−310	−320	−340	−360	−380	−410	−460
	−370	−390	−430	−470	−510	−560	−570	−640	−660	−730	−760	−860
a13	−270	−270	−280	−290	−300	−310	−320	−340	−360	−380	−410	−460
	−410	−450	−500	−560	−630	−700	−710	−800	−820	−920	−950	−1090
c12	−60	−70	−80	−95	−110	−120	−130	−140	−150	−170	−180	−200
	−160	−190	−230	−275	−320	−370	−380	−440	−450	−520	−530	−600
d6	−20	−30	−40	−50	−65	−80	−80	−100	−100	−120	−120	−145
	−26	−38	−49	−61	−78	−96	−96	−119	−119	−142	−142	−170
e6	−14	−20	−25	−32	−40	−50	−50	−60	−60	−72	−72	−85
	−20	−28	−34	−43	−53	−66	−66	−79	−79	−94	−94	−110
e7	−14	−20	−25	−32	−40	−50	−50	−60	−60	−72	−72	−85
	−24	−32	−40	−50	−61	−75	−75	−90	−90	−107	−107	−125
e8	−14	−20	−25	−32	−40	−50	−50	−60	−60	−72	−72	−85
	−28	−38	−47	−59	−73	−89	−89	−106	−106	−126	−126	−148
f5	−6	−10	−13	−16	−20	−25	−25	−30	−30	−36	−36	−43
	−10	−15	−19	−24	−29	−36	−36	−43	−43	−51	−51	−61
f6	−6	−10	−13	−16	−20	−25	−25	−30	−30	−36	−36	−43
	−12	−18	−22	−27	−33	−41	−41	−49	−49	−58	−58	−68
f7	−6	−10	−13	−16	−20	−25	−25	−30	−30	−36	−36	−43
	−16	−22	−28	−34	−41	−50	−50	−60	−60	−71	−71	−83
g5	−2	−4	−5	−6	−7	−9	−9	−10	−10	−12	−12	−14
	−6	−9	−11	−14	−16	−20	−20	−23	−23	−27	−27	−32
g6	−2	−4	−5	−6	−7	−9	−9	−10	−10	−12	−12	−14
	−8	−12	−14	−17	−20	−25	−25	−29	−29	−34	−34	−39
g7	−2	−4	−5	−6	−7	−9	−9	−10	−10	−12	−12	−14
	−12	−16	−20	−24	−28	−34	−34	−40	−40	−47	−47	−54
h5	0	0	0	0	0	0	0	0	0	0	0	0
	−4	−5	−6	−8	−9	−11	−11	−13	−13	−15	−15	−18
h6	0	0	0	0	0	0	0	0	0	0	0	0
	−6	−8	−9	−11	−13	−16	−16	−19	−19	−22	−22	−25
h7	0	0	0	0	0	0	0	0	0	0	0	0
	−10	−12	−15	−18	−21	−25	−25	−30	−30	−35	−35	−40
h8	0	0	0	0	0	0	0	0	0	0	0	0
	−14	−18	−22	−27	−33	−39	−39	−46	−46	−54	−54	−63
h9	0	0	0	0	0	0	0	0	0	0	0	0
	−25	−30	−36	−43	−52	−62	−62	−74	−74	−87	−87	−100
h10	0	0	0	0	0	0	0	0	0	0	0	0
	−40	−48	−58	−70	−84	−100	−100	−120	−120	−140	−140	−160

Continuation of table, see Page 440.

140 160	160 180	180 200	200 225	225 250	250 280	280 315	315 355	355 400	400 450	450 500
−520 −920	−580 −980	−660 −1120	−740 −1 200	−820 −1 280	−920 −1440	−1 050 1 570	−1 200 −1770	−1350 −1920	−1 500 −2 130	−1650 −2 280
−520 −1 150	−580 −1 210	−660 −1380	−740 −1460	−820 −1 540	−920 −1730	−1050 −1860	−1200 −2 090	−1350 −2 240	−1500 −2 470	−1650 −2 620
−210 −610	−230 −630	−240 −700	−260 −720	−280 −740	−300 −820	−330 −850	−360 −930	−400 −970	−440 −1 070	−480 −1 110
−145 −170	−145 −170	−170 −199	−170 −199	−170 −199	−190 −222	−190 −222	−210 −246	−210 −246	−230 −270	−230 −270
−85 −110	−85 −110	−100 −129	−100 −129	−100 −129	−110 −142	−110 −142	−125 −161	−125 −161	−135 −175	−135 −175
−85 −125	−85 −125	−100 −146	−100 −146	−100 −146	−110 −162	−110 −162	−125 −182	−125 −182	−135 −198	−135 −198
−85 −148	−85 −148	−100 −172	−100 −172	−100 −172	−110 −191	−110 −191	−125 −214	−125 −214	−135 −232	−135 −232
−43 −61	−43 −61	−50 −70	−50 −70	−50 −70	−56 −79	−56 −79	−62 −87	−62 −87	−68 −95	−68 −95
−43 −68	−43 −68	−50 −79	−50 −79	−50 −79	−56 −88	−56 −88	−62 −98	−62 −98	−68 −108	−68 −108
−43 −83	−43 −83	−50 −96	−50 −96	−50 −96	−56 −108	−56 −108	−62 −119	−62 −119	−68 −131	−68 −131
−14 −32	−14 −32	−15 −35	−15 −35	−15 −35	−17 −40	−17 −40	−18 −43	−18 −43	−20 −47	−20 −47
−14 −39	−14 −39	−15 −44	−15 −44	−15 −44	−17 −49	−17 −49	−18 −54	−18 −54	−20 −60	−20 −60
−14 −54	−14 −54	−15 −61	−15 −61	−15 −61	−17 −69	−17 −69	−18 −75	−18 −75	−20 −83	−20 −83
0 −18	0 −18	0 −20	0 −20	0 −20	0 −23	0 −23	0 −25	0 −25	0 −27	0 −27
0 −25	0 −25	0 −29	0 −29	0 −29	0 −32	0 −32	0 −36	0 −36	0 −40	0 −40
0 −40	0 −40	0 −46	0 −46	0 −46	0 −52	0 −52	0 −57	0 −57	0 −63	0 −63
0 −63	0 −63	0 −72	0 −72	0 −72	0 −81	0 −81	0 −89	0 −89	0 −97	0 −97
0 −100	0 −100	0 −115	0 −115	0 −115	0 −130	0 −130	0 −140	0 −140	0 −155	0 −155
0 −160	0 −160	0 −185	0 −185	0 −185	0 −210	0 −210	0 −230	0 −230	0 −250	0 −250

Continuation of table, ISO tolerances for shafts, from Page 438.

Tolerance classes	Nominal size range mm											
	over 1 incl. 3	3 6	6 10	10 18	18 30	30 40	40 50	50 65	65 80	80 100	100 120	120 140
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = es Lower limit deviation = ei					
h11	0	0	0	0	0	0	0	0	0	0	0	0
	−60	−75	−90	−110	−130	−160	−160	−190	−190	−220	−220	−250
h13	−0	0	0	0	0	0	0	0	0	0	0	0
	−140	−180	−220	−270	−330	−390	−390	−460	−460	−540	−540	−630
j5	+2	+3	+4	+5	+5	+6	+6	+6	+6	+6	+6	+7
	−2	−2	−2	−3	−4	−5	−5	−7	−7	−9	−9	−11
j6	+4	+6	+7	+8	+9	+11	+11	+12	+12	+13	+13	+14
	−2	−2	−2	−3	−4	−5	−5	−7	−7	−9	−9	−11
j7	+6	+8	+10	+12	+13	+15	+15	+18	+18	+20	+20	+22
	−4	−4	−5	−6	−8	−10	−10	−12	−12	−15	−15	−18
js5	+2	+2,5	+3	+4	+4,5	+5,5	+5,5	+6,5	+6,5	+7,5	+7,5	+9
	−2	−2,5	−3	−4	−4,5	−5,5	−5,5	−6,5	−6,5	−7,5	−7,5	−9
js6	+3	+4	+4,5	+5,5	+6,5	+8	+8	+9,5	+9,5	+11	+11	+12,5
	−3	−4	−4,5	−5,5	−6,5	−8	−8	−9,5	−9,5	−11	−11	−12,5
js7	+5	+6	+7,5	+9	+10,5	+12,5	+12,5	+15	+15	+17,5	+17,5	+20
	−5	−6	−7,5	−9	−10,5	−12,5	−12,5	−15	−15	−17,5	−17,5	−20
k5	+4	+6	+7	+9	+11	+13	+13	+15	+15	+18	+18	+21
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
k6	+6	+9	+10	+12	+15	+18	+18	+21	+21	+25	+25	+28
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
k7	+10	+13	+16	+19	+23	+27	+27	+32	+32	+38	+38	+43
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
m5	+6	+9	+12	+15	+17	+20	+20	+24	+24	+28	+28	+33
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
m6	+8	+12	+15	+18	+21	+25	+25	+30	+30	+35	+35	+40
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
m7	+12	+16	+21	+25	+29	+34	+34	+41	+41	+48	+48	+55
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
n5	+8	+13	+16	+20	+24	+28	+28	+33	+33	+38	+38	+45
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
n6	+10	+16	+19	+23	+28	+33	+33	+39	+39	+45	+45	+52
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
n7	+14	+20	+25	+30	+36	+42	+42	+50	+50	+58	+58	+67
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
p5	+10	+17	+21	+26	+31	+37	+37	+45	+45	+52	+52	+61
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43
p6	+12	+20	+24	+29	+35	+42	+42	+51	+51	+59	+59	+68
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43
p7	+16	+24	+30	+36	+43	+51	+51	+62	+62	+72	+72	+83
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43

140	160	180	200	225	250	280	315	355	400	450
160	180	200	225	250	280	315	355	400	450	500
0	0	0	0	0	0	0	0	0	0	0
−250	−250	−290	−290	−290	−320	−320	−360	−360	−400	−400
0	0	0	0	0	0	0	0	0	0	0
−630	−630	−720	−720	−720	−810	−810	−890	−890	−970	−970
+7	+7	+7	+7	+7	+7	+7	+7	+7	+7	+7
−11	−11	−13	−13	−13	−16	−16	−18	−18	−20	−20
+14	+14	+16	+16	+16	+16	+16	+18	+18	+20	+20
−11	−11	−13	−13	−13	−16	−16	−18	−18	−20	−20
+22	+22	+25	+25	+25	+26	+26	+29	+29	+31	+31
−18	−18	−21	−21	−21	−26	−26	−28	−28	−32	−32
+9	+9	+10	+10	+10	+11,5	+11,5	+12,5	+12,5	+13,5	+13,5
−9	−9	−10	−10	−10	−11,5	−11,5	−12,5	−12,5	−13,5	−13,5
+12,5	+12,5	+14,5	+14,5	+14,5	+16	+16	+18	+18	+20	+20
−12,5	−12,5	−14,5	−14,5	−14,5	−16	−16	−18	−18	−20	−20
+20	+20	+23	+23	+23	+26	+26	+28,5	+28,5	+31,5	+31,5
−20	−20	−23	−23	−23	−26	−26	−28,5	−28,5	−31,5	−31,5
+21	+21	+24	+24	+24	+27	+27	+29	+29	+32	+32
+3	+3	+4	+4	+4	+4	+4	+4	+4	+5	+5
+28	+28	+33	+33	+33	+36	+36	+40	+40	+45	+45
+3	+3	+4	+4	+4	+4	+4	+4	+4	+5	+5
+43	+43	+50	+50	+50	+56	+56	+61	+61	+68	+68
+3	+3	+4	+4	+4	+4	+4	+4	+4	+5	+5
+33	+33	+37	+37	+37	+43	+43	+46	+46	+50	+50
+15	+15	+17	+17	+17	+20	+20	+21	+21	+23	+23
+40	+40	+46	+46	+46	+52	+52	+57	+57	+63	+63
+15	+15	+17	+17	+17	+20	+20	+21	+21	+23	+23
+55	+55	+63	+63	+63	+72	+72	+78	+78	+86	+86
+15	+15	+17	+17	+17	+20	+20	+21	+21	+23	+23
+45	+45	+51	+51	+51	+57	+57	+62	+62	+67	+67
+27	+27	+31	+31	+31	+34	+34	+37	+37	+40	+40
+52	+52	+60	+60	+60	+66	+66	+73	+73	+80	+80
+27	+27	+31	+31	+31	+34	+34	+37	+37	+40	+40
+67	+67	+77	+77	+77	+86	+86	+94	+94	+103	+103
+27	+27	+31	+31	+31	+34	+34	+37	+37	+40	+40
+61	+61	+70	+70	+70	+79	+79	+87	+87	+95	+95
+43	+43	+50	+50	+50	+56	+56	+62	+62	+68	+68
+68	+68	+79	+79	+79	+88	+ 88	+98	+98	+108	+108
+43	+43	+50	+50	+50	+56	+56	+62	+62	+68	+68
+83	+83	+96	+96	+96	+108	+108	+119	+119	+131	+131
+43	+43	+50	+50	+50	+56	+56	+62	+62	+68	+68

ISO tolerances for holes The following table shows a selection of ISO tolerances for holes and their corresponding limit deviations:

Tolerance classes	Nominal size range mm													
	over 3	6	10	18	30	40	50	65	80	100	120	140	160	180
	incl. 6	10	18	30	40	50	65	80	100	120	140	160	180	200
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = ES Lower limit deviation = EI							
A11	+345	+370	+400	+430	+470	+480	+530	+550	+600	+630	+710	+770	+830	+950
	+270	+280	+290	+300	+310	+320	+340	+360	+380	+410	+460	+520	+580	+660
C11	+145	+170	+205	+240	+280	+290	+330	+340	+390	+400	+450	+460	+480	+530
	+70	+80	+95	+110	+120	+130	+140	+150	+170	+180	+200	+210	+230	+240
D10	+78	+98	+120	+149	+180	+180	+220	+220	+260	+260	+305	+305	+305	+355
	+30	+40	+50	+65	+80	+80	+100	+100	+120	+120	+145	+145	+145	+170
E6	+28	+34	+43	+53	+66	+66	+79	+79	+94	+ 94	+110	+110	+110	+129
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
E7	+32	+40	+50	+61	+75	+75	+90	+90	+107	+107	+125	+125	+125	+146
	+20	+25	+32	+40	+50	+ 50	+60	+60	+72	+72	+85	+85	+85	+100
E9	+50	+61	+75	+92	+112	+112	+134	+134	+159	+159	+185	+185	+185	+215
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
E10	+68	+83	+102	+124	+150	+150	+180	+180	+212	+212	+245	+245	+245	+285
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
F6	+18	+22	+27	+33	+41	+41	+49	+49	+58	+58	+68	+68	+68	+79
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
F7	+22	+28	+34	+41	+50	+50	+60	+60	+71	+71	+83	+83	+83	+96
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
F8	+28	+35	+43	+53	+64	+64	+76	+76	+90	+90	+106	+106	+106	+122
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
G6	+12	+14	+17	+20	+25	+25	+29	+29	+34	+34	+39	+39	+39	+44
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
G7	+16	+20	+24	+28	+34	+34	+40	+40	+47	+47	+54	+54	+54	+61
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
G8	+22	+27	+33	+40	+48	+48	+56	+56	+66	+66	+77	+77	+77	+87
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
H6	+8	+9	+11	+13	+16	+16	+19	+19	+22	+22	+25	+25	+25	+29
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H7	+12	+15	+18	+21	+25	+25	+30	+30	+35	+35	+40	+40	+40	+46
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H8	+18	+22	+27	+33	+39	+39	+46	+46	+54	+54	+63	+63	+63	+72
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H9	+30	+36	+43	+52	+62	+62	+74	+74	+87	+87	+100	+100	+100	+115
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H10	+48	+58	+70	+84	+100	+100	+120	+120	+140	+140	+160	+160	+160	+185
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H11	+75	+90	+110	+130	+160	+160	+190	+190	+220	+220	+250	+250	+250	+290
	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continuation of table, see Page 444.

200	225	250	280	315	355	400	450	500	560	630	710	800	900
225	250	280	315	355	400	450	500	560	630	710	800	900	1000
+1030	+1110	+1240	+1370	+1560	+1710	+1900	+2050	–	–	–	–	–	–
+740	+820	+920	+1050	+1200	+1350	+1500	+1650	–	–	–	–	–	–
+550	+570	+620	+650	+720	+760	+840	+880	–	–	–	–	–	–
+260	+280	+300	+330	+360	+400	+440	+480	–	–	–	–	–	–
+355	+355	+400	+400	+440	+440	+480	+480	+540	+540	+610	+610	+680	+680
+170	+170	+190	+190	+210	+210	+230	+230	+260	+260	+290	+290	+320	+320
+129	+129	+142	+142	+161	+161	+175	+175	+189	+189	+210	+210	+226	+226
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+146	+146	+162	+162	+182	+182	+198	+198	+215	+215	+240	+240	+260	+260
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+215	+215	+240	+240	+265	+265	+290	+290	+320	+320	+360	+360	+400	+400
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+285	+285	+320	+320	+355	+355	+385	+385	+425	+425	+480	+480	+530	+530
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+79	+79	+88	+88	+98	+98	+108	+108	+120	+120	+130	+130	+142	+142
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+96	+96	+108	+108	+119	+119	+131	+131	+146	+146	+160	+160	+176	+176
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+122	+122	+137	+137	+151	+151	+165	+165	+186	+186	+205	+205	+226	+226
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+44	+44	+49	+49	+54	+54	+60	+60	+66	+66	+74	+74	+82	+82
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+61	+61	+69	+69	+75	+75	+83	+83	+92	+92	+104	+104	+116	+116
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+87	+87	+98	+98	+107	+107	+117	+117	+132	+132	+149	+149	+166	+166
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+29	+29	+32	+32	+36	+36	+40	+40	+44	+44	+50	+50	+56	+56
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+46	+46	+52	+52	+57	+57	+63	+63	+70	+70	+80	+80	+90	+90
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+72	+72	+81	+81	+89	+89	+97	+97	+110	+110	+125	+125	+140	+140
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+115	+115	+130	+130	+140	+140	+155	+155	+175	+175	+200	+ 200	+230	+230
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+185	+185	+210	+210	+230	+230	+250	+250	+280	+280	+320	+320	+360	+360
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+290	+290	+320	+320	+360	+360	+400	+400	+440	+440	+500	+500	+560	+560
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continuation of table, ISO tolerances for holes, from Page 442.

Tolerance classes	Nominal size range mm over 3 incl. 6	6 10	10 18	18 30	30 40	40 50	50 65	65 80	80 100	100 120	120 140	140 160	160 180
	Limit deviations (1 μm = 0,001 mm) μm					Upper limit deviation = ES Lower limit deviation = EI							
J6	+5	+5	+6	+8	+10	+10	+13	+13	+16	+16	+18	+18	+18
	−3	−4	−5	−5	−6	−6	−6	−6	−6	−6	−7	−7	−7
J7	+6	+8	+10	+12	+14	+14	+18	+18	+22	+22	+26	+26	+26
	−6	−7	−8	−9	−11	−11	−12	−12	−13	−13	−14	−14	−14
J8	+10	+12	+15	+20	+24	+24	+28	+28	+34	+34	+41	+41	+41
	−8	−10	−12	−13	−15	−15	−18	−18	−20	−20	−22	−22	−22
JS6	+4	+4,5	+5,5	+6,5	+8	+8	+9,5	+9,5	+11	+11	+12,5	+12,5	+12,5
	−4	−4,5	−5,5	−6,5	−8	−8	−9,5	−9,5	−11	−11	−12,5	−12,5	−12,5
JS7	+6	+7,5	+9	+10,5	+12,5	+12,5	+15	+15	+17,5	+17,5	+20	+20	+20
	−6	−7,5	−9	−10,5	−12,5	−12,5	−15	−15	−17,5	−17,5	−20	−20	−20
JS8	+9	+11	+13,5	+16,5	+19,5	+19,5	+23	+23	+27	+27	+31,5	+31,5	+31,5
	−9	−11	−13,5	−16,5	−19,5	−19,5	−23	−23	−27	−27	−31,5	−31,5	−31,5
K6	+2	+2	+2	+2	+3	+3	+4	+4	+4	+4	+4	+4	+4
	−6	−7	−9	−11	−13	−13	−15	−15	−18	−18	−21	−21	−21
K7	+3	+5	+6	+6	+7	+7	+9	+9	+10	+10	+12	+12	+12
	−9	−10	−12	−15	−18	−18	−21	−21	−25	−25	−28	−28	−28
K8	+5	+6	+8	+10	+12	+12	+14	+14	+16	+16	+20	+20	+20
	−13	−16	−19	−23	−27	−27	−32	−32	−38	−38	−43	−43	−43
M6	−1	−3	−4	−4	−4	−4	−5	−5	−6	−6	−8	−8	−8
	−9	−12	−15	−17	−20	−20	−24	−24	−28	−28	−33	−33	−33
M7	0	0	0	0	0	0	0	0	0	0	0	0	0
	−12	−15	−18	−21	−25	−25	−30	−30	−35	−35	−40	−40	−40
M8	+2	+1	+2	+4	+5	+5	+5	+5	+6	+6	+8	+8	+8
	−16	−21	−25	−29	−34	−34	−41	−41	−48	−48	−55	−55	−55
N6	−5	−7	−9	−11	−12	−12	−14	−14	−16	−16	−20	−20	−20
	−13	−16	−20	−24	−28	−28	−33	−33	−38	−38	−45	−45	−45
N7	−4	−4	−5	−7	−8	−8	−9	−9	−10	−10	−12	−12	−12
	−16	−19	−23	−28	−33	−33	−39	−39	−45	−45	−52	−52	−52
N8	−2	−3	−3	−3	−3	−3	−4	−4	−4	−4	−4	−4	−4
	−20	−25	−30	−36	−42	−42	−50	−50	−58	−58	−67	−67	−67
P6	−9	−12	−15	−18	−21	−21	−26	−26	−30	−30	−36	−36	−36
	−17	−21	−26	−31	−37	−37	−45	−45	−52	−52	−61	−61	−61
P7	−8	−9	−11	−14	−17	−17	−21	−21	−24	−24	−28	−28	−28
	−20	−24	−29	−35	−42	−42	−51	−51	−59	−59	−68	−68	−68
P8	−12	−15	−18	−22	−26	−26	−32	−32	−37	−37	−43	−43	−43
	−30	−37	−45	−55	−65	−65	−78	−78	−91	−91	−106	−106	−106
R6	−12	−16	−20	−24	−29	−29	−35	−37	−44	−47	−56	−58	−61
	−20	−25	−31	−37	−45	−45	−54	−56	−66	−69	−81	−83	−86
R7	−11	−13	−16	−20	−25	−25	−30	−32	−38	−41	−48	−50	−53
	−23	−28	−34	−41	−50	−50	−60	−62	−73	−76	−88	−90	−93

180 200	200 225	225 250	250 280	280 315	315 355	355 400	400 450	450 500	500 560	560 630	630 710	710 800	800 900	900 1000
+22 −7	+22 −7	+22 −7	+25 −7	+25 −7	+29 −7	+29 −7	+33 −7	+33 −7	– –	– –	– –	– –	– –	– –
+30 −16	+30 −16	+30 −16	+36 −16	+36 −16	+39 −18	+39 −18	+43 −20	+43 −20	– –	– –	– –	– –	– –	– –
+47 −25	+47 −25	+47 −25	+55 −26	+55 −26	+60 −29	+60 −29	+66 −31	+66 −31	– –	– –	– –	– –	– –	– –
+14,5 −14,5	+14,5 −14,5	+14,5 −14,5	+16 −16	+16 −16	+18 −18	+18 −18	+20 −20	+20 −20	+22 −22	+22 −22	+25 −25	+25 −25	+28 −28	+28 −28
+23 −23	+23 −23	+23 −23	+26 −26	+26 −26	+28,5 −28,5	+28,5 −28,5	+31,5 −31,5	+31,5 −31,5	+35 −35	+35 −35	+40 −40	+40 −40	+45 −45	+45 −45
+36 −36	+36 −36	+36 −36	+40,5 −40,5	+40,5 −40,5	+44,5 −44,5	+44,5 −44,5	+48,5 −48,5	+48,5 −48,5	+55 −55	+55 −55	+62,5 −62,5	+62,5 −62,5	+70 −70	+70 −70
+5 −24	+5 −24	+5 −24	+5 −27	+5 −27	+7 −29	+7 −29	+8 −32	+8 −32	0 −44	0 −44	0 −50	0 −50	0 −56	0 −56
+13 −33	+13 −33	+13 −33	+16 −36	+16 −36	+17 −40	+17 −40	+18 −45	+18 −45	0 −70	0 −70	0 −80	0 −80	0 −90	0 −90
+22 −50	+22 −50	+22 −50	+25 −56	+25 −56	+28 −61	+28 −61	+29 −68	+29 −68	0 −110	0 −110	0 −125	0 −125	0 −140	0 −140
−8 −37	−8 −37	−8 −37	−9 −41	−9 −41	−10 −46	−10 −46	−10 −50	−10 −50	−26 −70	−26 −70	−30 −80	−30 −80	−34 −90	−34 −90
0 −46	0 −46	0 −46	0 −52	0 −52	0 −57	0 −57	0 −63	0 −63	−26 −96	−26 −96	−30 −110	−30 −110	−34 −124	−34 −124
+9 −63	+9 −63	+9 −63	+9 −72	+9 −72	+11 −78	+11 −78	+11 −86	+11 −86	−26 −136	−26 −136	−30 −155	−30 −155	−34 −174	−34 −174
−22 −51	−22 −51	−22 −51	−25 −57	−25 −57	−26 −62	−26 −62	−27 −67	−27 −67	−44 −88	−44 −88	−50 −100	−50 −100	−56 −112	−56 −112
−14 −60	−14 −60	−14 −60	−14 −66	−14 −66	−16 −73	−16 −73	−17 −80	−17 −80	−44 −114	−44 −114	−50 −130	−50 −130	−56 −146	−56 −146
−5 −77	−5 −77	−5 −77	−5 −86	−5 −86	−5 −94	−5 −94	−6 −103	−6 −103	−44 −154	−44 −154	−50 −175	−50 −175	−56 −196	−56 −196
−41 −70	−41 −70	−41 −70	−47 −79	−47 −79	−51 −87	−51 −87	−55 −95	−55 −95	−78 −122	−78 −122	−88 −138	−88 −138	−100 −156	−100 −156
−33 −79	−33 −79	−33 −79	−36 −88	−36 −88	−41 −98	−41 −98	−45 −108	−45 −108	−78 −148	−78 −148	−88 −168	−88 −168	−100 −190	−100 −190
−50 −122	−50 −122	−50 −122	−56 −137	−56 −137	−62 −151	−62 −151	−68 −165	−68 −165	−78 −188	−78 −188	−88 −213	−88 −213	−100 −240	−100 −240
−68 −97	−71 −100	−75 −104	−85 −117	−89 −121	−97 −133	−103 −139	−113 −153	−119 −159	−150 −194	−155 −199	−175 −225	−185 −235	−210 −266	−220 −276
−60 −106	−63 −109	−67 −113	−74 −126	−78 −130	−87 −144	−93 −150	−103 −166	−109 −172	−150 −220	−155 −225	−175 −255	−185 −265	−210 −300	−220 −310

Systems of fits

Systems of fits are intended to assist in restricting the large number of possible tolerance classes for defining a particular fit, in order to save costs on production and measuring equipment.

Through the appropriate combination of external and internal tolerance zone positions, it is possible to achieve various fits:

- clearance fits
- transition fits
- interference fits

Shaft basis and hole basis

The choice of system of fits is based on the area of application:

- System of fits "shaft basis":
  Fits where the fundamental deviation of the shaft is zero (upper deviation is zero).
  Position of tolerance zone (fundamental deviation identifier) for the shaft always "h", see Figure 12, Page 447
- System of fits "hole basis":
  Fits where the fundamental deviation of the hole is zero (lower deviation is zero).
  Position of tolerance zone (fundamental deviation identifier) for the hole always "H", see Figure 13, Page 447

In mating an outer part and inner part of the same nominal size with the maximum and minimum sizes indicated by the relevant upper and lower limit deviations, the following fit types can be obtained:

- **Clearance fit:**
  Fit where, after mating the parts, clearance always results.
- **Transition fit:**
  Fit where, after mating the parts, clearance or interference results (depending on the actual sizes).
- **Interference fit:**
  Fit where, after mating the parts, interference always results (and pressure occurs between the fit surfaces).

The following diagrams show the ISO systems of fits “shaft basis” and “hole basis”.

Figure 12
System of fits “shaft basis”

① Upper limit of size of the hole ULS_B with tolerance class A9
② Lower limit of size of the hole LLS_B with tolerance class A9
③ Nominal size
④ Housing
⑤ Shaft
⑥ Clearance fit
⑦ Transition fit
⑧ Interference fit
⑨ Tolerance zone position not present in this tolerance grade

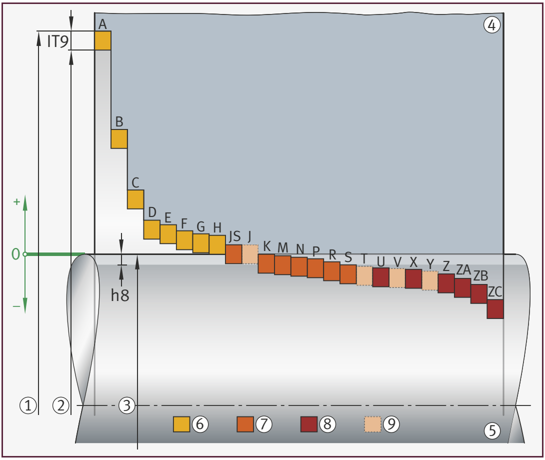
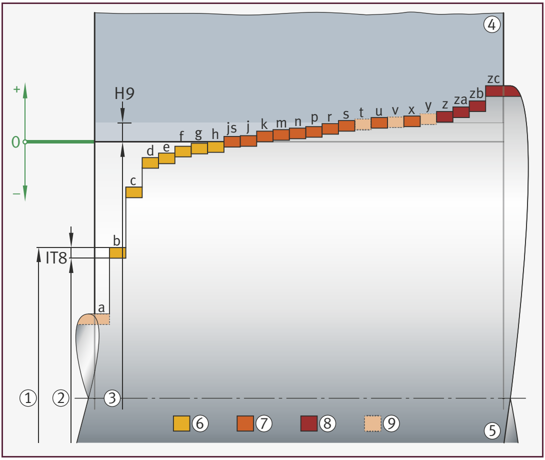


Figure 13
System of fits “hole basis”

① Upper limit of size of the shaft ULS_W with tolerance class b8
② Lower limit of size of the shaft LLS_W with tolerance class b8
③ Nominal size
④ Housing
⑤ Shaft
⑥ Clearance fit
⑦ Transition fit
⑧ Interference fit
⑨ Tolerance zone position not present in this tolerance grade



Calculation of fits Fits and fit tolerances are calculated as described below.

Maximum fit The maximum fit P_o is calculated as follows:

Equation 1

$$P_o = ULS_B - LLS_W = ES - ei$$

The calculation is thus: maximum hole size minus the minimum shaft size. The results are interpreted as follows:

Equation 2

> 0	Maximum clearance
< 0	Minimum interference

Minimum fit The minimum fit P_u is calculated as follows:

Equation 3

$$P_u = LLS_B - ULS_W = EI - es$$

The calculation is thus: minimum hole size minus the maximum shaft size. The results are interpreted as follows:

Equation 4

> 0	Minimum clearance
< 0	Maximum interference

In the case of clearance fits, the calculated values for the maximum and minimum fit are always positive while, in the case of interference fits, they are always negative.

Fit tolerance The fit tolerance P_T is the sum of the dimensional tolerances of both form elements that give the fit. It is an absolute value without a mathematical sign.

The fit tolerance is calculated as follows:

Equation 5

$$\begin{aligned} P_T &= P_o - P_u = (ULS_B - LLS_W) - (LLS_B - ULS_W) \\ &= (ES - EI) + (es - ei) \end{aligned}$$

Application of ISO fits

Examples of the application of ISO fits are given below:

Hole basis	Application	Shaft basis
	Clearance fits	
H11/a11	Parts with **very large clearance** and **large tolerance**: locomotive and waggon construction, coupling pins, agricultural machinery	A11/h11
H11/c11	Parts with **large clearance** and **large tolerance**: agricultural and household machinery	C11/h11
H10/d9	Parts with **very abundant clearance**: transmission shafts, stuffing box parts, loose pulleys, countershafts	D10/h9
H8/e8	Parts with **abundant clearance**: machine tool shafts with several supports, plain bearings	E8/h8
H7/f7	Parts with **significant clearance**: main machine tool bearings, sliding sleeves on shafts, pistons in cylinders	F7/h7
H7/g6	**Without significant clearance**, capable of movement: sliding gear wheels, movable coupling parts, valve lever bearing arrangement	G7/h6
H7/h6	Lubricated by hand, **capable of upward movement**: sleeve in the tailstock, centring flanges for couplings and pipelines	H7/h6
H6/h5	**Very small mean clearance**: for parts not moving against each other	H6/h5
	Transition fits	
H7/j6	**Joining by hand by means of light blows**: for belt pulleys, gear wheels and bearing bushes that can be easily dismounted	J7/h6
H7/k6	**Effective joining by means of hand-held hammer**: for belt pulleys, couplings and flywheels with a feather key joint	K7/h6
H7/m6	**Joining by means of hand-held hammer difficult but possible:** belt pulleys for single mounting only, couplings and gear wheels on electric motor shafts	M7/h6
H7/n6	**Joining possible by means of press**: for armatures on motor shafts and gear rings on gear wheels, bearing bushes in hubs	N7/h6
	Interference fits	
H7/r6 H7/s6	**Joining possible under high pressure or by heating:** circlip rings on flake graphite cast iron hubs, bearing bushes in housings (s6 for larger diameters, r6 for smaller diameters)	R7/h6 S7/h6
H8/u8 H8/x8	**Joining only achievable by means of press or temperature difference:** for transmission of high circumferential or longitudinal forces by frictional locking	U8/h8 X8/h8

Rolling bearing tolerances and ISO tolerances for shafts and housings

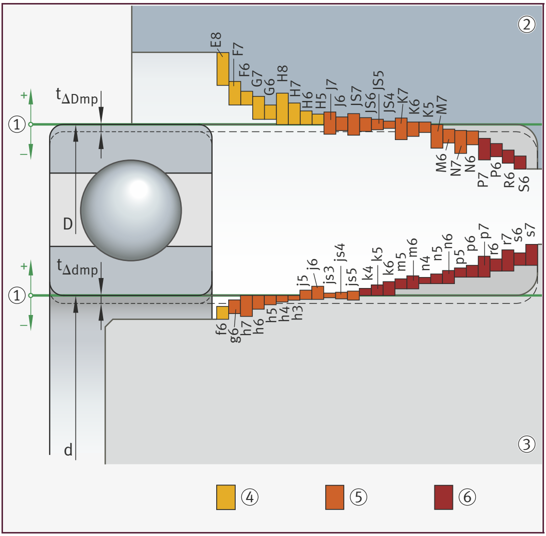
The tolerances for rolling bearings are defined for radial rolling bearings in accordance with ISO 492:2023 and for axial rolling bearings in accordance with ISO 199:2023.
These give tolerances for the deviation of the bearing bore as $t_{\Delta dmp}$ and the deviation of the bearing outside diameter as $t_{\Delta Dmp}$. In both cases, the upper limit deviation of these tolerance zones are additional to the nominal size. The lower limit deviations (in both cases towards minus) are defined by the tolerance classes detailed in ISO 492 and ISO 199.

In combination with the ISO tolerances for shafts and holes in accordance with ISO 286, this gives approximately the fits shown in the following diagram.

Figure 14
Fits for rolling bearings

$t_{\Delta Dmp}$ = deviation of mean bearing outside diameter
$t_{\Delta dmp}$ = deviation of mean bearing bore diameter
D = nominal bearing outside diameter
d = nominal bearing bore diameter

① Zero line
② Housing
③ Shaft
④ Clearance fit
⑤ Transition fit
⑥ Interference fit



Mounting fits for rolling bearings

Mounting fits for rolling bearings are selected as a function of the conditions of rotation. The conditions of rotation indicate the movement of one bearing ring with respect to the load direction.

For further information on the conditions of rotation of rolling bearings, see the chapter Design elements, section Conditions of rotation, Page 633.

Tolerances of form, orientation, location and run-out in drawings

Indications in drawings

The standard ISO 1101:2017 defines methods of indicating tolerances for form, orientation, location and run-out in drawings.

Symbols for toleranced characteristics

For toleranced characteristics, the following symbols are defined:

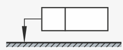
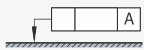
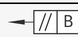
Types of elements and tolerances		Toleranced characteristic	Symbol
Elements without datum	Form tolerances	Straightness	⏤
		Flatness	⏥
		Roundness (circularity)	○
		Cylindricity (cylindrical form)	⌭
		Line profile	⌒
		Surface profile	⌓
Elements with datum	Orientation tolerances	Parallelism	∥
		Perpendicularity	⊥
		Angularity	∠
	Orientation and location tolerances	Line profile	⌒
		Surface profile	⌓
Elements with/without datum	Location tolerances	Position	⌖
Elements with datum		Concentricity and coaxiality	◎
		Symmetry	⌯
	Run-out tolerances	Run-out Circular radial run-out, circular axial run-out	↗
		Total run-out Total radial run-out, total axial run-out	⌰

Additional symbols The following additional symbols are defined in ISO 1101:2017:

Description	Symbol
Modifiers for the combination of tolerance zones (including ISO 1660, ISO 2692 and ISO 5458)	
Combined zone	CZ
Separate zones	SZ
Modifiers for unequal tolerance zones (including ISO 1660, ISO 2692 and ISO 5458)	
Specified tolerance zone offset	UZ
Modifiers for constraints	
Unspecified linear tolerance zone offset (offset zone)	OZ
Unspecified angular tolerance zone offset (variable angle)	VA
Modifiers for associated toleranced features	
Minimax (Chebyshev) feature	Ⓒ
(Gaussian) least squares feature	Ⓖ
Minimum circumscribed feature	Ⓝ
Tangent feature	Ⓣ
Maximum inscribed feature	Ⓧ
Modifiers for derived toleranced features	
Derived feature	Ⓐ
Projected tolerance zone	Ⓟ
Modifiers for the association of reference features for geometrical evaluation	
Minimax (Chebyshev) feature without constraint	C
Minimax (Chebyshev) feature with external material constraint	CE
Minimax (Chebyshev) feature with internal material constraint	CI
Least squares (Gaussian) feature without constraint	G
Least squares (Gaussian) feature with external material constraint	GE
Least squares (Gaussian) feature with internal material constraint	GI
Minimum circumscribed feature	N
Maximum inscribed feature	X

Continuation of table, see Page 453.

Continuation of table, Additional symbols, from Page 452.

Description	Symbol
Modifiers for parameters	
Total range of deviations	T
Peak height	P
Valley depth	V
Standard deviation	Q
Modifiers for toleranced features	
Between	
United feature	UF
Minor diameter	LD
Major diameter	MD
Pitch diameter	PD
All around (profile)	
All over (profile)	
Tolerance indicators	
Geometrical specification indication without datums section	
Geometrical specification indication with datums section[1)]	
Auxiliary indications of features	
Any cross-section	ACS
Intersection plane indicator[1)]	
Orientation plane indicator[1)]	
Direction feature indicator[1)]	
Collection plane indicator[1)]	
Symbol for the theoretically exact dimension (TED)	
Theoretically exact dimension (TED)[1)]	$\boxed{50}$

Continuation of table, see Page 454.

[1)] The letters, values and characteristic symbols in these symbols are examples.

Continuation of table, Additional symbols, from Page 453.

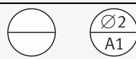
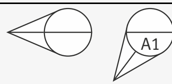
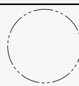
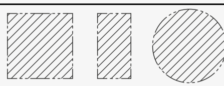
Description	Symbol
Modifiers for the material requirement (in accordance with ISO 2692)	
Maximum material requirement	Ⓜ
Least material requirement	Ⓛ
Reciprocity requirement	Ⓡ
Modifier for the free state condition (in accordance with ISO 10579)	
Free state condition (non-rigid parts)	Ⓕ
Modifier for the size tolerance (in accordance with ISO 14405-1)	
Envelope requirement	Ⓔ

Indications and modifiers for datums, see also the table Datum features and datum point symbols.

Datums Datums are defined in the standard ISO 5459:2011.

Datum features and datum point symbols

For datums in drawings, ISO 5459:2011 defines the following datum features and datum point symbols (letters or values in these symbols are examples only):

Description	Symbol
Datum feature indicator	
Datum feature identifier	Capital letter (A, B, C, AA etc.)
Single datum target frame	
Movable datum target frame	
Datum target point	
Closed datum target line	
Non-closed datum target line	
Datum target area	

Modifier symbols for datums

The following modifier symbols can, in accordance with ISO 5459:2011, be associated with the datum letter:

Description	Symbol
Pitch diameter	[PD]
Major diameter	[MD]
Minor diameter	[LD]
Any cross-section	[ACS]
Any longitudinal section	[ALS]
Contacting feature	[CF]
Distance variable (for a common datum)	[DV]
(Situation feature of type) Point	[PT]
(Situation feature of type) Straight line	[SL]
(Situation feature of type) Plane	[PL]
For orientation constraint only	><
Projected (for secondary or tertiary datums)	Ⓟ
Least material requirement (in accordance with ISO 2692)	Ⓛ
Maximum material requirement (in accordance with ISO 2692)	Ⓜ

Tolerance indicator and auxiliary indications

The indication of a geometrical product specification comprises a tolerance indicator, the optional plane and feature indications as well as optional adjacent indications, see Figure 15.

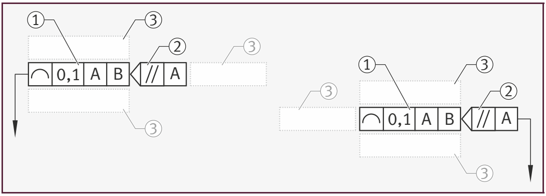


Figure 15
Components of a geometrical specification

① Tolerance indicator
② Plane and feature indicator
③ Adjacent indications

The tolerance requirements are indicated in a rectangular frame which is divided into two or more sections. This tolerance indicator contains, from left to right, the following compartments:

- Symbol section:
  with the symbol for the characteristic to be toleranced
- Section for zone, feature and characteristic:
  with the tolerance value in the unit applied to dimensioning. This value is preceded by the symbol "∅" if the tolerance zone is circular or cylindrical or the notation "S∅" if the tolerance zone is spherical. Further indications with complementary modifiers are possible, see table Section for zone, feature and characteristic.
- Datum section:
  if necessary, the letter or letters that identify the datum feature or features.

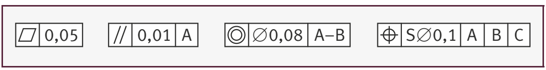


Figure 16
Tolerance indicators

If it is necessary to specify more than one tolerance characteristic for a feature, the tolerance indications should be stacked:

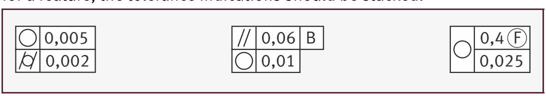


Figure 17
Multiple tolerance indications per feature

Section for zone, feature and characteristic

Complementary modifiers for particular requirements are indicated within the tolerance indicator in accordance with the tolerance value in the "section for zone, feature and characteristic"; several such requirements may be present at the same time in the same tolerance frame.

The following table, see Page 457, shows the grouping of the modifiers and the sequence in which the modifiers are indicated.

With the exception of “Width and extent”, all modifiers are optional. Between indications with different numbering (no. 1 to 11), spaces are required (except before letters in circles, columns no. 6, 7, 10, 11).

Tolerance zone					Toleranced feature				Characteristic			
					Filter[1]							
Shape	Width and extent	Combination	Specified offset	Constraint	Type	Indices	Ass. tol. feature	Derived feature	Association	Parameter	Material condition	State
∅ S∅	0,02 0,02–0,01 0,1/75 0,1/75×75 0,2/∅4 0,2/75×30° 0,3/10°×30°	CZ SZ	UZ+0,2 UZ–0,3 UZ+0,1:+0,2 UZ+0,2:–0,3 UZ–0,2:-0,3	OZ VA ><	G S etc.	0,8 –250 0,8–250 500 –15 500–15 etc.	Ⓒ Ⓖ Ⓝ Ⓣ Ⓧ	Ⓐ Ⓟ Ⓟ25 Ⓟ32-7	C CE CI G GE GI X N	P V T Q	Ⓜ Ⓛ Ⓡ	Ⓕ
1		**2**[2]	**3**	**4**[2]	**5**		**6**	**7**[2]	**8**	**9**	**10**[2]	**11**

Source: ISO 1101:2017.

[1] Filters: see ISO 1101.

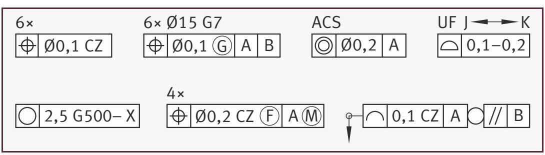
[2] More than one of the listed modifiers can be applied at the same time.

Auxiliary indications of features

Auxiliary indications of features (intersection plane indicator, orientation plane indicator, collection plane indicator and/or lastly the direction feature indicator) are placed next to the tolerance indicator in compartment ②, see Figure 15, Page 456.

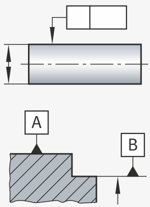
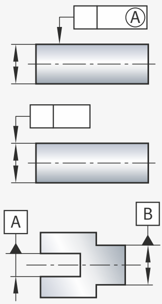
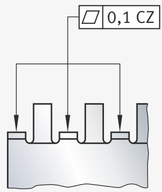
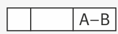
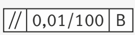
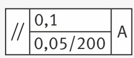
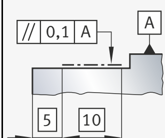
If a tolerance applies to more than one feature, the number of features is indicated with the symbol “×” above the tolerance indicator and can be supplemented by the dimension of the feature. Further optional indications for describing the characteristics of the feature within the tolerance zone are also placed in compartment ③, see Figure 15, Page 456, (preferably above): Dimensional tolerance indications, variable width specification with the aid of the “between” symbol, additionally UF, ACS, LD, MD, PD. The “all around” arrow is placed at the tolerance indicator instead of the standard arrow if, for example, the profile characteristic applies to the entire contour line:

Figure 18
Additional tolerance indications



Features, datums and restrictions

Toleranced features, datum features and datum indicators or restrictive instructions are indicated in drawings as follows:

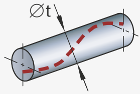
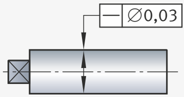
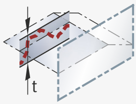
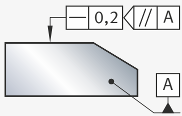
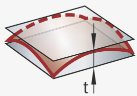
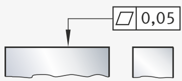
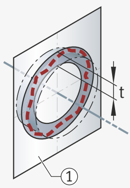
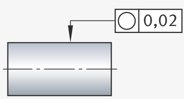
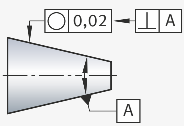
Toleranced features and datum features		
Line or area 	Axis or median plane 	Combined zone 
Single datum feature 	Multiple datum features with a fixed ranked-order (primary/secondary/tertiary) 	Datums of equal significance forming a common datum feature 
Tolerance on a restricted length 	Smaller tolerance on a restricted length 	Tolerance on a restricted part of the feature 

Drawing indication and definitions

The standard ISO 1101:2017 contains detailed definitions of form, orientation, location and run-out tolerances and their symbolic language.

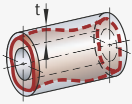
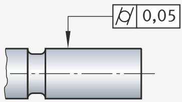
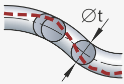
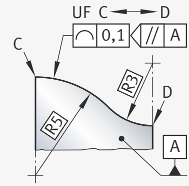
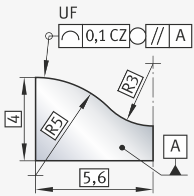
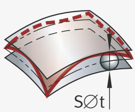
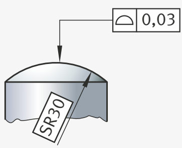
The following table gives a compilation of these definitions.
An example is described for each toleranced characteristic.
All further possible combinations can be derived from these examples.

The corresponding representation following the table shows An example of the indication of form, orientation, location and run-out tolerances in technical drawings.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
—	Straightness			The extracted median (actual) line of the cylinder to which the tolerance applies shall lie within a cylindrical tolerance zone of diameter t = 0,03 mm.
				The extracted (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained in any plane under consideration parallel to the datum A (intersection plane indicator) within two parallel straight lines at a distance of t = 0,2 mm apart.
▱	Flatness			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,05 mm apart.
○	Roundness			The extracted (actual) circumferential line to which the tolerance applies shall lie within a tolerance zone that is contained in any cross-section within two concentric circles which are at a radial distance of t = 0,02 mm apart.
				Auxiliary indication on taper (in this case the directional feature indicator to perpendicularity): The circles are perpendicular to the datum axis A. (1) Any cross-sectional plane (any cross-section)

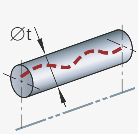
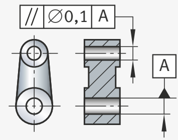
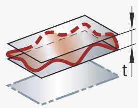
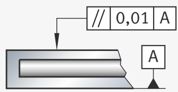
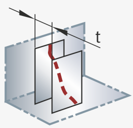
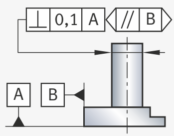
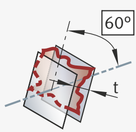
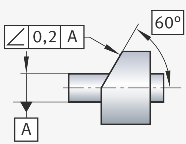
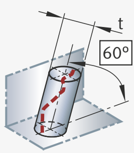
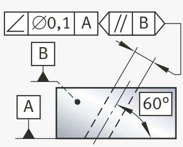
Continuation of table, see Page 460.

Continuation of table, Drawing indication and definitions, from Page 459.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
⌭	Cylindricity (cylindrical form)			The extracted cylindrical (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two coaxial cylinders which have a radius differential of t = 0,05 mm.
⌒	Line profile		 	The extracted (actual) profile line to which the tolerance applies shall lie within a tolerance zone that is contained within any parallel plane to the datum plane A by two lines that enclose circles of diameter t = 0,1 mm The centre points of these circles are located on a theoretically geometrically exact line. The extracted profile line must be continuous. Auxiliary indications in the examples substantiate the tolerance specification.
⌓	Surface profile			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two surfaces that enclose spheres of diameter t = 0,03 mm. The centre points of these circles are located on a theoretically geometrically exact surface.

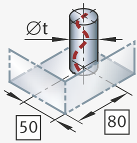
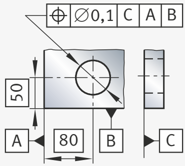
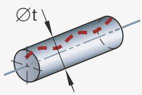
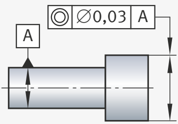
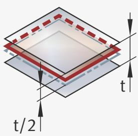
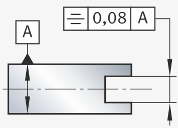
Continuation of table, see Page 461.

Continuation of table, Drawing indication and definitions, from Page 460.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
//	Parallelism			The extracted median (actual) line to which the tolerance applies shall lie within a cylindrical tolerance zone parallel to the datum axis A which is of the diameter t = 0,1 mm.
				The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes to the datum plane A which are at a distance of t = 0,01 mm apart.
⊥	Perpendicularity			The extracted median (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,1 mm apart. The planes are perpendicular to the datum A and parallel to the secondary datum B (orientation plane indicator).
∠	Angularity			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two inclined parallel planes at a theoretically exact angle to the datum which are at a distance of t = 0,2 mm apart.
				The extracted median (actual) line of the hole to which the tolerance applies shall lie within a cylindrical tolerance zone of diameter t = 0,1 mm. The tolerance zone lies parallel to the datum plane B (orientation plane indicator) and is inclined at a theoretically exact angle to the datum plane A.

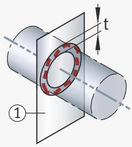
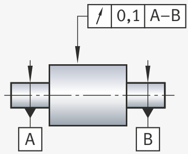
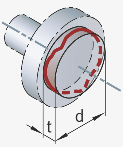
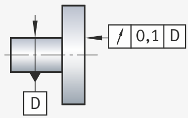
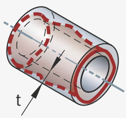
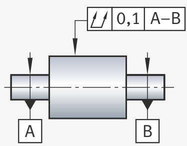
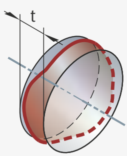
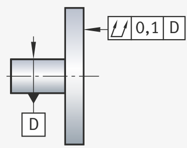
Continuation of table, see Page 462.

Continuation of table, Drawing indication and definitions, from Page 461.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
⌖	Position			The extracted median (actual) line of the hole to which the tolerance applies shall lie within a cylindrical tolerance zone of diameter t = 0,1 mm, whose axis coincides with the theoretically exact location of the axis of the bore in relation to the datum planes C, A and B.
◎	Coaxiality, concentricity			The extracted median (actual) line of the large cylinder shall lie within a cylindrical tolerance zone with the diameter t = 0,03 mm whose axis coincides with the datum axis A.
⌯	Symmetry			The extracted (actual) median plane of the slot to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,08 mm apart and symmetrical to the datum medium plane A.

Continuation of table, see Page 463.

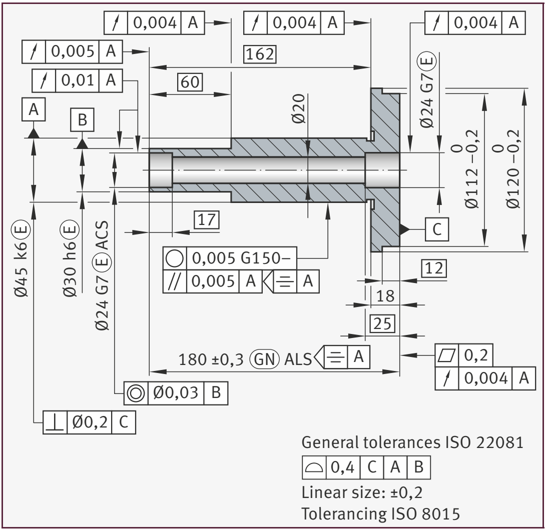
Continuation of table, Drawing indication and definitions, from Page 462.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
↗	Circular radial run-out			The extracted (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained in any cross-section perpendicular to the datum axis A–B, within two concentric circles at a radial distance of t = 0,1 mm whose centre point lies on the datum axis. ① Cross-sectional plane with a variable position along the datum axis.
↗	Circular axial run-out			The extracted (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained in any ring-shaped cross-section of a variable diameter d whose axis coincides with the datum axis D, within two circles at an axial distance of t = 0,1 mm.
⌰	Total radial run-out			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two coaxial cylinders which are at a radial distance of t = 0,1 mm, whose axis coincides with the common datum line A–B.
⌰	Total axial run-out			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,1 mm apart, that are perpendicular to datum axis D.

The following representation shows an example of the indication of form, orientation, location and run-out tolerances in technical drawings.

Figure 19
Drawing with tolerances of form, orientation, location and run-out

Dimensioning not complete



Tolerancing principle

The tolerancing principle controls the relationship between tolerances of dimension, form, orientation, location and run-out. If the principle of independence applies, this means that the tolerances must be captured and assessed independently of each other (ISO 8015:2011).
If the envelope requirement in accordance with ISO 14405 (Linear Size ISO 14405 Ⓔ) applies, this means that all deviations in dimension, form, orientation, location and run-out lie within an envelope specified by the dimensional tolerance. If no other determinations are made, the principle of independence applies.

Principle of independence

Under the principle of independence, tolerances of dimension, form, orientation, location and run-out must be observed independently of each other and each tolerance must be checked independently if no mutual relationship is indicated. If only linear size ISO 14405 is indicated in or near the title block and there are no characteristics that revoke independence (modification symbols) on the dimensional feature, the two-point size always applies (ISO 17450-1).

If the dimensional tolerance is defined as a two-point size, the local actual dimension must lie within the tolerance limits. This does not, however, ensure that no other geometrical deviations can be present.

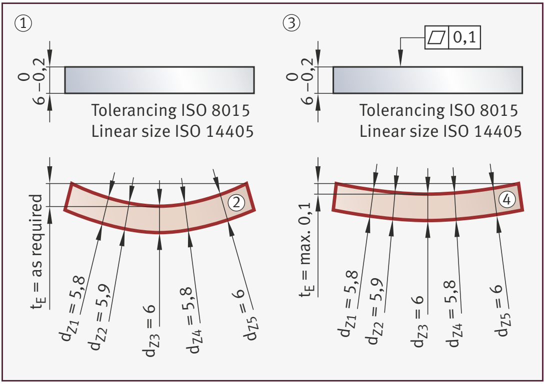
For example, if there is no special tolerance for the flatness, the requirements of the drawing in Figure 20 ①, Page 465, will still be fulfilled by a sheet metal piece with a considerable geometrical deviation whose local actual dimensions conform to the tolerance limits.

If the geometrical deviation is restricted, a special tolerance must be applied to the flatness, see Figure 20 ③ and ④.

Figure 20
Drawing with principle of independence

d_Z = local actual size (two-point size)
t_E = flatness tolerance

① Drawing without flatness tolerance
② Permissible sheet metal piece with considerable geometrical deviation
③ Drawing with flatness tolerance
④ Sheet metal piece fulfils flatness tolerance



Unless otherwise specified (e.g. by a standard) or the following statement is indicated in or near the title block of the drawing, the principle of independence applies: Tolerancing ISO 8015, Linear Size ISO 14405 (or simply: Linear Size ISO 14405).

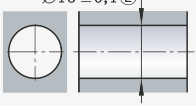
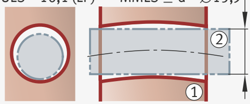
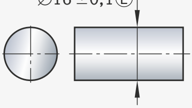
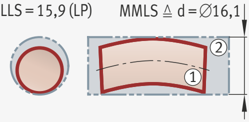
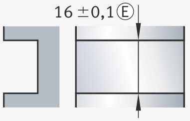
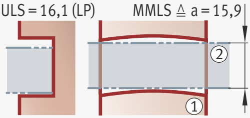
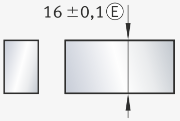
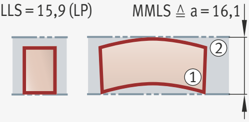
Unless a particular drawing indication is present, the principle of independence also applies worldwide in accordance with ISO 14405-1. If drawings contain the following in or near the title block: Tolerancing ASME Y14.5, then the envelope requirement applies here.

Envelope requirement

If the principle of independence is defined for a drawing, the envelope requirement should be defined for sizes (former description: fit surfaces) in order to ensure mating capability if no tolerances of form, orientation, location and run-out are indicated. This stipulates that dimensional, geometrical and parallelism tolerances are in a particular relationship with each other. The application of the envelope requirement for the form element is defined by the symbol Ⓔ after the dimensional tolerance of the form element.

The envelope requirement is checked as a two-point size and by means of a gauge (envelope). This can also be carried out using a coordinate measuring machine.
The envelope requirement stipulates that the form element may not pierce the geometrically ideal envelope with a maximum material limit of size (MMLS) and that, at the same time, the local two-point size (LP) may not at any point lie outside the tolerance limits, see ISO 14405-1:2016. Deviations of geometry and parallelism are thus also included automatically. According to the standard, the envelope requirement is the use of a combination of the two-point size and the minimum circumscribed lower limit of size LLS (in the case of a shaft or block) or the maximum inscribed upper limit of size ULS (in the case of a hole or slot).

The following table shows the geometrically ideal envelope for various form elements and the geometrical deviations subject to the envelope requirement:

Form element	Example drawing	Permissible workpiece, geometry of envelope	Included deviation
Circular cylinder (hole) ① Permissible workpiece ② Geometry of envelope	Hole  Ø16 ±0,1Ⓔ	Envelope: mandrel with d = LLS  ULS = 16,1 (LP) MMLS ≙ d = Ø15,9	Straightness, roundness, parallelism, cylindricity
Circular cylinder (shaft) ① Permissible workpiece ② Geometry of envelope	Shaft  Ø16 ±0,1Ⓔ	Envelope: sleeve with d = ULS  LLS = 15,9 (LP) MMLS ≙ d = Ø16,1	Straightness, roundness, parallelism, cylindricity
Parallel, flat inner surfaces ① Permissible workpiece ② Geometry of envelope	Slot  16 ±0,1Ⓔ	Envelope: 2 parallel planes with a = LLS  ULS = 16,1 (LP) MMLS ≙ a = 15,9	Straightness, flatness, parallelism
Parallel, flat outer surfaces ① Permissible workpiece ② Geometry of envelope	Block  16 ±0,1Ⓔ	Envelope: 2 parallel planes with a = ULS  LLS = 15,9 (LP) MMLS ≙ a = 16,1	Straightness, flatness, parallelism

In accordance with ISO 1938-1:2015, the maximum material limit of size (MMLS) is the limit of size that corresponds to the maximum material condition of the feature of size. The least material limit of size (LMLS) is the limit of size that corresponds to the least material condition of the feature of size. MMLS and LMLS contain in each case the numerical value of the size and the defined assocation criteria.

For external sizes (shaft, block), the following applies:

- MMLS corresponds to the upper limit of size ULS; the material must lie within an envelope with d = ULS or a = ULS.
- LMLS (two-point size) corresponds to the lower limit of size LLS and must not fall short of the specified value at any point on the shaft or block.

For internal sizes (hole, slot), the following applies:

- MMLS corresponds to the lower limit of size LLS; the material must lie outside an envelope with d = LLS or a = LLS.
- LMLS (two-point size) corresponds to the upper limit of size ULS and must not be exceeded at any point in the hole or slot.

The envelope requirement is applied to the entire technical drawing if the following appears in or near the title block:
Linear Size ISO 14405 Ⓔ. A drawing indication for the individual sizes of the form elements using the symbol Ⓔ is not necessary then.
The envelope requirement applies to all form elements such as cylinders or parallel planes.

General rule

In German drawings without reference to the standard (DIN) ISO 8015, the envelope requirement applied automatically to all cylinders and parallel planes up to the year 2010 in accordance with the standard DIN 7167:1987 without further special indications on the drawing.
After a revision of the standard ISO 8015 and application of the standard ISO 14405-1, the principle of independence has applied as a general rule with effect from 2010 without reference to the standard ISO 8015.

General tolerances

Application of general tolerances

According to the basic understanding of the GPS system, it is the designer's responsibility to provide a full and clear description of a component according to its function, i.e. to give full specifications. General geometrical specifications and general size specifications can be used to reduce the amount of information contained in technical product documentation (TPD) to a minimum.

However, when using general geometrical specifications or general size specifications, there is a risk that important functional requirements will be overlooked or that unnecessarily tight tolerances will be selected in relation to the functional requirement.

All features must be specified clearly and in full.

General tolerances in accordance with ISO 22081

ISO 22081:2021 provides methods for the precise indication of general geometrical specifications, general size specifications, and the rules governing their application. Specific tolerance values, as defined in the past for cutting manufacturing processes in ISO 2768-1:1989 and ISO 2768-2:1989, for example, are not provided in ISO 22081.
ISO 22081 defines two types of general specifications:

- general geometrical specifications, the surface profile
- general size specifications, i.e. linear size and angular size

These specifications can only be applied to integral features (including features of size) such as cylinders, spheres and planes, but not to derived features (centre line, median plane, centre point) or integral lines. A datum system is required that blocks all six degrees of freedom. These datum features should be specified individually.

ISO 22081:2021 is part of the GPS standards system.
According to ISO 8015, the function of the component is regarded as fulfilled if the component lies within the specification limits.

Historical standard ISO 2768-2

The aim of general tolerances according to ISO 2768-1:1989 and ISO 2768-2:1989 was to simplify the drawing and to illustrate the accuracy normally expected of workshops. However, due to non-conformity with the GPS system, ISO 2768-2:1989 has been withdrawn and replaced by ISO 22081:2021.

Geometrical product specifications are only unambiguous and complete if all features are specified in a three-dimensional datum system with location tolerances. Missing datums in ISO 2768-2 create room for interpretation and thus ambiguity. According to ISO 2768-1, distances with ± tolerances can be specified, which leads to ambiguity, see ISO 14405-2.

As ISO 2768-2:1989 has been withdrawn and directly replaced by ISO 22081, the former standard can only be cited as a dated reference: ISO 2768-2:1989.

General tolerance values in accordance with DIN 2769 as an addition to ISO 22081

ISO 22081:2021 is restricted to defining the concept and does not provide specific tolerance values; this is left to the user.

German standard DIN 2769:2023, which provides cross-technology and cross-material value tables for general tolerances and tolerance classes, can also be used in addition to ISO 22081:2021.
Thus, with the selection of a tolerance class, the relevant accuracy normally expected of workshops and specific to the process can be taken into account in accordance with the ISO GPS system.

In DIN 2769, general tolerance values are selected for the surface profile according to the code or the size of the minimum circumscribed sphere of the part, see table General tolerance values for surface profiles, Page 470. For sizes, general tolerance values are selected according to the nominal size and tolerance class, see tables General tolerance values for linear sizes, Page 470, and General tolerance values for angular sizes, Page 470.

If product-specific or process-specific general tolerance standards exist, such as for castings or plastic, these should be used.

In the past, for example, values were specified for cutting manufacturing processes in ISO 2768-1:1989 and ISO 2768-2:1989, however, these standards are not GPS-compliant and lead to ambiguous specifications when used in conjunction with the ISO GPS system.

General tolerance values for surface profiles

The standard DIN 2769 defines general tolerance values for surface profiles for geometrical tolerancing, which are also used for distances. The general tolerance values for surface profiles are defined by the code and the code letters for the tolerance class:

Tolerance class	General tolerance values for ball diameter SD of the minimum circumscribed ball of the part mm							
	> 0 ≦ 3	> 3 ≦ 6	> 6 ≦ 30	> 30 ≦ 120	> 120 ≦ 400	> 400 ≦ 1000	> 1000 ≦ 2 000	> 2 000 ≦ 4 000
Code letter	Code							
	1	2	3	4	5	6	7	8
A	0,1	0,1	0,2	0,3	0,4	0,6	1	2
B	0,2	0,2	0,4	0,6	1	1,6	2,4	4
C	0,3	0,6	1	1,6	2,4	4	6	8
D	0,4	1	2	3	5	8	12	16

General tolerance values for linear sizes

General tolerance values for linear sizes are defined in DIN 2769 in tolerance classes a to d:

Tolerance class	General tolerance values for nominal size ranges mm							
Code letter	> 0 ≦ 3	> 3 ≦ 6	> 6 ≦ 30	> 30 ≦ 120	> 120 ≦ 400	> 400 ≦ 1000	> 1000 ≦ 2 000	> 2 000 ≦ 4 000
a	±0,05	±0,05	±0,1	±0,15	±0,2	±0,3	±0,5	±1
b	±0,1	±0,1	±0,2	±0,3	±0,5	±0,8	±1,2	±2
c	±0,2	±0,3	±0,5	±0,8	±1,2	±2	±3	±4
d	±0,3	±0,5	±1,0	±1,5	±2,5	±4	±6	±8

General tolerance values for angular sizes

The general tolerance values for angular sizes are defined in DIN 2769 in tolerance classes 1 to 3:

Tolerance class	General tolerance values for the length of the shorter angle side mm				
	> 0 ≦ 10	> 10 ≦ 50	> 50 ≦ 120	> 120 ≦ 400	> 400
1	±1°	±0°30′	±0°20′	±0°10′	±0°5′
2	±1°30′	±1°	±0°30′	±0°15′	±0°10′
3	±3°	±2°	±1°	±0°30′	±0°20′

Indications in drawings

The use of general tolerances is referenced on the drawing by the indication "General tolerances ISO 22081" near the title block. The general tolerance can be indicated as an individual value or with reference to a document, such as a company standard or DIN 2769.

In the application of DIN 2769, general tolerance values for surface profiles, linear size or angular size can be found as individual values or with reference to the tolerance class in the tables under section General tolerance values in accordance with DIN 2769 as an addition to ISO 22081, Page 470.

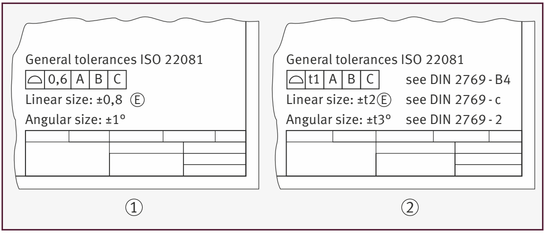


Figure 21
Application example from ISO 22081 and additionally DIN 2769

1 With individual values and envelope requirement
2 With reference to DIN 2769

Design elements

Definitions and principal functions

Design elements are machine elements of widely varying complexity that are always used in an identical or similar form in technical applications, where they fulfil an identical or similar function. They are among the most important elements used by design engineers to realise solutions.

Function-oriented approach

In accordance with this definition, it is possible to structure the very wide spectrum of machine elements in terms of their function, see Figure 1. This makes it easier for engineers to access this wide range in their design work on and with the aid of machine elements.

Classification according to principal function

This chapter is structured in accordance with the principal functions of design elements as they are found repeatedly in technical practice.

Design elements are described substantially in accordance with the following scheme:

- Characteristics
  A characteristic is a feature of a system that is perceived as a property by its manifestation.[1)]
  Characteristics describe the structure, geometry and quality of the product, in the way that these can be directly influenced by design engineers.
- Properties
  A property is a result determined for an object on the basis of observations, measurement results and generally accepted statements, etc.[1)]
  Properties describe the behaviour of the product as it results from the interaction with other design elements and under the influence of operating conditions. Properties are the product of all characteristics and therefore cannot be influenced directly by the design engineer.

Overview through classification schemes

It would contradict the character of this Pocket Guide to give a detailed description of all common machine elements. For this reason, classification schemes are used to give an overview of the totality of design elements.

The detailed description focusses essentially on the elements that could traditionally be found in earlier editions of the Schaeffler Technical Pocket Guide.

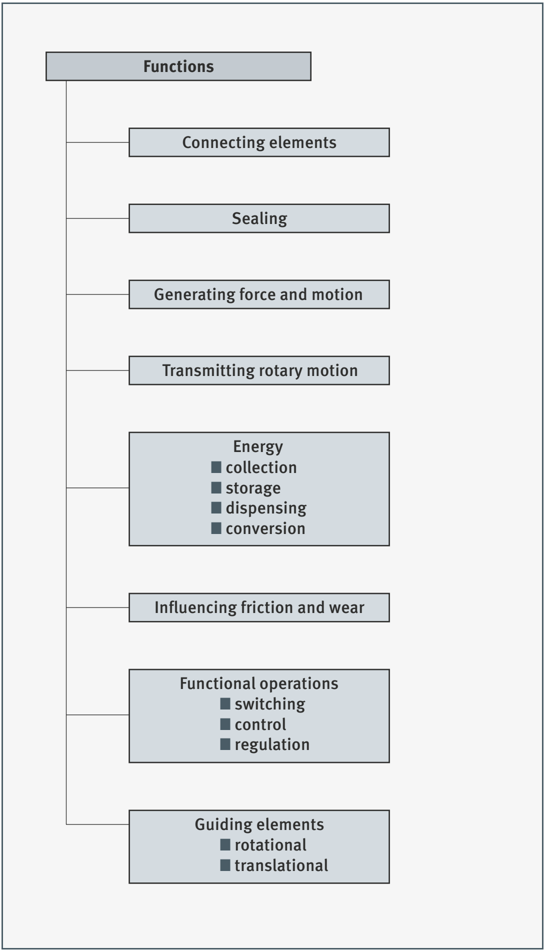
The design elements with the function "Guiding elements (rotational and translational)" are covered in comprehensive detail.

[1)] Source: in accordance with VDI Guideline 2221.

Principal functions

The principal functions of the design elements are shown in Figure 1.

Figure 1
Classification according to principal functions – overview

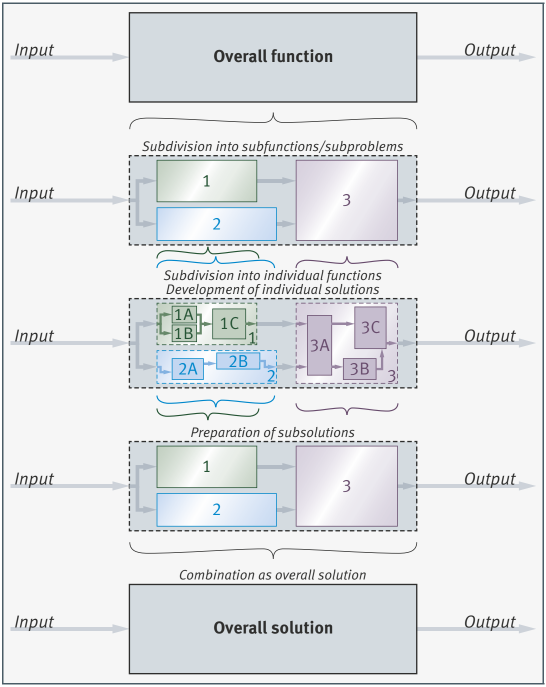


Functional structure

Thinking and working on a function-oriented basis is an essential principle in modern development methodology.

From the starting point of the list of requirements, each technical system must fulfil an overall function that can be subdivided into subfunctions (functional structure). Through this "breaking down" into subordinate subfunctions, it can be ensured that the complex overall problem is classified into smaller subtasks that can be resolved more easily, giving subsolutions that can later be recombined to give an overall solution for a technical system.

Figure 2
Functional structure
Source: in accordance with VDI Guideline 2221

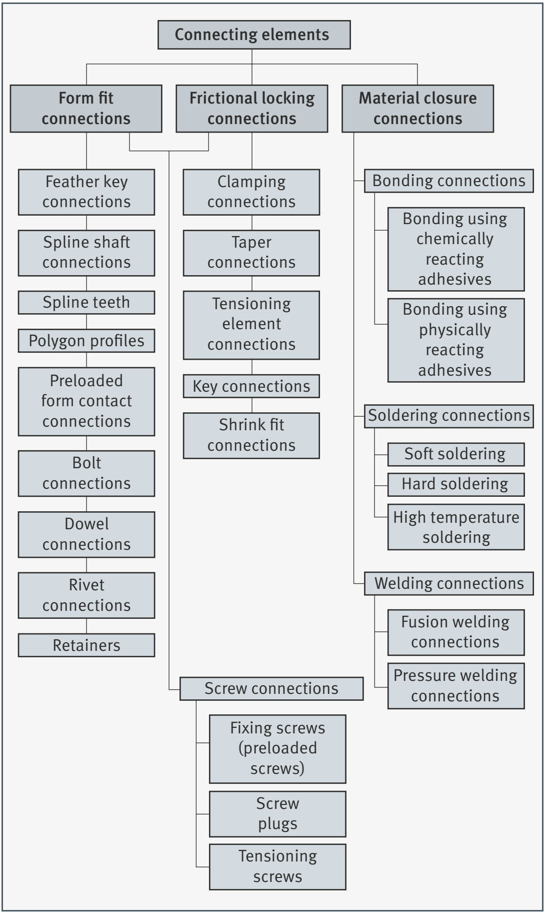


Working on a function-oriented basis also ensures that solutions from various domains (mechanics, electronics, software) can be found and followed up.
In addition, individual and partial solutions can be developed sequentially or in parallel.

Connecting elements

Overview There is a large number of elements that can be used to connect design elements and combine design elements in more complex structures. The function of these connecting elements is to transmit forces and moments. Depending on the type of force transmission (operating principle), a distinction is made between connections based on material closure, form fit and frictional locking. Combinations of frictional locking and form fit are possible.

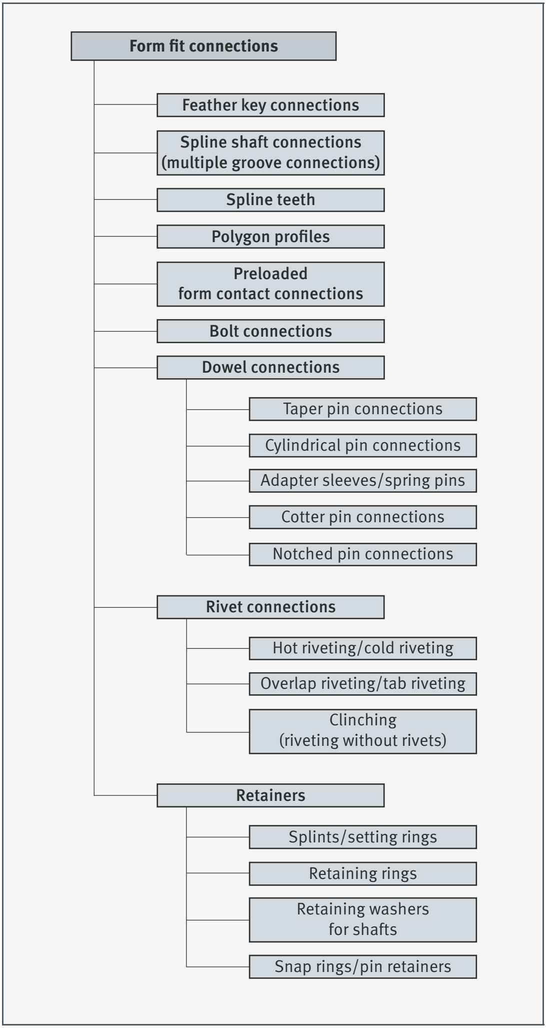
Figure 3
Connecting elements (form fit, frictional locking, material closure) – overview



Form fit connections

In the case of form fit connections, the connection is achieved by means of the form of the parts to be connected (= direct form fit) or by means of additional elements that act as "drivers" (= indirect form fit).

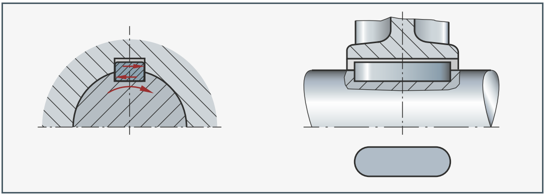
Figure 4
Form fit connections – overview



Feather key connections

The feather key connection is the most important representative type of shaft/hub connection based on form fit. However, it is not suitable for the transmission of shock type torques and high, alternating torques.

Figure 5
Feather key connection



Characteristics:

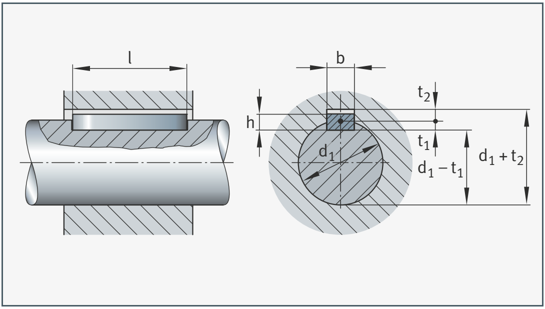
- Feather key is simultaneously present in the shaft keyway and the hub keyway.
- One feather key is normally used; up to a maximum of two can be arranged with an offset around the circumference of 120°.
- Keyways are parallel to the axis.
- A feather key used as a driver has a rectangular cross-section.
- Designs of feather key with a round end or square end are possible.
- The hub must be axially located.

Properties:

- For high torques
- Not suitable for changes of load direction and shocks
- Displacement of axis possible under certain preconditions (for indexing)
- Stress concentration on the shaft, due to notch effect
- Easy to mount and dismount, reusable

Figure 6
Key values of a feather key connection

b = feather key width
h = feather key height
d_1 = shaft diameter
t_1 = shaft keyway width
t_2 = hub keyway width
l = effective feather key length (for round end design, deduct b)



The feather key connection transmits via its flanks the circumferential force generated by the torque to be transmitted and is thus subjected to contact pressure.

The circumferential force F_u can be calculated as follows:

Equation 1

$$F_u = \frac{2 \cdot M_t}{d_1}$$

where M_t is the transmissible torque and d_1 is the shaft diameter, while contact pressure can be calculated as follows:

Equation 2

$$p = \frac{F_u}{(h - t_1) l \cdot i} \quad \text{or} \quad p \approx \frac{F_u}{0{,}45 \cdot h \cdot l \cdot i}$$

to DIN 6892 method C

Legend

l mm
Effective feather key length

i
Number of feather keys

if i = 2
use only 75% of l.

Guide values for the permissible flank contact pressures can be found in the following table. For shaft calculation, take account of the reduction in strength as a result of the feather key connections (β_{kt} = 1,3 ... 2,0; β_{kb} = 2,1 ... 3,2).[1)]

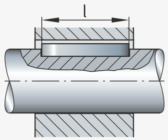
Material	Permissible contact pressure p_{per} under load type N/mm^2	
	Static	Pulsating
Steel, unhardened	100 ... 200	70 ... 150
Steel, hardened	150 ... 250	100 ... 170
Cast steel	100 ... 150	80 ... 100
Cast iron, malleable cast iron	80 ... 100	60 ... 80
Copper alloy (bronze, brass)	40 ... 50	30 ... 40
AlCuMg, aged	100 ... 160	70 ... 100
AlMg, AlMn, AlMgSi, aged	80 ... 150	60 ... 90
G AlSi, G AlSiMg	60 ... 70	40 ... 50

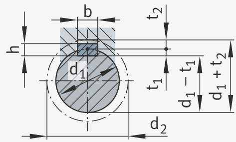
In general, the higher values apply in the case of higher values for yield stress, fracture strength and hardness of the materials, while the lower values apply accordingly in the case of lower strength values.

1) The values apply in the range R_m = 400 ... 1 200 N/mm^2 (in accordance with DIN 743).

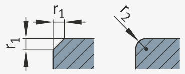
Feather keys, keyways, deep design

The following table shows feather keys, keyways (deep design) in accordance with DIN 6885 Part 1/Part 2:

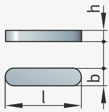




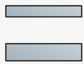
Edge break (all sides), chamfer or rounding (at manufacturer's discretion)



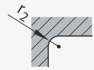
Type A, round end[1)]



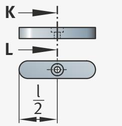
Type B, square end[1)]



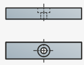
Rounding of keyway base for shaft and hub



Type C, round end Size 8 × 7 and above, with hole for 1 retaining screw above stepped line

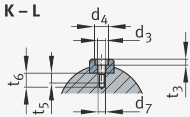


Type B, square end Size 8 × 7 and above, with hole for 1 retaining screw above stepped line



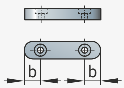
Hole for retaining screw

K – L

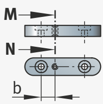


Type E, round end, with holes for 2 retaining screws below the stepped line

8 × 7 and 10 × 8

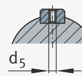


Size 12 × 8 and above, additionally with threaded hole for 1 or 2 extraction screws



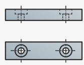
Hole for extraction screw

M – N

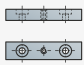


Type F, round end, with holes for 2 retaining screws below the stepped line

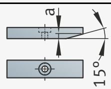
8 × 7 and 10 × 8



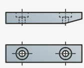
Size 12 × 8 and above, additionally with threaded hole for 1 or 2 extraction screws



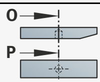
Type G, square end, with chamfer and hole for 1 retaining screw



Type H, square end, with chamfer and hole for 2 retaining screws

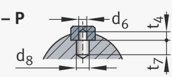


Type J, square end, with chamfer and hole for 1 adapter sleeve



Hole for adapter sleeve

O – P



Designation of feather key, type A, width b = 12 mm, height h = 8 mm, length l = 56 mm:
feather key A 12 × 8 × 56 DIN 6885.

Material: Part 1 for h ≤ 25 mm and Part 2 for all sizes E295, Part 1 for h ≥ 25 mm E335.

1) If feather keys of type A and B are to be supplied with holes for extraction screws (S), this must be specified at the time of ordering. In this case, the designation is: feather key AS 12 × 8 × 56 DIN 6885.

For feather keys (high design) in accordance with DIN 6885 Part 1/Part 2[1), the following values apply.

Feather key, cross-section (wedge steel to DIN 6880)		Width b	4	5	6
		Height h	4	5	6
Shaft diameter d_1		over	10	12	17
		incl.	12	17	22
Shaft	b, tight fit P9	loose fit N9	4	5	6
	t_1 with back clearance		2,5	3	3,5
Hub	b, tight fit P9	loose fit JS9	4	5	6
	t_2 in the case of back clearance		1,8	2,3	2,8
	for interference[2)]		1,2	1,7	2,2
	a		–	–	–
	$d_2 = d_1 +$ [3)]		4	5	6
	Feather key r_1	min./max.	0,16/0,25	0,25/0,40	0,25/0,40
	Keyway r_2	max./min.	0,16/0,08	0,25/0,16	0,25/0,16
	Shaft t_1		3	3,8	4,4
	Hub t_2		1,1	1,3	1,7
	$d_2 = d_1 +$ [1)]		3	3,5	4
	l[4)]		(10)	(12)	(16)
		over	8	10	14
		incl.	45	56	70
	Stepping of l	6 8 10 12 14 16 18 20 22 25 28 32			
	Feather key	d_3	–	–	–
		d_4	–	–	–
		d_5	–	–	–
		d_6 H12	–	–	–
		t_3	–	–	–
		t_4	–	–	–
	Shaft	d_1	–	–	–
		d_6	–	–	–
		t_5	–	–	–
		t_6	–	–	–
		t_7	–	–	–
Screw to DIN 84, DIN 7984 or DIN 6912, adapter sleeve to DIN 1481		–	–	–	–

[1)] For Part 2 (type A, C and E only), the dimensions t_1, t_2 and d_2 in the part enclosed by broad lines apply; all other dimensions such as feather keys are in accordance with Part 1.

[2)] t_2 in the case of interference is intended for exceptional cases in which the feather key is reworked (adapted).

[3)] d_2 is the smallest diameter (inside dimension) of parts which can be slid concentrically over the feather key.

[4)] The values stated within () are the smallest lengths of the feather keys according to Part 2 where these do not coincide with Part 1.

[5)] Only up to 250 for feather keys in accordance with Part 2.

8 7	10 8	12 8	14 9	16 10	18 11	20 12	22 14	25 14
22 30	30 38	38 44	44 50	50 58	58 65	65 75	75 85	85 95
8	10	12	14	16	18	20	22	25
4	5	5	5,5	6	7	7,5	9	9
8	10	12	14	16	18	20	22	25
3,3	3,3	3,3	3,8	4,3	4,4	4,9	5,4	5,4
2,4	2,4	2,4	2,9	3,4	3,4	3,9	4,4	4,4
3 8	3 8	3 8	3,5 9	4 11	4,5 11	5 12	5,5 14	5,5 14
0,25/0,40 0,25/0,16	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,6/0,8 0,6/,04	0,6/0,8 0,6/,04	0,6/0,8 0,6/,04
5,4 1,7 4,5	6 2,1 5,5	6 2,1 6	6,5 2,6 7	7,5 2,6 8	8 3,1 8,5	8 4,1 11	10 4,1 12	10 4,1 12
(20) 18 90	(25) 22 110	(32) 28 140	(40) 36 160	– 45 180	– 50 200	– 56 220	– 63 250	– 70 280[5)]
36 40	45 50	56 63	70 80 90	100 110	125 140	160 180	200 220	250 280[5)]
3,4 6 M3 4	3,4 6 M3 4	4,5 8 M4 5	5,5 10 M5 6	5,5 10 M5 6	6,6 11 M6 8	6,6 11 M6 8	6,6 11 M6 8	9 15 M8 10
2,4 4	2,4 4	3,2 5	4,1 6	4,1 6	4,8 7	4,8 8	4,8 8	6 10
M3 4,5	M3 4,5	M4 5,5	M5 6,5	M5 6,5	M6 9	M6 9	M6 9	M8 11
4 7 M3 × 8	5 8 M3 × 10	6 10 M4 × 10	6 10 M5 × 10	6 10 M5 × 10	7 12 M6 × 12	6 11 M6 × 12	8 13 M6 × 16	9 15 M8 × 16
4 × 8	4 × 8	5 × 10	6 × 12	6 × 12	8 × 16	8 × 16	8 × 16	10 × 20

Spline shaft connections

Characteristics:

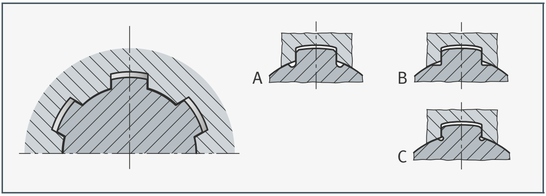
- Direct form contact, separable connection
- The shaft and hub are provided with a large number of splines and grooves (6, 8, 10, ...).
- The splines are integral with the shaft, the grooves are integral with the hub.
- The splines and grooves lie parallel to the axis.

Properties:

- For high, alternating and abruptly acting torques
- For alternating direction of rotation
- Displacement of axis possible under load (for indexing), otherwise axial location of the hub is necessary
- Stress concentration on the shaft, due to notch effect
- Simple mounting and dismounting

Figure 7
Spline shaft connection

A, B, C = various spline shaft types in accordance with DIN 5471



Spline teeth

Characteristics:

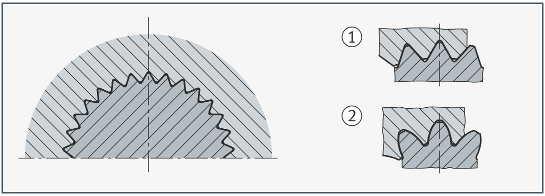
- Direct form contact, separable connection
- A large number of teeth are integrated in the shaft and hub.
- The teeth lie parallel to the axis.
- The design normally has a spline angle of 60°, however other variants such as those with involute teeth are used.
- Simple mounting and dismounting

Properties:

- For transmission of moderate torques
- For alternating direction of rotation
- Less stress concentration than in the case of a spline shaft
- Separable connection, axial retention necessary
- Centring only by tooth flanks and thus restricted running accuracy

Figure 8
Spline teeth

① Notch tooth profile
② Involute tooth profile



Polygon connections

Characteristics:

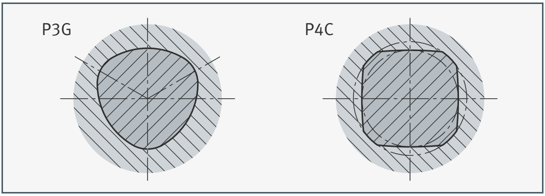
- Direct form contact, separable connection
- Non-circular profiles with defined profile geometry, standardised as P3G and P4C

Properties:

- Highly suitable for transmission of high torques as well as alternating and shock type torques
- Profile P4C very suitable for axial displacement capability under load
- Very little stress concentration with tight fits
- Very smooth running due to self-centring
- High manufacturing costs
- Simple mounting and dismounting

Figure 9
Polygon connection

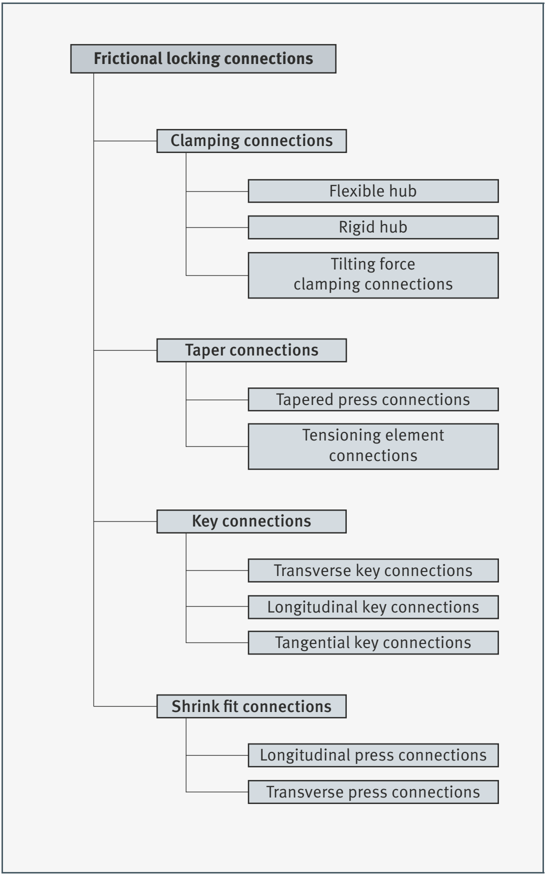
P3G = profile in accordance with DIN 32711
P4C = profile in accordance with DIN 32712



Frictional locking connections

In the case of these connections, forces are transmitted by frictional locking through pressing or clamping.

Figure 10
Frictional locking connections – overview



Tapered press connections

Characteristics:

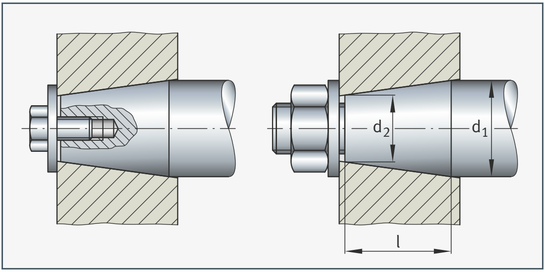
- The necessary joint pressure is achieved by sliding the hub in an axial direction onto the conical shaft seat.
- Axial tensioning is achieved by means of a screw or nut.
- Taper ratio $C = (d_1 - d_2)/l$
- Application of standardised taper ratios in accordance with DIN 254, for example:
  - tapered shaft ends 1:10
  - metric tool tapers 1:20
  - adapter sleeves 1:12 or 1:30

Properties:

- Suitable for high torques, changes in load direction and shocks
- Very smooth running due to precise concentric seat
- Support of high axial forces possible
- No axial displacement of hub possible
- No precise axial positioning of hub possible
- Hub can be displaced in the direction of rotation
- High manufacturing costs, but simple mounting

Figure 11
Taper ratio

d_1, d_2, l = taper dimensions



Axially preloaded tapered press joint

In the case of the axially preloaded tapered press joint, the joint pressure p_F is generated by sliding the hub in an axial direction onto the conical shaft seat with the force F_A.

Figure 12
Axially preloaded tapered press joint

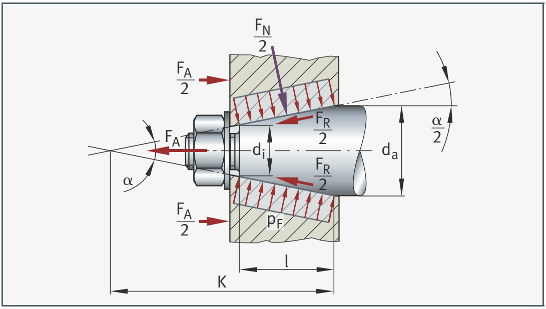
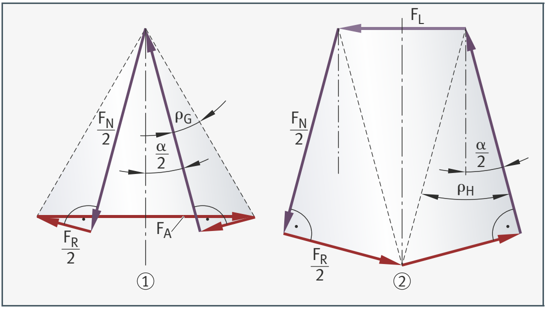


Figure 13
Forces in the axially preloaded tapered press joint

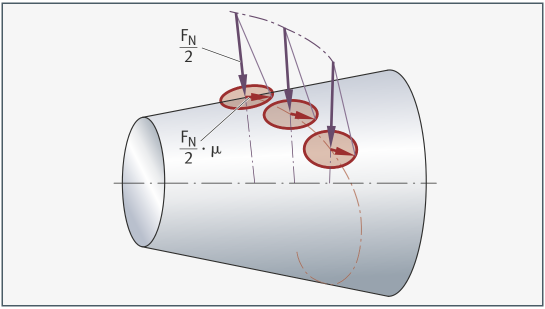
① Sliding-on process
② Dismounting



Taper to DIN 1448/49	$d_a/K = 1{:}10$, $\alpha = 5{,}7258° = 5°43'29''$
Sliding-on force (axial force)	$F_A = F_N \cdot [\sin(\alpha/2) + \mu_G \cdot \cos(\alpha/2)]$
Normal force	$F_N = p_F \cdot d_m \cdot \pi \cdot l/\cos(\alpha/2)$; $d_m = (d_a + d_i)/2$
Joint pressure	$p_F = F_N \cdot \cos(\alpha/2)/(d_m \cdot \pi \cdot l)$
Transmissible circumferential force	$F_U = \mu \cdot F_N/S$ S = safety against slipping, S = 1,2...1,5
Transmissible torque	$M_t = F_U \cdot d_m/2$
Dismounting force	$F_L = F_N \cdot [\sin(\alpha/2) + \mu_H \cdot \cos(\alpha/2)]$

In an axially preloaded tapered press joint, there is an equilibrium between the axial force F_A and the resultant of the normal force component F_N sin (α/2) and the friction component F_N μ cos (α/2).

Figure 14
Forces acting on an axially preloaded tapered press joint during the sliding-on process under initial torque loading



During the first torque transmission, a circumferential force F_U is added to the forces and the equilibrium created by the joining process is altered. Since an additional circumferential force F_U is now acting additionally, the friction vector is rotated out of the axis direction into the direction of the resultant formed by F_U and $\mu_G \cdot F_N$, with the result that the axial sliding-on force is now counteracted by only one component of the friction force. As a result, the force F_A pushes the hub helically further onto the tapered stud until the new equilibrium state is reached. At the same time, the axial force F_A decreases while there is an increase in F_N and consequently p_F. This results in a connection with increased safety against slipping.

If an additional feather key (see DIN 1448/1449) or Woodruff key is used in the tapered press joint, this alone transmits the entire torque since it prevents the helical sliding-on process. In technical terms, a combination of a tapered press joint (with frictional locking) and a feather key (with form fit) is therefore not appropriate.

Cylindrical press connections

In the case of cylindrical press connections, the shaft and hub have an interference fit. The contact pressure necessary for frictional locking is achieved by the elastic deformation of both components after joining. Depending on the type of mounting, a distinction is made between longitudinal press connections, transverse press connections and oil press connections.

Characteristics:

- The shaft and hub have an interference fit before joining.
- Frictional locking through elastic deformation of the shaft and hub
- No additional mechanical connecting elements
- No cross-sectional weakening
- For axial positioning, a means of restricting position (such as a shaft shoulder) is advantageous.

Properties:

- Large forces and moments can be transmitted
- Circumferential and longitudinal forces can be transmitted
- For alternating direction of rotation and abrupt operation
- Simple and economical manufacture
- Difficult dismounting
- Optimum force transmission under uniform flow of forces
- High geometrical strength and operational strength

According to DIN EN ISO 286-1, an interference fit is defined as a fit where the maximum dimension of the hole (hub) is smaller than the minimum dimension of the shaft, in which case interference is then present. After joining, this interference between the hub and the shaft leads to a press fit with a normal force acting on the joined faces. The normal force generates an adhesion force by means of which longitudinal forces (parallel to the axis) and circumferential forces or torques (forces that act tangentially in the joint) can be transmitted from one part to another.

The joining process is carried out as follows:

- longitudinal press fit: longitudinal pressing of the inner part
- transverse press fit:
  - contraction of the outer part (with prior heating)
  - expansion of the inner part (with prior cooling)
  - expansion of the inner part and contraction of the outer part

The joining temperature necessary for the transverse press fit can be determined on the basis of the interference U necessary for transmission of the torque or the axial force. In order to ensure reliable mounting, a mounting clearance S_M (advantageous value: $S_M = U/2$) must be taken into consideration.

If the outer part is heated, the following overtemperature for example is necessary for mounting:

Equation 3

$$\Delta T = \frac{U + S_M}{\alpha \cdot d_{Ai}}$$

Legend

d_{Ai} Inside diameter of outer part	Steel $\alpha = (11 \dots 12)\ 10^{-6}\ 1/K$[1]
α Coefficient of thermal expansion	Cast iron $\alpha = (9 \dots 10)\ 10^{-6}\ 1/K$[1]
ΔT Overtemperature	Aluminium $\alpha = (23 \dots 24)\ 10^{-6}\ 1/K$[1]

[1] α values are only valid for heating.

As a temperature source for heating of the outer part or cooling of the inner part, the following facilities can be used.

Heating facility

Heating facility	Area of application	Notes
Electric heating plates	Volume production parts (normally small)	Heating frequently incomplete, risk of local overheating!
Electric heating cores	Sleeves and hubs	Achievable joining temperature: up to ≈ 50 °C
Bath heating	Outer parts on whose joining surfaces oil may be present during joining	Natural organic heat transfer media up to 300 °C; paraffin-based or silicone-based oils up to 400 °C
Hot air ovens or hot air chambers	Outer parts whose joining surfaces must be dry and devoid of oxide layers	Normally for heating temperatures up to 400 °C; up to 650 °C possible in special ovens

The upper temperature is restricted by the increased loss of strength of the materials used.

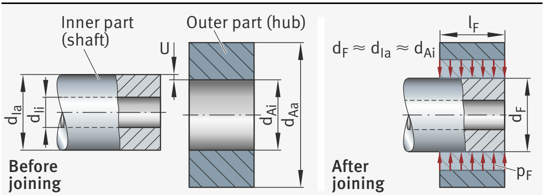
Means of supercooling

Means of supercooling	Chemical formula	Boiling point of gas	Notes
Carbon dioxide snow or dry ice	CO_2	–78,4 °C	Joined part cools relatively slowly; cooling speed is increased when spirit is used as a heat transfer medium. Addition of trichloroethylene prevents icing of the surfaces of the joined part
Liquid nitrogen	N_2	–195,8 °C	Ensure good ventilation during use in closed rooms! Otherwise, no particular risks

Due to the extreme risk of explosion, the use of liquid oxygen or liquid air is not advisable.

Calculation of cylindrical press fits

The following tables show the calculation of a cylindrical press fit for elastic load. In the case of an elasto-plastic load, see DIN 7190-1.



Diameter conditions	
Inner part I (hollow shaft)	$Q_I = d_{Ii}/d_{Ia} \approx d_{Ii}/d_F < 1$
(solid shaft)	$Q_I = 0$
Outer part A	$Q_A = d_{Ai}/d_{Aa} \approx d_F/d_{Aa} < 1$
Interference for the press fit determined from the fit data	$U = d_{Ia} - d_{Ai}$
Smoothing dimension (Rz = mean roughness depth)	$G = 0{,}4\,(Rz_{Ia} + Rz_{Ai})$
Effective adhesion dimension	$Z = U - G$
Constriction of the inner part as a result of the joint pressure	$\Delta d_{Ia} = -\dfrac{p_F\, d_F}{E_I}\left(\dfrac{1+Q_I^{\,2}}{1-Q_I^{\,2}} - \nu_I\right)$
Expansion of the outer part as a result of the joint pressure	$\Delta d_{Ai} = \dfrac{p_F d_F}{E_A}\left(\dfrac{1+Q_A^{\,2}}{1-Q_A^{\,2}} + \nu_A\right)$

Continuation of table, see Page 491.

Continuation of table, Calculation of cylindrical press fits, from Page 490.

When the press fit is joined, the adhesion dimension is transformed into a constriction of the inner part and an expansion of the outer part, where the relationship is as follows:

$$Z = |\Delta d_{Ia}| + |\Delta d_{Ai}|$$

$$Z = p_F \left[\frac{d_F}{E_I} \left(\frac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) + \frac{d_F}{E_A} \left(\frac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) \right]$$

From this relationship, the connection between the effective adhesion dimension and the joint pressure is derived. The smallest joint pressure results from the minimum interference of the fit data for the press fit.

Axial force transmission	$F_A = p_F \cdot d_F \cdot \pi \cdot l_F \cdot \mu / S$
Necessary joint pressure	$p_{F\,req} = F_A \cdot S/(d_F \cdot \pi \cdot l_F \cdot \mu)$
Torque transmission	$M_t = p_F \cdot d_F \cdot \pi \cdot l_F \cdot \mu \cdot (d_F/2)/S$
Necessary joint pressure	$p_{F\,req} = 2\,M_t \cdot S/(d_F^2 \cdot \pi \cdot l_F \cdot \mu)$

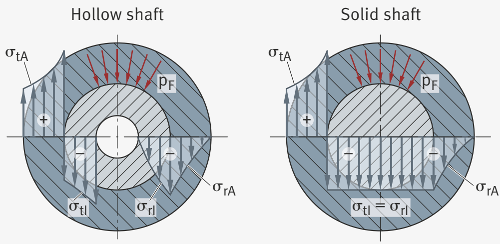
Type of press fits and stresses			Coefficients of adhesion μ (DIN 7190)	
			Dry	Lubricated
Coefficients of adhesion for transverse press fits in longitudinal and transverse directions				
Steel/steel combination	Hydraulic oil joint	Joined using mineral oil	–	0,12
		Degreased press surfaces, joined using glycerine	0,18	–
	Shrinkage joint	Heating up to 300 °C	–	0,14
		Degreased press surfaces, heating up to 300 °C	0,20	–
Steel/cast iron combination	Hydraulic oil joint	Joined using mineral oil	–	0,16
		Degreased press surfaces	0,16	–
Coefficients of adhesion for longitudinal press fits				
Shaft material		Chromium steel	–	–
Hub material		E335	0,11	0,08
		S235 JRG2	0,10	0,07
		EN-GJL-259	0,12 ... 0,14	0,06

Continuation of table, see Page 492.

Continuation of table, Calculation of cylindrical press fits, from Page 491.

Safety against slipping	
Transverse press joint	$S = 1{,}5 \ldots 2{,}0$
Longitudinal press joint	$S = 2{,}0 \ldots 2{,}5$

The stress profiles in the outer and inner parts of the cylindrical press joint (loading of thick-walled pipes under internal and external pressure) can be found in the compilation of the most important load types.



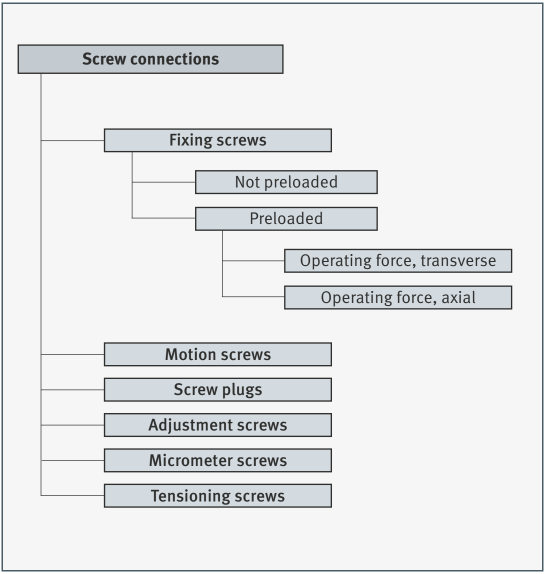
The loads that are critical for the press joint generally occur on the inner edge of the outer part (hub). In the case of a hollow shaft, it is also necessary to check the stress σ_{tIi} on the inner edge of the inner part.

Stresses on the inner edge of the outer part	
Tangential stress	$\sigma_{tAi} = p_F \dfrac{1+Q_A^2}{1-Q_A^2}$
Radial stress	$\sigma_{rAi} = -p_F$
Equivalent stress (DEH)	$\sigma_{vAi} = \sqrt{\sigma_{tAi}^2 + \sigma_{rAi}^2 - \sigma_{tAi} \cdot \sigma_{rAi}}$ $\sigma_{vAi} = \dfrac{2 \cdot p_F}{1-Q_A^2} < \sigma_{per}$

Screw connections

Screw connections are a combination of form fit and frictional locking.

Figure 15
Screw connections – overview



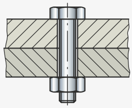
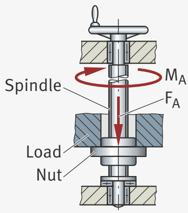
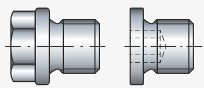
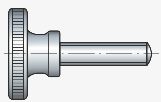
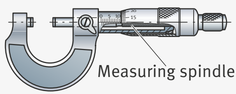
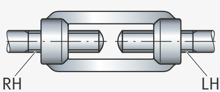
Form fit and frictional locking connections

Screw connections are based on a combination of a screw or grub screw with an external thread and a component with an internal thread (normally a nut) where form fit between the two is achieved in the thread.

In the thread, which can be seen in an unwound form as a skewed plane, rotation of the screw relative to the nut gives sliding of the thread flanks of the screw on the thread flanks of the nut and thereby longitudinal motion.

Design elements

Depending on the intended purpose, a distinction is made between various types of screw design.

Screw connection	Description
Fixing screw 	■ Separable connection of components ■ Design as through screw connection or fit screw connection possible
Motion screw 	■ Conversion of rotary motion into longitudinal motion and vice versa ■ Generation of high forces (design as ball screw possible for reduction of friction)
Screw plug 	■ Closing off of filling or outlet openings (sealing screw)
Adjustment screw 	■ Alignment of devices and instruments ■ Example: knurled screw
Micrometer screw 	■ Length measurement in micron range
Tensioning screw 	■ Generation of tensioning forces ■ Example: turnbuckle RH = right hand thread LH = left hand thread

Fixing screws

Screws are the most widely used machine elements for joining components. In comparison with welded, soldered, bonded and riveted joints, components can be separated non-destructively and joined again. As a fixing element, the screw must perform the task of connecting components under the preload force applied during assembly and of maintaining this connection in the event of static and dynamic loads.

The screw connection offers the advantages of simple assembly, non-destructive separability and ability to transmit large forces. In contrast, there is the drawback that the **highly notched** screw may suffer fatigue fracture under dynamic loads or that an inadmissible breakdown of the preload force may occur in the joint as the result of settling phenomena at the contact points, or as a result of the nut working loose from the screw. A screw connection subjected to high loads will survive or fail depending on the ability of its screws to maintain or lose the preload force applied during assembly. Very often, the cause of a fatigue fracture in a screw can be attributed to a prior reduction in the preload force. It is therefore imperative that a screw connection should be carefully designed and calculated.

Through screw connections

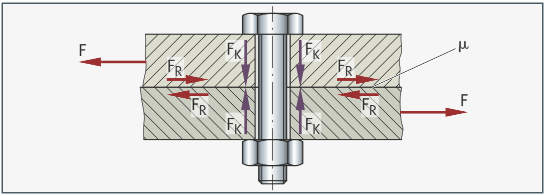
Characteristics:

- Frictional locking connection
- Clamping force generated by tightening of the nut
- Screw subjected to tensile load (torsion due to the tightening torque)
- Securing of screw necessary
- Additional centring necessary

Properties:

- Separable connection
- Suitable for large forces
- Easy to fit
- Stress concentration due to hole in the flange (notch effect)

Figure 16
Through screw connection



Fit screw connections

Characteristics:

- Form fit and frictional locking connection
- Screw subjected to shear load and contact pressure (torsion due to the tightening torque)
- Centring by means of fit between screw shank and flange
- Securing of screw necessary

Properties:

- Separable connection
- Suitable for centring
- Easy to fit
- Stress concentration due to hole in the flange (notch effect)
- Significantly more expensive compared to through screw connection

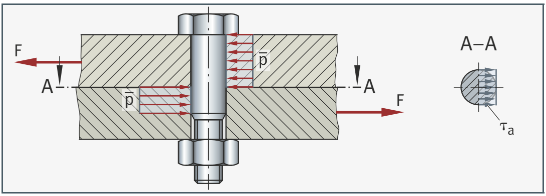
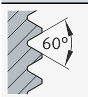
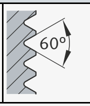
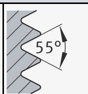
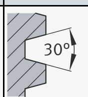
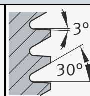
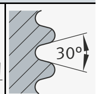


Figure 17
Fit screw connection

Base forms of the most common threads

Certain thread types defined in the corresponding DIN standards have proven themselves in accordance with the various conditions of use.

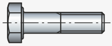
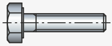
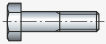
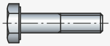
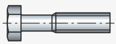
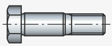
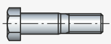
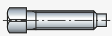
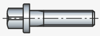
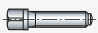
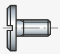
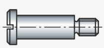
The base forms of the most common threads are shown in the following table.

Fixing thread			**Motion thread**		
					
Metric ISO thread		Whitworth pipe thread	Trapezoidal thread	Buttress thread	Round thread
Coarse pitch thread	Fine pitch thread				
DIN 13 DIN 14		DIN 2999 DIN 3858	DIN 103 DIN 263 DIN 380	DIN 513 DIN 2781	DIN 405 DIN 15403 DIN 20400

Unless otherwise indicated, these are right hand threads. A left hand thread is always marked with LH (= left hand).
The following sections specifically cover the use of screws as fixing elements.

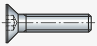
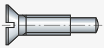
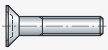
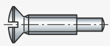
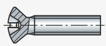
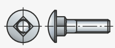
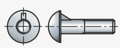
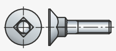
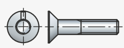
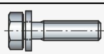
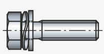
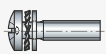
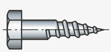
Overview of standardised screws

The following table gives an overview of standardised screws.

	DIN EN ISO 4014 DIN EN ISO 8765	Hexagon head screws Metric thread, metric fine pitch thread
	DIN EN ISO 4017 DIN EN ISO 8676	Hexagon head screws Full thread
	DIN EN ISO 4016	(Unfinished) hexagon head screws for steel structures
	DIN EN ISO 4018	(Unfinished) hexagon head screws Full thread
	DIN EN 14399-4	Hexagon head screws with large widths across flats
	DIN 561	Hexagon head set screws with full dog point
	DIN 564	Hexagon head set screws with half dog point and flat cone point
	DIN 24015 DIN 2510	Hexagon head screws with thin shank
	DIN 609	Hexagon fit screws
	DIN 7968	Hexagon fit screws for steel structures
	DIN 479	Square head screws with short dog point
	DIN 478	Square head screws with collar
	DIN 480	Square head screws with collar and short dog point with rounded end
	DIN EN ISO 4762 DIN 6912	Hexagon socket head cap screws
	DIN EN ISO 1207	Slotted cheese head screws
	DIN EN ISO 1580	Slotted pan head screws
	DIN 920	Slotted pan head screws with small head
	DIN 921	Slotted pan head screws with large head
	DIN 922	Slotted pan head screws with small head and full dog point
	DIN 923	Slotted pan head screws with shoulder
	DIN EN ISO 7045	Pan head screws with cross recess

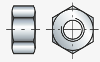
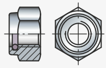
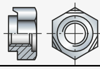
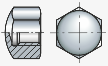
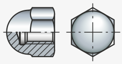
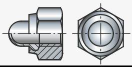
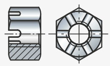
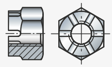
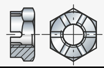
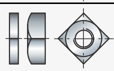
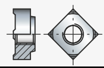
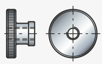
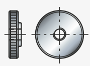
Continuation of table, see Page 498.

Continuation of table, Overview of standardised screws, from Page 497.

	DIN EN ISO 10642	Slotted countersunk head screws
	DIN EN ISO 2009	Slotted countersunk flat head screws
	DIN 925	Slotted countersunk head screws with full dog point
	DIN 7969	Slotted countersunk head screws (for steel structures)
	DIN EN ISO 7046	Countersunk flat head screws with cross recess
	DIN EN ISO 2010	Slotted raised countersunk head screws
	DIN 924	Slotted raised countersunk head screws with full dog point
	DIN EN ISO 7047	Slotted cross recess raised countersunk head screws
	DIN 603	Mushroom head square neck screws
	DIN 607	Cup head nib screws
	DIN 605 DIN 608	Flat countersunk square neck screws
	DIN 604	Flat countersunk nib screws
	DIN 404	Slotted capstan screws
	DIN EN ISO 10644 DIN EN ISO 10673	Screw and washer assemblies
	DIN 6900-2 DIN 6904	Screw and washer assemblies Curved spring washers
	DIN 6900-4 DIN 6907	Screw and washer assemblies Serrated lock washers
	DIN EN ISO 1479	Self-tapping screws
	DIN 7513 DIN 7516	Thread cutting screws
	DIN 571	Wood screws

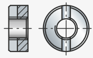
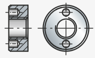
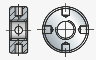
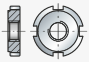
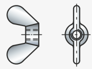
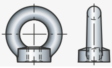
Overview of standardised nuts

The following table gives an overview of standardised nuts.

	DIN EN ISO 4032 DIN EN ISO 4034 DIN EN ISO 4035	(Unfinished) hexagon nuts Hexagon nuts Metric thread, metric fine pitch thread Flat hexagon nuts
	DIN EN ISO 7040 DIN EN ISO 7042 DIN EN ISO 10511	Hexagon nuts Self-locking
	DIN 929	Hexagon weld nuts
	DIN 917	Cap nuts Low type
	DIN 1587	Cap nuts High type
	DIN 986	Cap nuts Self-locking
	DIN 935 to M10	Hexagon castle nuts Metric thread, metric fine pitch thread
	DIN 935 to M10	Hexagon castle nuts Metric thread, metric fine pitch thread
	DIN 979	Hexagon thin castle nuts
	DIN 557 DIN 562	(Unfinished) square nuts Flat square nuts
	DIN 928	Square weld nuts
	DIN 466	Knurled nuts, high type
	DIN 467	Knurled nuts, flat type

Continuation of table, see Page 500.

Continuation of table, Overview of standardised nuts, from Page 499.

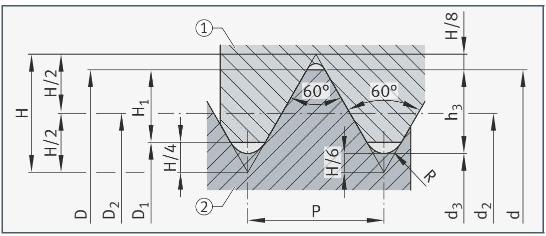
	DIN 546	Slotted round nuts
	DIN 547	Round nuts with drilled holes in one face
	DIN 548 DIN 1816	Round nuts with set pin holes in side
	DIN 981 DIN 1804	Locknuts
	DIN 315	Wing nuts
	DIN 582	Lifting eye nuts

Metric ISO threads

The following diagram and the subsequent table describe metric ISO threads in accordance with DIN 13-1 for coarse pitch threads with nominal thread diameter from 1 to 52 mm.
The following example shows a metric ISO thread M12 where d = D = 12 mm:

Figure 18
Parameters for metric ISO thread

① Nut thread
② Bolt thread



Legend

$D_1 = d - 2\,H_1$

$d_2 = D_2 = d - 0{,}649\,52\,P$

$d_3 = d - 1{,}226\,87\,P$

$H = 0{,}866\,03\,P$

$H_1 = 0{,}541\,27\,P$

$h_3 = 0{,}613\,43\,P$

$$R = \frac{H}{6} = 0{,}144\,34\,P$$

Table of coarse pitch threads with nominal thread diameter from 1 to 52 mm:

Nominal thread diameter d = D			Pitch	Flank diameter	Core diameter		Thread depth		Rounding
Series 1	Series 2	Series 3	P	$d_2 = D_2$	d_3	D_1	h_3	H_1	R
1	–	–	0,25	0,838	0,693	0,729	0,153	0,135	0,036
–	1,1	–	0,25	0,938	0,793	0,829	0,153	0,135	0,036
1,2	–	–	0,25	1,038	0,893	0,929	0,153	0,135	0,036
–	1,4	–	0,3	1,205	1,032	1,075	0,184	0,162	0,043
1,6	–	–	0,35	1,373	1,171	1,221	0,215	0,189	0,051
–	1,8	–	0,35	1,573	1,371	1,421	0,215	0,189	0,051
2	–	–	0,4	1,740	1,509	1,567	0,245	0,217	0,058
–	2,2	–	0,45	1,908	1,648	1,713	0,276	0,244	0,065
2,5	–	–	0,45	2,208	1,948	2,013	0,276	0,244	0,065
3	–	–	0,5	2,675	2,378	2,459	0,307	0,271	0,072
–	3,5	–	0,6	3,110	2,764	2,850	0,368	0,325	0,087
4	–	–	0,7	3,545	3,141	3,424	0,429	0,379	0,101
–	4,5	–	0,75	4,013	3,580	3,688	0,460	0,406	0,108
5	–	–	0,8	4,480	4,019	4,134	0,491	0,433	0,115
6	–	–	1	5,350	4,773	4,917	0,613	0,541	0,144
–	–	7	1	6,350	5,773	5,917	0,613	0,541	0,144
8	–	–	1,25	7,188	6,466	6,647	0,767	0,677	0,015
–	–	9	1,25	8,168	7,466	7,647	0,767	0,677	0,144
10	–	–	1,5	9,026	8,160	8,376	0,920	0,812	0,217
–	–	11	1,5	10,026	9,160	9,376	0,920	0,812	0,217
12	–	–	1,75	10,863	9,853	10,106	1,074	0,947	0,253
–	14	–	2	12,701	11,546	11,835	1,227	1,083	0,289
16	–	–	2	14,701	13,546	13,835	1,227	1,083	0,289
–	18	–	2,5	16,376	14,933	15,294	1,534	1,353	0,361
20	–	–	2,5	18,376	16,933	17,294	1,534	1,353	0,361
–	22	–	2,5	20,376	18,933	19,294	1,534	1,353	0,361
24	–	–	3	22,051	20,319	20,752	1,840	1,624	0,433
–	27	–	3	25,051	23,319	23,752	1,840	1,624	0,433
30	–	–	3,5	27,727	25,706	26,211	2,147	1,894	0,505
–	33	–	3,5	30,727	28,706	29,211	2,147	1,894	0,505
36	–	–	4	33,402	31,093	31,670	2,454	2,165	0,577
–	39	–	4	36,402	34,093	34,670	2,454	2,165	0,577
42	–	–	4,5	39,077	36,479	37,129	2,760	2,436	0,650
–	45	–	4,5	42,077	39,479	40,129	2,760	2,436	0,650
48	–	–	5	44,752	41,866	42,587	3,067	2,706	0,722
–	52	–	5	48,752	45,866	46,587	3,067	2,706	0,722

Selection of coarse pitch and fine pitch threads

A selection of coarse pitch and fine pitch threads in accordance with DIN 13-2 is given below.

Nominal thread diameter			Pitch P for								
d = D			Coarse pitch thread	Fine pitch thread							
Series 1	Series 2	Series 3		4	3	2	1,5	1,25	1	0,75	0,5
1	–	–	0,25	–	–	–	–	–	–	–	–
1,2	–	–	0,25	–	–	–	–	–	–	–	–
–	1,4	–	0,3	–	–	–	–	–	–	–	–
1,6	–	–	0,35	–	–	–	–	–	–	–	–
–	1,8	–	0,35	–	–	–	–	–	–	–	–
2	–	–	0,4	–	–	–	–	–	–	–	–
–	2,2	–	0,45	–	–	–	–	–	–	–	–
2,5	–	–	0,45	–	–	–	–	–	–	–	–
3	–	–	0,5	–	–	–	–	–	–	–	–
–	3,5	–	0,6	–	–	–	–	–	–	–	–
4	–	–	0,7	–	–	–	–	–	–	–	0,5
5	–	–	0,8	–	–	–	–	–	–	–	0,5
6	–	–	1	–	–	–	–	–	–	0,75	0,5[1)]
8	–	–	1,25	–	–	–	–	–	1	0,75	0,5[1)]
10	–	–	1,5	–	–	–	–	1,25	1	0,75	–
12	–	–	1,75	–	–	–	1,5	1,25	1	–	–
–	14	–	2	–	–	–	1,5	1,25	1	–	–
–	–	15	–	–	–	–	1,5	–	1	–	–
16	–	–	2	–	–	–	1,5	–	1	–	–
–	–	17	–	–	–	–	–	–	1	–	–
–	18	–	2,5	–	–	2	1,5	–	1	–	–
20	–	–	2,5	–	–	2	1,5	–	1	–	–
–	22	–	2,5	–	–	2	1,5	–	1	–	–
24	–	–	3	–	–	2	1,5	–	1	–	–
–	–	25	–	–	–	–	1,5	–	–	–	–
–	–	26	–	–	–	–	1,5	–	–	–	–
–	27	–	3	–	–	2	1,5	–	–	–	–

Continuation of table, see Page 503.

1) Not included in ISO 261:1998.

Continuation of table, Selection of coarse pitch and fine pitch threads,
from Page 502.

Nominal thread diameter d = D			Pitch P for									
			Coarse pitch thread	Fine pitch thread								
Series 1	Series 2	Series 3		4	3	2	1,5	1,25	1	0,75	0,5
–	–	28	–	–	–	–	1,5	–	–	–	–
30	–	–	3,5	–	–	2	1,5	–	–	–	–
–	–	32	–	–	–	–	1,5	–	–	–	–
–	33	–	3,5	–	–	2	1,5	–	–	–	–
–	–	35	–	–	–	–	1,5	–	–	–	–
36	–	–	4	–	3	2	1,5	–	–	–	–
–	–	38	–	–	–	–	1,5	–	–	–	–
–	39	–	4	–	3	2	–	–	–	–	–
–	–	40	–	–	–	–	1,5	–	–	–	–
42	–	–	4,5	–	3	2	1,5	–	–	–	–
–	45	–	4,5	–	3	2	1,5	–	–	–	–
48	–	–	5	–	3	2	1,5	–	–	–	–
–	–	50	–	–	–	–	1,5	–	–	–	–
–	52	–	5	–	3	2	1,5	–	–	–	–
–	–	55	–	–	–	2	1,5	–	–	–	–
56	–	–	5,5	4	3	2	1,5	–	–	–	–
–	–	58	–	–	–	–	1,5	–	–	–	–
–	60	–	5,5	4	3	2	1,5	–	–	–	–
64	–	–	6	4	3	2	–	–	–	–	–
–	–	65	–	–	–	2	–	–	–	–	–
–	68	–	6	4	3	2	–	–	–	–	–

Grades for screws

Mechanical characteristics of screws are subdivided into grades. The grades are identified by two numbers that are separated by a full stop. The first number is 1/100 of the minimum tensile strength in N/mm^2. The second number is 10 times the ratio between the minimum yield strength and the minimum tensile strength of the screw material. The following table shows an excerpt from DIN EN ISO 898-1.

		Grades									
							8.8				
		4.6	4.8	5.6	5.8	6.8	≤M16	>M16[1)]	9.8	10.9	12.9
Tensile strength R_m N/mm^2	nom.	400	400	500	500	600	800	800	900	1000	1200
	min.	400	420	500	520	600	800	830	900	1040	1220
Yield strength R_{eL} or 0,2% proof stress $R_{p0,2}$ or 0,0048 d proof stress for whole screws R_{pf} MPa = N/mm^2	nom.	240	320	300	400	480	640	640	720	900	1080
	min.	240	340	300	420	480	640	660	720	940	1100
Elongation at fracture A %	nom.	22	–	20	–	–	12	12	10	9	8
Vickers hardness HV F ≥ 98 N	min.	120	130	155	160	190	250	255	290	320	385
	max.	220	220	220	220	250	320	335	360	380	435
Brinell hardness HBW F = 30 D^2	min.	114	124	147	152	181	245	250	286	316	380
	max.	209	209	209	209	238	316	331	355	375	429
Notched bar impact work (ISO-U) Joule	min.	–	–	27	–	–	27	27	27	27	–

1) For bolting of steel structures, the limit is 12 mm.

Marking

At or above M5, the marking of the grade is applied directly on the screw head (end face or cylindrical face) or, in the case of grub screws, on the shank. The markings must conform to the specifications in the following table. The full stop in the marking may be omitted in this case. Examples of marking are shown in Figure 19, Page 505.

	Grades								
	4.6	4.8	5.6	5.8	6.8	8.8	9.8	10.9	12.9
Marking of grade									
For screws with full load carrying capacity	4.6	4.8	5.6	5.8	6.8	8.8	9.8	10.9	12.9
For screws with reduced load carrying capacity	04.6	04.8	05.6	05.8	06.8	08.8	09.8	010.9	012.9

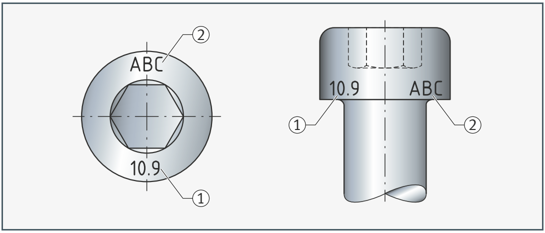


Figure 19
Examples of the marking of grades for screws of M5 and larger

① Marking of grade
② Manufacturer's symbol

In the case of smaller screws or a special head shape, marking may be carried out using symbols in accordance with the clockface system for marking of screws in DIN EN ISO 898-1.

Grades for nuts

The following table lists nuts with coarse pitch thread in accordance with DIN EN 20898-2.

Nut grade	Matching screw				Nut	
					Type 1	Type 2
	Grade			Value	Value	
4	3.6	4.6	4.8	>M16	>M16	–
5	3.6	4.6	4.8	≤M16	≤M39	–
	5.6	5.8	–	≤M39		
6	6.8			≤M39	≤M39	–
8	8.8			≤M39	≤M39	>M16 ≤M39
9	9.8			≤M16	–	≤M16
10	10.9			≤M39	≤M39	–
12	12.9			≤M39	≤M16	≤M39

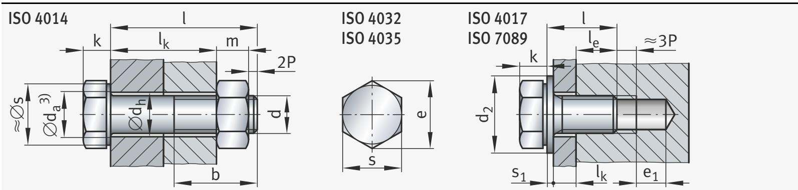
Nuts with a nominal height $\geq 0{,}8 \cdot D$ (effective thread height $\geq 0{,}6 \cdot D$) are identified by a number corresponding to the highest screw grade with which the nut may be matched.

Type 1: Nut height in accordance with DIN EN 20898-2, nominal height $\geq 0{,}8 \cdot D$

Type 2: Nut height in accordance with DIN EN 20898-2 (approx. 10% greater than Type 1)

A screw with a thread M5 to M39, matched with a nut of the corresponding grade, will give a connection that can be subjected to a load up to the test force defined for the screw without stripping of the thread.

Connections using hexagon head screws The following table shows design dimensions for connections using hexagon head screws in a selection taken from various DIN standards:



DIN EN ISO DIN EN DIN	4014, 4032 etc. 475, ISO 272		4014	4014	4017	4014	4014	4032	4035	 935	1234
Thread	Width across flats A/F	Width across corners	Height of head	Nominal length range	Nominal length range	Length of thread for $l \leqq 125$ mm	Length of thread for $l > 125 \ldots 200$ mm	Height of nut Type 1	Height of nut Low type	Hexagon castle nut	Splint
d	s	e	k	l[1]	l[1]	b	b	m[2]	m	h	$d_1 \times l_1$
M3	5,5	6,01	2	20 ... 30	6 ... 30	12	18	2,4	1,8	–	–
M4	7	7,66	2,8	25 ... 40	8 ... 40	14	20	3,2	2,2	5	1 × 10
M5	8	8,79	3,5	25 ... 50	10 ... 50	16	22	4,7	2,7	6	1,2 × 12
M6	10	11,05	4	30 ... 60	12 ... 60	18	24	5,2	3,2	7,5	1,6 × 14
M8	13	14,38	5,3	40 ... 80	16 ... 80	22	28	6,8	4	9,5	2 × 16
M10	16	17,77	6,4	45 ... 100	20 ... 100	26	32	8,4	5	12	2,5 × 20
M12	18	20,03	7,5	50 ... 120	25 ... 120	30	36	10,8	6	15	3,2 × 22
M14	21	23,38	8,8	60 ... 140	30 ... 140	34	40	12,8	7	16	3,2 × 25
M16	24	26,75	10	65 ... 160	30 ... 200	38	44	14,8	8	19	4 × 28
M20	30	33,53	12,5	80 ... 200	40 ... 200	46	52	18	10	22	4 × 36
M24	36	39,98	15	90 ... 240	50 ... 200	54	60	21,5	12	27	5 × 40
M30	46	51,28	18,7	110 ... 300	60 ... 200	66	72	25,6	15	33	6,3 × 50
M36	55	61,31	22,5	140 ... 360	70 ... 200	–	84	31	18	38	6,3 × 63

All dimensions in mm; exception: A_p: mm^2.

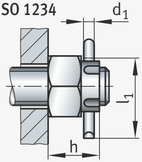
Source: Roloff/Matek, Maschinenelemente, Vieweg+Teubner, 24. Auflage 2021.

[1] Stepping of length l:
... 6 8 10 12 16 20 25 30 35 40 45 50 55 60 65 70 80 90 100 110 120 130 140 150 160 180 200 220 240 260 280 300 320 340 ... 500.

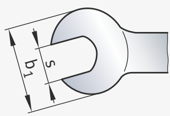
[2] Higher resistance to stripping due to larger nut heights in accordance with DIN EN ISO 4033 with $m/d \approx 1$.

[3] The transition diameter d_a restricts the maximum transition from the radius to the flat head seat. In accordance with DIN 267-2, the following applies in general for the product classes A(m) and B(mg) up to M18: d_a = through hole "medium" + 0,2 mm and for M20 to M39: d_a = through hole "medium" + 0,4 mm. For the product class C(g), the same equations apply with the through hole "coarse".

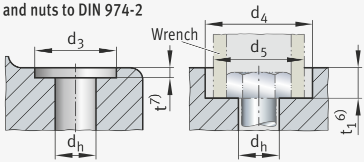
DIN 935
ISO 1234



DIN 3110



Counterbores for normal hexagon head screws and nuts to DIN 974-2



7089, 7090		20273				76	3129	3110	974-2				
Washers		Through hole[4] Series Fine	Medium	Coarse	Head or nut seating surface	Blind hole transition (standard)	Socket wrench insert Outside diameter	Open end wrench Width	Series 1 For socket wrenches, socket wrench inserts to DIN 3124	Series 2 For offset ring wrenches, socket wrench inserts to DIN 3129	Series 3 For countersinks under restricted space conditions	For screws to ISO 4014 and ISO 4017 without support parts	For nuts to ISO 4032 and ISO 4035 without support parts and thread overhang
d_2	s_1	d_h	d_h	d_h	A_p[5]	e_1	d_5	b_1	d_4	d_4	d_3	t_1	t_1
7	0,5	3,2	3,4	3,6	7,5	2,8	9,7	19	11	11	9	2,6	2,8
9	0,8	4,3	4,5	4,8	11,4	3,8	12,8	20	13	15	10	3,4	3,6
10	1	5,3	5,5	5,8	13,6	4,2	15,3	22	15	18	11	4,1	5,1
12	1,6	6,4	6,6	7	28	5,1	17,8	27	18	20	13	4,6	5,6
16	1,6	8,4	9	10	42	6,2	21,5	34	24	26	18	6,1	7,4
20	2	10,5	11	12	72,3	7,3	27,5	38	28	33	22	7,3	9
24	2,5	13	13,5	14,5	73,2	8,3	32,4	44	33	36	26	8,4	11,4
28	2,5	15	15,5	16,5	113	9,3	36,1	49	36	43	30	9,7	13,4
30	3	17	17,5	18,5	157	9,3	42,9	56	40	46	33	10,9	15,4
37	3	21	22	24	244	11,2	50,4	66	46	54	40	13,4	18,4
44	4	25	26	28	356	13,1	64,2	80	58	73	48	16,1	22,3
56	4	31	33	35	576	15,2	76,7	96	73	82	61	20,1	26,6
66	5	37	39	42	856	16,8	87,9	–	82	93	73	23,9	32

4) For screws of the mainly used product class A(m), to be designed using the series “medium” so that $d_h \approx d_a$.

5) Ring-shaped seating surface determined with minimum diameter d_w of the seating surface and through hole of series “medium”. Possible reduction by the chamfering of the through hole.

6) The countersink depth for flush closing is determined from the sum of the maximum values of the screw head height and the height of the support parts with the addition of: 0,4 mm for M3 to M6; 0,6 mm for M8 to M20; 0,8 mm for M24 to M27 and 1,0 mm at or greater than M30.
The countersink depth on the nut side must be defined including the overhang of the screw end in a suitable manner.

7) t needs to be no larger than necessary for the production by machining of a circular surface perpendicular to the axis of the through hole.

Thread runout and thread undercut

Thread runouts and thread undercuts are described in DIN 76-1.
This standard is valid for screws and design parts with a metric ISO thread (coarse pitch/fine pitch thread) in accordance with DIN 13-1 and DIN ISO 261:1999.

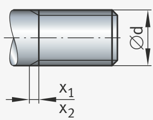
For the thread undercut on external and internal threads, type letters are included. As a result, there is no need to enter the dimensions.
If no type letters are stated, standard case A or standard case C applies.
Example of the designation of a thread undercut of type B:
thread undercut DIN 76 – B.

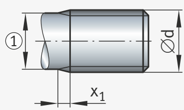
Metric external threads

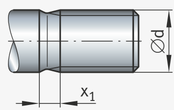
The following two tables show thread runouts and thread undercuts for metric external threads in accordance with DIN 76-1.

Thread runout

x_1 Standard case
x_2 Short
① Shank diameter = flank diameter

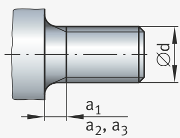


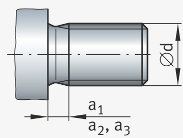




Spacing between the last full thread turn and the seating surface
(for parts with full thread)

a_1 Standard case
x_2 Short
a_3 Long

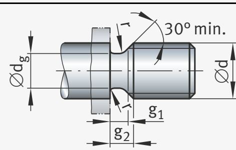




Thread undercut

Type A:
g_1 and g_2
Standard case

Type B:
g_1 and g_2
Short



Thread pitch P	Nominal thread diameter d (coarse pitch thread)	Thread runout		Spacing			Thread undercut					
		x_1[1] max.	x_2[2] max.	a_1[3] max.	a_2[4] max.	a_3[5] max.	d_g	g_1 min.		g_2 max.		r approx.
		Standard	Short	Standard	Short	Long	h13[6]	A[7] Standard	B[8] Short	A[7] Standard	B[8] Short	
0,5	3	1,25	0,7	1,5	1	–	d – 0,8	1,1	0,5	1,75	1,25	0,2
0,6	3,5	1,5	0,75	1,8	1,2	–	d – 1	1,2	0,6	2,1	1,5	0,4
0,7	4	1,75	0,9	2,1	1,4	–	d – 1,1	1,5	0,8	2,45	1,75	0,4
0,75	4,5	1,9	1	2,25	1,5	–	d – 1,2	1,6	0,9	2,6	1,9	0,4
0,8	5	2	1	2,4	1,6	3,2	d – 1,3	1,7	0,9	2,8	2	0,4
1	6; 7	2,5	1,25	3	2	4	d – 1,6	2,1	1,1	3,5	2,5	0,6
1,25	8	3,2	1,6	3,75	2,5	5	d – 2	2,7	1,5	4,4	3,2	0,6
1,5	10	3,8	1,9	4,5	3	6	d – 2,3	3,2	1,8	5,2	3,8	0,8
1,75	12	4,3	2,2	5,25	3,5	7	d – 2,6	3,9	2,1	6,1	4,3	1

[1] The thread runout x_1 is always valid if no other indications are given in the individual standards and drawings.

[2] The thread runout x_2 is only valid for the cases in which a short thread runout is necessary for technical reasons.

[3] The spacing a_1 is always valid if no other indications are given in the individual standards and drawings.

[4] The spacing a_2 is valid for slotted screws and crosshead screws and those cases in which a short spacing is necessary for technical reasons.

[5] The spacing a_3 is only valid for screws in product class C.

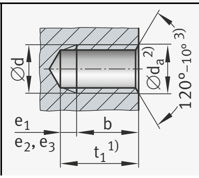
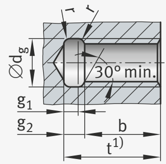
[6] Tolerance class h12 for threads up to nominal diameter 3 mm.

[7] The thread undercut of type A is always valid if no other indications are given in the individual standards and drawings. In a variation from ISO 4755, g_2 = 3,5P applies instead of 3P.

[8] The thread undercut of type B (short) is only valid for special cases in which a short thread undercut is necessary for technical reasons. This thread undercut requires special tools for thread production. It is not included in ISO 4755.

Metric internal threads (threaded blind holes)

The following two tables show thread runouts and thread undercuts for metric internal threads (threaded blind holes) in accordance with DIN 76-1.

Threaded blind hole with thread runout		Threaded blind hole with thread undercut	
e_1 Standard case e_2 Short e_3 Long		Type C Standard case Type D Short	

1) Permissible deviation for the calculated dimension t, t_1: $+0{,}5 \cdot P$.

2) $d_{a\,min} = 1 \cdot d$; $d_{a\,max} = 1{,}05 \cdot d$; countersunk diameter for nuts: see dimensional standards.

3) The special cases 90°, 60°, ... must be indicated in the drawing. Recommendation of 60° for set screws with thread runout and for centring holes, cylindrical countersink for stud bolts made from light metal.

Thread pitch P	Nominal thread diameter d (coarse pitch thread)	Thread runout including blind hole overhang			Thread undercut					
		e_1[1)]	e_2[2)]	e_3[3)]	d_g	g_1 min.		g_2 max.		r approx.
		Guide values				C[4)]	D[5)]	C[4)]	D[5)]	
		Standard	Short	Long	H13	Standard	Short	Standard	Short	
0,5	3	2,8	1,8	4,5	d + 0,3	2	1,25	2,7	2	0,2
0,6	3,5	3,4	2,1	5,4	d + 0,3	2,4	1,5	3,3	2,4	0,4
0,7	4	3,8	2,4	6,1	d + 0,3	2,8	1,75	3,8	2,75	0,4
0,75	4,5	4	2,5	6,4	d + 0,3	3	1,9	4	2,9	0,4
0,8	5	4,2	2,7	6,8	d + 0,3	3,2	2	4,2	3	0,4
1	6; 7	5,1	3,2	8,2	d + 0,5	4	2,5	5,2	3,7	0,6
1,25	8	6,2	3,9	10	d + 0,5	5	3,2	6,7	4,9	0,6
1,5	10	7,3	4,6	11,6	d + 0,5	6	3,8	7,8	5,6	0,8
1,75	12	8,3	5,2	13,3	d + 0,5	7	4,3	9,1	6,4	1

1) The thread runout e_1 is always valid if no other indications are given in the individual standards and drawings.

2) The thread runout e_2 is only valid for the cases in which a short overhang is necessary for technical reasons.

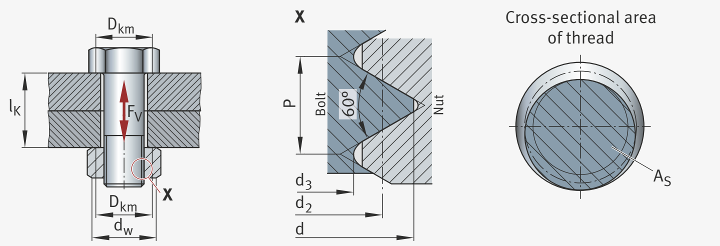
3) The thread runout e_3 is only valid for the cases in which a long overhang is necessary for technical reasons.

4) The thread undercut of type C is always valid if no other indications are given in the individual standards and drawings.

5) The thread undercut of type D (short) is only valid for special cases in which a short thread undercut is necessary for technical reasons.

General calculation of fixing screws

Screw connections can be calculated with the aid of the following relationships:



Nominal thread diameter Flank diameter Core diameter Pitch	d d_2 d_3 P See DIN 13 page 1 ... 12 or table, Page 501
Pitch angle	$\tan \varphi = P/(d_2 \cdot \pi)$; $\varphi = \arctan P/(d_2 \cdot \pi)$
Cross-sectional area of thread	$A_S = \frac{\pi}{4}\left(\frac{d_2 + d_3}{2}\right)^2$
Mean head (nut) seat ∅	$D_{km} = (d_w + d_h)/2$, see Page 530
Coefficients of friction in thread (vee thread)	$\mu_G{}' = \tan \rho_G{}' = \frac{\mu_G}{\cos(\beta/2)}$ $\rho_G{}'$ = friction angle
in head (nut) seating surface	μ_K
Tightening torque	$M_A = M_G + M_K$ $M_A = F_V \frac{d_2}{2}\left[\tan(\varphi + \rho_G{}') + \mu_K \frac{D_{km}}{d_2}\right]$
Loosening torque	$M_L = F_V \frac{d_2}{2}\left[\tan(-\varphi + \rho_G{}') + \mu_K \frac{D_{km}}{d_2}\right]$
Thread torque during tightening	$M_G = F_V \frac{d_2}{2} \tan(\varphi + \rho_G{}')$
Frictional torque on head (nut) seating surface	$M_K = F_V \cdot \mu_K \frac{D_{km}}{2}$

Continuation of table, see Page 512.

Continuation of table, General calculation of fixing screws, from Page 511.

Normal stress in thread	$\sigma_N = F_V/A_S$
Torsional stress in thread	$\tau = \frac{M_G}{W_P}$ where $W_P = \frac{\pi\left[(d_2 + d_3)/2\right]^3}{16}$
Equivalent stress in thread during tightening	$\sigma_V = \sqrt{\sigma^2{}_N + 3\tau^2} \leq \nu \cdot R_e$
Utilisation of yield stress	$\nu = 0{,}6 \ldots 0{,}9$
Coefficients of friction depending on surface treatment and lubrication[1)] (μ_G in thread, μ_K in head seating surface)	$\mu_G = 0{,}08 \ldots 0{,}20$ $\mu_K = 0{,}08 \ldots 0{,}20$

1) For values, see VDI Guideline 2230: Systematic calculation of high duty bolted joints.

Coefficients of friction μ_G and μ_K

In the case of various surface and lubrication conditions, the values for the coefficients of friction μ_G are as follows:

μ_G Thread					External thread (screw)								
	Material				Steel								
		Surface			Black annealed or phosphated				Electroplated zinc (Zn6)		Electroplated cadmium (Cd6)		Ad-hesive
				Thread production method	Rolled			Cut	Cut or rolled				
Thread	Material	Surface	Thread production method		Dry	Oiled	MoS_2	Oiled	Dry	Oiled	Dry	Oiled	Dry
Internal thread (nut)	Steel	Bright	Cut	Dry	0,12 to 0,18	0,10 to 0,16	0,08 to 0,12	0,10 to 0,16	–	0,10 to 0,18	–	0,08 to 0,14	0,16 to 0,25
		Electroplated zinc			0,10 to 0,16	–	–	–	0,12 to 0,20	0,10 to 0,18	–	–	0,14 to 0,25
		Electroplated cadmium			0,08 to 0,14	–	–	–	–	–	0,12 to 0,16	0,12 to 0,14	–
	GG/GTS	Bright			–	0,10 to 0,18	–	0,10 to 0,18	–	0,10 to 0,18	–	0,08 to 0,16	–

In the case of various surface and lubrication conditions, the values for the coefficients of friction μ_G are as follows:

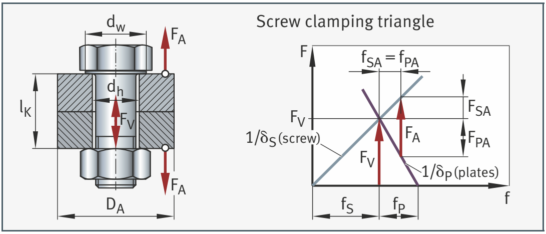
μ_K	Seating surface			Screw head									
Seating surface	Material			Steel									
		Surface		Black annealed or phosphated						Electroplated zinc (Zn6)		Electroplated cadmium (Cd6)	
			Production	Pressed			Turned		Ground	Pressed			
			Lubrication	Dry	Oiled	MoS_2	Oiled	MoS_2	Oiled	Dry	Oiled	Dry	Oiled
Mating position	Steel	Bright	Cut	–	0,16 to 0,22	–	0,10 to 0,18	–	0,16 to 0,22	0,10 to 0,18	–	0,08 to 0,16	–
	Steel	Bright	Machined by cutting	0,12 to 0,18	0,10 to 0,18	0,08 to 0,12	0,10 to 0,18	0,08 to 0,12	– –	0,10 to 0,18		0,08 to 0,16	0,08 to 0,14
	Steel	Electroplated zinc	Machined by cutting	0,10 to 0,16		–	0,10 to 0,16	–	0,10 to 0,18	0,16 to 0,20	0,16 to 0,18	–	–
	Steel	Electroplated cadmium	Machined by cutting	0,08 to 0,16						–	–	0,12 to 0,20	0,12 to 0,14
	GG/GTS	Bright	Ground	–	0,10 to 0,18	–	–	–	0,10 to 0,18			0,08 to 0,16	–
	GG/GTS	Bright	Machined by cutting	–	0,14 to 0,20	–	0,10 to 0,18	–	0,14 to 0,22	0,10 to 0,18	0,10 to 0,16	0,08 to 0,16	–

Lubrication of the seating surface (mating position): Dry

Systematic calculation of heavily loaded screw connections

The following section presents the calculation of heavily preloaded screw connections (HV connections) in accordance with VDI 2230.

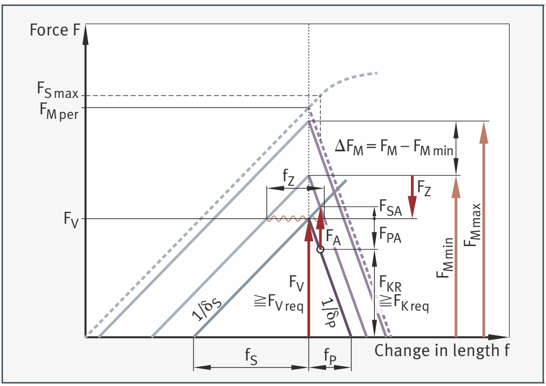
Figure 20
Screw connection and tensioning diagram



Typical applications include:

- flange connections in pipework construction
- location of crown wheels, clutch discs, etc. (force locking transmission of traction forces)
- location of the cylinder head and connecting rod in the engine (flange connections with simultaneous occurrence of static and dynamic forces)

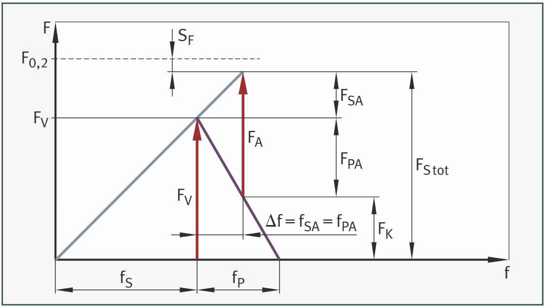
Figure 21
Dimensioning: Principal and other important values



The tensioning forces must be continuously high enough that, under the influence of operating forces, there is no unilateral lifting at the parting line and no displacement movement. The preload forces required in this case can be a multiple of the operating forces occurring.

Figure 22
Screw clamping triangle in operation

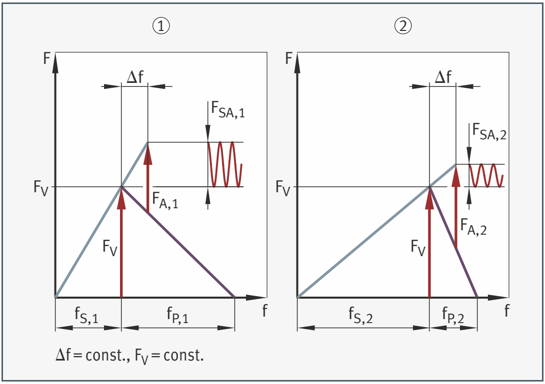
F_{PA} = proportion of the operating force that relieves parts of load
F_{SA} = proportion of the operating force that places additional load on screws
F_K = clamping force



The durability of a screw connection is all the greater, the more compliant the screw and the more rigid the flange.

Figure 23
Influence of the rigidity of screw and flange on the screw clamping triangle

① Rigid screw, compliant flange
② Compliant screw, rigid flange: advantageous for durability



The basis for screw calculation is the dimensioning formula:

Equation 4

$$\begin{aligned} F_{M\,max} &= \alpha_A \cdot F_{M\,min} \\ &= \alpha_A \cdot \left[F_{K\,req} + (1 - \Phi) \cdot F_A + F_Z + \Delta F_{Vth} \right] \end{aligned}$$

Values and designations

The following table shows a selection of suitable values and designations for the calculation of heavily preloaded screw connections.

Value	Designation
A_D	Sealing surface (largest parting line area minus the through hole for the screw)
A_P	Area of screw head or nut seating surface
A_S	Cross-sectional area of screw thread in accordance with DIN 13-28
A_0	Smallest applicable cross-sectional area of screw
D_A	Substitute diameter of main body in the parting line; if the parting line area deviates from a circular form, a mean diameter must be used
D_{Km}	Effective diameter for the frictional torque in the screw head or nut seating surface
d	Screw diameter = thread outside diameter (nominal diameter)
d_h	Bore diameter of clamped parts
d_w	Outside diameter of the flat head seating surface of the screw (at the entry of the radius transition from the head); in general, outside seating diameter
d_0	Diameter at the smallest applicable cross-section of the screw
d_2	Pitch diameter of screw thread
F_A	Axial force; a component of the freely aligned operating force F_B aligned to the screw axis and proportionally related to a screw
F_B	Freely aligned operating force at a connection
F_K	Clamping force
$F_{K\,req}$	Clamping force required for sealing functions, frictional locking and connection of the unilateral lifting at the parting line
F_{KP}	Minimum clamping force for ensuring sealing function
F_{KQ}	Minimum clamping force for transmission of a transverse force and/or a torque by frictional locking
F_{KR}	Residual clamping force at the parting line at application or alleviation of load by F_{PA} and after settling in operation
F_M	Mounting preload force
ΔF_M	Differential between mounting preload force F_M and minimum preload force $F_{M\,min}$

Continuation of table, see Page 517.

Source: VDI 2230.

Continuation of table on Values and designations from Page 516.

Value	Designation
F_{Mm}	Mean mounting preload force
$F_{M\ max}$	Maximum mounting preload force for which a screw must be designed in order that, despite inaccuracy of the tightening method and the anticipated settling rates in operation, the necessary clamping force at the connection is achieved and maintained
$F_{M\ min}$	Requisite minimum mounting preload force; smallest mounting preload force that can occur at $F_{M\ max}$ as a result of inaccuracy of the tightening method and maximum friction
$F_{M\ Tab}$	Tabular value for mounting preload force (see table for R7 – Determining the mounting stress $F_{M\ per}$ and checking the screw size on Page 524)
$F_{M\ per}$	Permissible mounting preload force
F_{mGM}	Stripping force of nut or internal thread
F_{mGS}	Stripping force of pin thread
F_{mS}	Breaking force of the free loaded screw thread
F_{PA}	Proportion of the axial force that changes the load on the clamped parts, additional plate force
F_Q	Transverse force; an operating force aligned perpendicular to the screw axis or its component under a freely aligned operating force F_B
F_S	Screw force
F_{SA}	Axial additional screw force
F_{SAo}	Upper (maximum) axial additional screw force
F_{SAu}	Lower (minimum) axial additional screw force
F_V	Preload force, general
ΔF_{Vth}	Change in the preload force due to a temperature other than room temperature; additional thermal force
$\Delta F'_{Vth}$	Change in the preload force due to a temperature other than room temperature (simplified); approximated additional thermal force
F_Z	Preload force loss due to settling in operation
f	Elastic length change under a force F
f_Z	Plastic deformation through settling, settling rate
k_τ	Reduction coefficient
l_K	Clamping length

Continuation of table, see Page 518.
Source: VDI 2230.

Continuation of table on Values and designations from Page 517.

Value	Designation
M_A	Tightening torque in mounting for preloading a screw to F_M
M_G	Active part of the tightening torque in the thread (thread torque)
M_K	Frictional torque on the seating surface of the screw head or nut
M_Y	Torque about the screw axis
P	Pitch of thread
p	Contact pressure
p_G	Limit contact pressure, maximum permissible pressure under screw head, nut or washer
p_i	Internal pressure to be sealed off
p_M	Contact pressure in the mounted state
q_F	Number of force-transmitting (F_Q) internal parting lines involved in any sliding/shearing of the screw
q_M	Number of torque-transmitting (M_Y) internal parting lines involved in any sliding
$R_{p0,2}$	0,2% proof stress of screw in accordance with DIN EN ISO 898-1
Rz	Averaged roughness depth from at least two individual measurement distances
r_a	Frictional radius on the clamped parts under the influence of M_Y
W_P	Polar section modulus of screw cross-section
α_A	Tightening factor
δ_P	Elastic compliance of clamped parts under concentric clamping and concentric loading
δ_S	Elastic compliance of screw
μ_G	Friction coefficient in thread
μ_K	Friction coefficient in head seat
μ_T	Friction coefficient in parting line
σ_a	Fatigue loading of screw
$\sigma_{red,B}$	Equivalent stress in operating condition
σ_z	Tensile stress in screw in operating condition
τ	Torsional stress in thread due to M_G
Φ	Force ratio, relative compliance ratio
Φ_{en}	Force ratio under concentric clamping and eccentric force application via the clamped parts

Source: VDI 2230.

Calculation in accordance with VDI 2230 is carried out in individual steps (R0 to R13).

R0 – Determining the nominal diameter
Selection of screw type and grade. With the aid of the following table, this gives a screw diameter for the first draft (estimate of the diameter range of the screws).

Force	Nominal diameter d mm		
	Grade		
N	8.8	10.9	12.9
250	–	–	–
400	–	–	–
630	–	–	–
1 000	3	3	3
1 600	3	3	3
2 500	4	3	3
4 000	5	4	4
6 300	6	5	4
10 000	8	6	5
16 000	10	8	6
25 000	12	10	8
40 000	14	12	10
63 000	16	14	12
100 000	20	18	16
160 000	24	22	20
250 000	30	27	24
400 000	36	33	30
630 000	–	39	36

Source: VDI 2230.

R1 – Determining the tightening factor α_A

The tightening factor α_A takes account of the scatter in the achievable mounting preload force between $F_{M\,min}$ and $F_{M\,max}$:

Equation 5

$$\alpha_A = \frac{F_{M\,max}}{F_{M\,min}}$$

It is determined with the aid of guide values from the following table. The scatter is calculated as follows:

Equation 6

$$\frac{\Delta F_M}{2 \cdot F_{Mm}} = \frac{\alpha_A - 1}{\alpha_A + 1}$$

Table for determining the tightening factor α_A:

Tightening factor α_A	Scatter %	Tightening method	Setting method	Comments
1,1 to 1,2	±5 to ±9	Tightening with control or inspection of elongation by means of ultrasound	Sound run time	■ Calibration values necessary ■ If $l_K/d < 2$, pay attention to progressive increase in errors. ■ Smaller errors with direct mechanical coupling, larger errors with indirect coupling
1,1 to 1,3	±5 to ±13	Mechanical elongation by pressure screws arranged in the nut or screw head	Specification for elongation of the screw, setting by means of the extraction torque of the pressure screws	■ Hardened seating washer for supporting the pressure screws ■ from approx. M24
1,2 to 1,5	±9 to ±20	Mechanical elongation by multi-piece nut with threaded bush	Torque of tightening tool	■ Substantially torsion-free screw mounting ■ from approx. M30

Continuation of table, see Page 521.

Source: VDI 2230.

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

Continuation of table on R1 – Determining the tightening factor α_A from Page 520.

Tightening factor α_A	Scatter %	Tightening method	Setting method	Comments
1,1 to 1,5	±5 to ±20	Tightening with mechanical elongation measurement or inspection	Direct method: setting by means of length measurement Indirect method: axial clearance to control pin used up	■ Precise determination of the proportional axial elastic compliance of the screw necessary ■ Scatter essentially dependent on the accuracy of the measurement method ■ Calibration necessary for low values ■ If $l_K/d < 2$, pay attention to progressive increase in errors
1,1 to 1,4	±5 to ±17	Hydraulic friction-free and torsion-free tightening	Setting by means of compression or elongation measurement or prevailing angle of nut	■ If $l_K/d \geqq 5$, smaller values can be achieved, in the case of machined screw and plates $\alpha_A = 1{,}05$ is possible ■ In the case of standard screws and nuts $\alpha_A \geqq 1{,}2$ ■ Smaller clamping length ratios lead to larger α_A values. ■ Spring-back losses occur that are not considered in the tightening factor. ■ Application from M20
1,2 to 2,0	±9 to ±33	Impulse wrench with hydraulic impulse cell, torque-controlled and/or angle-controlled	Setting by means of rotation angle or prevailing torque	■ Small values only with presetting to the screw joint by means of rotation angle, compressed air servo valve and impulse count ■ In special cases, over-elastic mounting is also possible
1,2 to 1,4	±9 to ±17	Yield strength-controlled tightening, motorised or manual	Specification of relative torque/ angle coefficient	The scatter in preload force is essentially determined by the scatter in yield strength of the screw batch fitted. The screws are dimensioned in this case for $F_{M\,min}$; design of the screws for $F_{M\,max}$ with the tightening factor α_A is therefore omitted for these tightening methods.
1,2 to 1,4	±9 to ±17	Angle-controlled tightening, motorised or manual	Test-based determination of initial tightening torque and rotation angle (steps)	
1,4 to 1,6	±17 to ±23	Torque-controlled tightening using hydraulic tool	Setting by means of pressure measurement	At or above approx. M30

Continuation of table, see Page 522.

Source: VDI 2230.

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

Continuation of table on R1 – Determining the tightening factor α_A from Page 521.

<table>
<tr><th>Tightening factor
α_A</th><th>Scatter
%</th><th>Tightening method</th><th>Setting method</th><th colspan="2">Comments</th></tr>
<tr><td>1,4
to
1,6</td><td>±17
to
±23</td><td>Torque-controlled tightening
with a torque wrench, signal-emitting wrench or motorised screwdriver
with dynamic torque measurement</td><td>Test-based determination of the nominal tightening torques on the original screw mounting part
(for example by elongation measurement of the screw)</td><td>Low values:
Large number of setting or control tests (for example 20) required;
small scatter of outputted torque (for example ±5%) necessary</td><td rowspan="3">Low values:
■ for small rotation angles, (relatively rigid connections)
■ for relatively low hardness of the mating position[1]
■ for mating positions without a tendency to “fretting”
(for example with phosphate coating or adequate lubrication)
High values:
■ for large rotation angles, (relatively compliant connections)
as well as fine pitch threads
■ for high hardness of the mating position combined with a rough surface</td></tr>
<tr><td>1,6
to
2,0[2]</td><td>±23
to
±33</td><td rowspan="2">Torque-controlled tightening
with a torque wrench, signal-emitting wrench or motorised screwdriver
with dynamic torque measurement</td><td rowspan="2">Determination of the nominal tightening torque by estimation of the friction coefficient (high influence of surface and lubrication conditions)</td><td rowspan="2">■ Low values:
for measuring torque wrench with uniform tightening and for precision screwdriver
■ High values:
for signal-emitting or buckling torque wrenches</td></tr>
<tr><td>1,7
to
2,5[3]</td><td>±26
to
±43</td></tr>
<tr><td>2,5
to
4</td><td>±43
to
±60</td><td>Tightening
with impact wrench, “choke” wrench or impulse wrench;
tightening by hand</td><td>Setting of the screwdriver by the retightening torque that results from the nominal tightening torque (for the estimated friction coefficient) and an allowance;
hand tightening on a subjective basis</td><td colspan="2">Low values:
■ for large number of setting tests (retightening torque)
■ on horizontal axis of screw characteristic diagram
■ for clearance-free impulse transmission
Method only suitable for preliminary tightening;
with tightening by hand,
risk of stretching with M10 and smaller</td></tr>
</table>

Source: VDI 2230.

1) Mating position: clamped part whose surface is in contact with the tightening element of the connection (screw head or nut).

2) Friction coefficient class B

3) Friction coefficient class A

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

R2 – Determining the requisite minimum clamping force

This gives frictional locking for transmission of the transverse force F_Q and/or a torque M_Y about the screw axis:

Equation 7

$$F_{KQ} = \frac{F_{Q\ max}}{q_F \cdot \mu_{T\ min}} + \frac{M_{Y\ max}}{q_M \cdot r_a \cdot \mu_{T\ min}}$$

or for sealing against a medium:

Equation 8

$$F_{KP} = A_D \cdot p_{i\ max}$$

R3 – Subdivision of operating force

Force ratio:

Equation 9

$$\Phi = \frac{F_{SA}}{F_A}$$

R4 – Changing the preload force

Due to settling, there is a preload force loss:

Equation 10

$$F_Z = \frac{f_Z}{\delta_S + \delta_P}$$

Guide values for the settling rates can be found in the following table.

Mean roughness depth in accordance with ISO 4287[1)] Rz µm	Load	Guide values for settling rates		
		in thread µm	per head or nut seating surface µm	per internal parting line µm
<10	Tension/compression	3	2,5	1,5
	Thrust	3	3	2
10 to <40	Tension/compression	3	3	2
	Thrust	3	4,5	2,5
40 to <160	Tension/compression	3	4	3
	Thrust	3	6,5	3,5

Source: VDI 2230.

1) Mean value of the maximum roughness depths Rt from at least two individual measurement distances. If five individual measurement distances are used, Rz corresponds with good approximation to the old R_z in accordance with DIN 4768.

R5 – Determining the minimum mounting preload force

The minimum mounting preload force $F_{M\,min}$ is determined as follows:

Equation 11

$$F_{M\,min} = F_{K\,req} + \left(1 - \Phi^{*}_{en}\right) \cdot F_{A\,max} + F_{Z} + \Delta F'_{Vth}$$

R6 – Determining the maximum mounting preload force

The maximum mounting preload force $F_{M\,max}$ is determined as follows:

Equation 12

$$F_{M\,max} = \alpha_{A} \cdot F_{M\,min}$$

R7 – Determining the mounting stress $F_{M\,per}$ and checking the screw size

With 90% utilisation of the minimum yield strength $R_{p0,2\,min}$, the mounting preload force based on $F_{M\,per} = F_{M\,Tab}$ can be taken from the following table. The table is valid for coarse pitch threads; for further tables, see VDI 2230.

Coarse pitch thread															
Size	Grade	Mounting preload force $F_{M\,Tab}$ kN							Tightening torque M_A Nm						
		μ_G =							$\mu_K = \mu_G$ =						
		0,08	0,10	0,12	0,14	0,16	0,20	0,24	0,08	0,10	0,12	0,14	0,16	0,20	0,24
M4	8.8	4,6	4,5	4,4	4,3	4,2	3,9	3,7	2,3	2,6	3,0	3,3	3,6	4,1	4,5
	10.9	6,8	6,7	6,5	6,3	6,1	5,7	5,4	3,3	3,9	4,6	4,8	5,3	6,0	6,6
	12.9	8,0	7,8	7,6	7,4	7,1	6,7	6,3	3,9	4,5	5,1	5,6	6,2	7,0	7,8
M5	8.8	7,6	7,4	7,2	7,0	6,8	6,4	6,0	4,4	5,2	5,9	6,5	7,1	8,1	9,0
	10.9	11,1	10,8	10,6	10,3	10,0	9,4	8,8	6,5	7,6	8,6	9,5	10,4	11,9	13,2
	12.9	13,0	12,7	12,4	12,0	11,7	11,0	10,3	7,6	8,9	10,0	11,2	12,2	14,0	15,5
M6	8.8	10,7	10,4	10,2	9,9	9,6	9,0	8,4	7,7	9,0	10,1	11,3	12,3	14,1	15,6
	10.9	15,7	15,3	14,9	14,5	14,1	13,2	12,4	11,3	13,2	14,9	16,5	18,0	20,7	22,9
	12.9	18,4	17,9	17,5	17,0	16,5	15,5	14,5	13,2	15,4	17,4	19,3	21,1	24,2	26,8
M7	8.8	15,5	15,1	14,8	14,4	14,0	13,1	12,3	12,6	14,8	16,8	18,7	20,5	23,6	26,2
	10.9	22,7	22,5	21,7	21,1	20,5	19,3	18,1	18,5	21,7	24,7	27,5	30,1	34,7	38,5
	12.9	26,6	26,0	25,4	24,7	24,0	22,6	21,2	21,6	25,4	28,9	32,2	35,2	40,6	45,1
M8	8.8	19,5	19,1	18,6	18,1	17,6	16,5	15,5	18,5	21,6	24,6	27,3	29,8	34,3	38,0
	10.9	28,7	28,0	27,3	26,6	25,8	24,3	22,7	27,2	31,8	36,1	40,1	43,8	50,3	55,8
	12.9	33,6	32,8	32,0	31,1	30,2	28,4	26,6	31,8	37,2	42,2	46,9	51,2	58,9	65,3
M10	8.8	31,0	30,3	29,6	28,8	27,9	26,3	24,7	36	43	48	54	59	68	75
	10.9	45,6	44,5	43,4	42,2	41,0	38,6	36,2	53	63	71	79	87	100	110
	12.9	53,3	52,1	50,8	49,4	48,0	45,2	42,4	73	73	83	93	101	116	129
M12	8.8	45,2	44,1	43,0	41,9	40,7	38,3	35,9	63	73	84	93	102	117	130
	10.9	66,3	64,8	63,2	61,5	59,8	56,3	52,8	92	108	123	137	149	172	191
	12.9	77,6	75,9	74,0	72,0	70,0	65,8	61,8	108	126	144	160	175	201	223

Continuation of table, see Page 525.

Source: VDI 2230.

Continuation of table on R7 – Determining the mounting stress $F_{M\ per}$ and checking the screw size from Page 524.

Coarse pitch thread															
Dimension	Grade	Mounting preload force $F_{M\ Tab}$ kN							Tightening torque M_A Nm						
		μ_G =							$\mu_K = \mu_G$ =						
		0,08	0,10	0,12	0,14	0,16	0,20	0,24	0,08	0,10	0,12	0,14	0,16	0,20	0,24
M14	8.8	62,0	60,6	59,1	57,5	55,9	52,6	49,3	100	117	133	148	162	187	207
	10.9	91,0	88,9	86,7	84,4	82,1	77,2	72,5	146	172	195	218	238	274	304
	12.9	106,5	104,1	101,5	98,8	96,0	90,4	84,4	171	201	229	255	279	321	356
M16	8.8	84,7	82,9	80,9	78,8	76,6	72,2	67,8	153	180	206	230	252	291	325
	10.9	124,4	121,7	118,8	115,7	112,6	106,1	99,6	224	264	302	338	370	428	477
	12.9	145,5	142,4	139	135,4	131,7	124,1	116,6	262	309	354	395	433	501	558
M18	8.8	107	104	102	99	96	91	85	220	259	295	329	360	415	462
	10.9	152	149	145	141	137	129	121	314	369	421	469	513	592	657
	12.9	178	174	170	165	160	151	142	367	432	492	549	601	692	769
M20	8.8	136	134	130	127	123	116	109	308	363	415	464	509	588	655
	10.9	194	190	186	181	176	166	156	438	517	592	661	725	838	933
	12.9	227	223	217	212	206	194	182	513	605	692	773	848	980	1092
M22	8.8	170	166	162	158	154	145	137	417	495	567	634	697	808	901
	10.9	242	237	231	225	219	207	194	595	704	807	904	993	1151	1284
	12.9	283	277	271	264	257	242	228	696	824	945	1057	1162	1347	1502
M24	8.8	196	192	188	183	178	168	157	529	625	714	798	875	1011	1126
	10.9	280	274	267	260	253	239	224	754	890	1017	1136	1246	1440	1604
	12.9	327	320	313	305	296	279	262	882	1041	1190	1329	1458	1685	1877
M27	8.8	257	252	246	240	234	220	207	772	915	1050	1176	1292	1498	1672
	10.9	367	359	351	342	333	314	295	1100	1304	1496	1674	1840	2134	2381
	12.9	429	420	410	400	389	367	345	1287	1526	1750	1959	2153	2497	2787
M30	8.8	313	307	300	292	284	268	252	1053	1246	1428	1597	1754	2031	2265
	10.9	446	437	427	416	405	382	359	1500	1775	2033	2274	2498	2893	3226
	12.9	522	511	499	487	474	447	420	1755	2077	2380	2662	2923	3386	3775
M33	8.8	389	381	373	363	354	334	314	1415	1679	1928	2161	2377	2759	3081
	10.9	554	543	531	517	504	475	447	2015	2392	2747	3078	3385	3930	4388
	12.9	649	635	621	605	589	556	523	2358	2799	3214	3601	3961	4598	5135
M36	8.8	458	448	438	427	415	392	368	1825	2164	2482	2778	3054	3541	3951
	10.9	652	638	623	608	591	558	524	2600	3082	3535	3957	4349	5043	5627
	12.9	763	747	729	711	692	653	614	3042	3607	4136	4631	5089	5902	6585
M39	8.8	548	537	525	512	498	470	443	2348	2791	3208	3597	3958	4598	5137
	10.9	781	765	748	729	710	670	630	3345	3975	4569	5123	5637	6549	7317
	12.9	914	895	875	853	831	784	738	3914	4652	5346	5994	6596	7664	8562

Source: VDI 2230.

In order that the screw size determined approximately in step R0 can be reused, the following must apply:

Equation 13

$$F_{M\ Tab} \geq F_{M\ max}$$

Otherwise, a larger screw diameter must be selected and calculations carried out again starting from step R2.

R8 – Determining the operating stress $\sigma_{red,B}$
For connections in which the yield strength of the screw is not to be exceeded under load, the maximum screw force should be as follows:

Equation 14

$$F_{S\ max} = F_{M\ per} + \Phi_{en}^{*} \cdot F_{A\ max} - \Delta F_{Vth}$$

For the maximum tensile stress, the following applies:

Equation 15

$$\sigma_{z\ max} = \frac{F_{S\ max}}{A_0}$$

and for the maximum torsional stress:

Equation 16

$$\tau_{max} = \frac{M_G}{W_P}$$

where $M_G = F_{M\ per} \dfrac{d_2}{2} \left(\dfrac{P}{\pi \cdot d_2} + 1{,}155\, \mu_{G\ min} \right)$

$$W_P = \frac{\pi}{16} d_0^3$$

For reduced stress or equivalent stress in operating condition (operating stress), the following applies:

Equation 17

$$\sigma_{red,B} = \sqrt{\sigma_{z\ max}^2 + 3\left(k_\tau \cdot \tau_{max}\right)^2}$$

The following must apply:

Equation 18

$$\sigma_{red,B} < R_{p0,2\ min}$$

R9 – Determining the continuous alternating stress σ_a
The fatigue behaviour is checked by means of:

Equation 19

$$\sigma_a = \frac{F_{SAo} - F_{SAu}}{2A_S}$$

R10 – Determining the maximum contact pressure p_{max}
The mounting preload force or maximum force in operation should not cause contact pressures that lead to creep processes and thus a loss in preload force.

For the mounted state, the following should apply:

Equation 20

$$p_{M\ max} = \frac{F_{M\ per}}{A_{P\ min}} \leq p_G$$

For yield strength-controlled or angle-controlled tightening methods, the following applies with the values for $F_{M\ Tab}$ from the table on R7 – Determining the mounting stress $F_{M\ per}$ and checking the screw size from Page 524:

Equation 21

$$p_{max} = \frac{F_{M\ Tab}}{A_{P\ min}} \cdot 1{,}4$$

R11 – Determining the minimum screw depth
In order to prevent failure due to stripping of the mating threads, adequate overlap is necessary between the screw and nut threads.

The following applies:

Equation 22

$$F_{mS} \leq \min\left(F_{mGM}, F_{mGS}\right)$$

R12 – Determining the security against sliding S_G and the shearing stress $\tau_{Q\,max}$

Transverse forces occurring in a screw connection must be transmitted by frictional locking.

For the minimum residual clamping force, the following applies:

Equation 23

$$F_{KR\,min} = \frac{F_{M\,per}}{\alpha_A} - \left(1 - \Phi^*_{en}\right) \cdot F_{A\,max} - F_Z - \Delta F_{Vth}$$

For the clamping force necessary to transmit the transverse forces, the following applies:

Equation 24

$$F_{KQ\,req} = \frac{F_{Q\,max}}{q_F \cdot \mu_{T\,min}} + \frac{M_{Y\,max}}{q_M \cdot r_a \cdot \mu_{T\,min}}$$

R13 – Determining the tightening torque M_A

The tightening torque is calculated as follows:

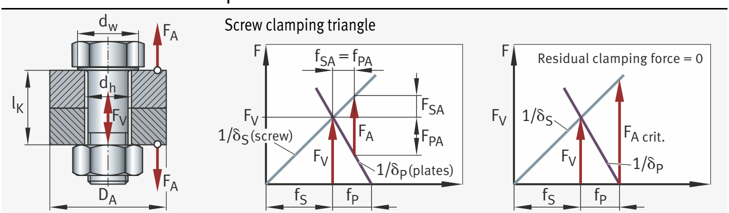
Equation 25

$$M_A = F_{M\,per}\left(0{,}16 \cdot P + 0{,}58 \cdot d_2 \cdot \mu_{G\,min} + \frac{D_{Km}}{2} \cdot \mu_{K\,min}\right)$$

For torque-controlled tightening, the torque required can be taken from the relevant tables.

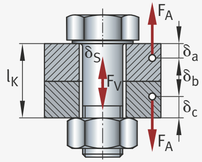
The following tables show some calculation examples.

Example 1:

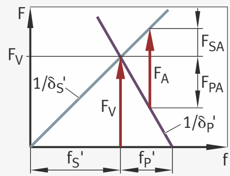


Application of a static operating force F_A under the screw head and the nut		
Additional loading of the screw due to F_A	$F_{SA} = F_A \cdot \delta_P/(\delta_S + \delta_P)$	
Load alleviation of clamped parts due to F_A	$F_{PA} = F_A \cdot \delta_S/(\delta_S + \delta_P)$	
Additional elongation of the screw due to F_A	$f_{SA} = F_{SA} \cdot \delta_S$	$f_{SA} = f_{PA}$
Spring-back of clamped parts due to F_A	$f_{PA} = F_{PA} \cdot \delta_P$	$f_{PA} = f_{SA}$
Critical operating force at which the residual clamping force of the parts becomes zero	$F_{A\,crit.} = F_V\,(1 + \delta_P/\delta_S)$	

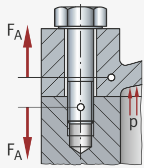
Example 2:



Screw clamping triangle

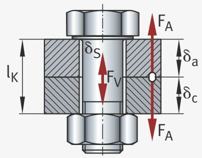


Example

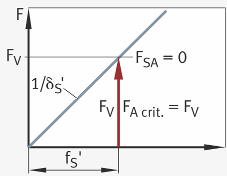


Application of the operating force F_A within the clamped parts	
Apparent elastic compliance of screw	$\delta_S' = \delta_S + \delta_a + \delta_c$
Apparent elastic compliance of clamped parts	$\delta_P' = \delta_b$
With δ_S' and δ_P', the relationships on the previous page apply	

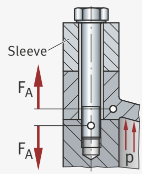
Example 3:



Screw clamping triangle

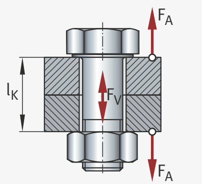


Example

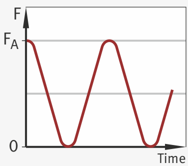


Application of the operating force F_A in the parting line of the clamped parts	
Apparent elastic compliance of screw	$\delta_S' = \delta_S + \delta_a + \delta_c$
Apparent elastic compliance of clamped parts	$\delta_P' \rightarrow 0$
Critical operating force at which the residual clamping force of the parts becomes zero	$F_{A\,crit.} = F_V$

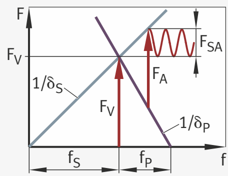
Example 4:



Pulsating load



Screw load



Pulsating load on the screw connection **Application of the operating force under the screw head and the nut**	
Oscillating load	$F_{dyn} = (F_V + F_{SA}/2) \pm F_{SA}/2$

Contact pressure at the head and nut seating surfaces

The contact pressure at the head and nut seating surfaces is calculated for standardised hexagon head and hexagon socket screws on the basis of utilisation of the yield stress of the screws.

Example 1: Seating surfaces for hexagon head screws to DIN EN ISO 4014 and nuts to DIN EN ISO 4032

Dimensions	Width across flats	Diameter of seating plate[1)]	Through hole[2)]	Seating surface	Cross-sectional area of thread	Contact pressure under head		
d	s_{max}	$d_{w\,min}$	d_h	A_P	A_S	$p = \frac{A_S}{A_P} \cdot 0{,}7\, R_{p0,2}$ $\frac{N}{mm^2}$		
mm	mm	mm	mm	mm^2	mm^2	**8.8**	**10.9**	**12.9**
M3	5,5	4,84	3,4	9,3	5,03	242	340	408
M4	7	6,2	4,5	14,3	8,78	275	387	465
M5	8	7,2	5,5	17,0	14,2	374	526	631
M6	10	8,88	6,6	27,7	20,1	325	457	548
M7	11	9,63	7,6	27,5	28,9	471	663	795
M8	13	11,63	9	42,6	36,6	385	541	648
M10	16	14,63	11	73,1	58	356	500	600
M12	18	16,63	13,5	74,1	84,3	510	717	860
M14	21	19,64	15,5	114,3	115	451	634	761
M16	24	22,49	17,5	156,7	157	449	631	757
M18	27	24,85	20	170,8	192	503	708	850
M20	30	27,7	22	222,5	245	493	694	832
M22	34	31,35	24	319,5	303	425	597	717
M24	36	33,25	26	337,4	353	469	659	791
M27	41	38	30	427,3	459	481	677	812
M30	46	42,75	33	580,1	561	433	609	731

1) In accordance with DIN 4032.
2) In accordance with DIN EN 20273.

Example 2: Seating surfaces for hexagon socket head screws to DIN EN ISO 4762, DIN 6912

Dimensions	Width across flats	Diameter of seating plate	Through hole[1]	Seating surface	Cross-sectional area of thread	Contact pressure under head		
d	s	$d_{w\,min}$[2]	d_h	A_P	A_S	$p = \frac{A_S}{A_P} \cdot 0{,}7\,R_{p0,2}$ $\frac{N}{mm^2}$		
mm	mm	mm	mm	mm^2	mm^2	**8.8**	**10.9**	**12.9**
M3	2,5	5,07	3,4	11,1	5,03	203	285	343
M4	3	6,53	4,5	17,6	8,78	223	314	377
M5	4	8,03	5,5	26,9	14,2	236	333	399
M6	5	9,38	6,6	34,9	20,1	258	363	435
M8	6	12,33	9	55,8	36,6	294	412	496
M10	8	15,33	11	89,5	58	290	408	490
M12	10	17,23	13,5	90	84,3	420	590	708
M14	12	20,17	15,5	131	115	393	552	664
M16	14	23,17	17,5	181	157	388	546	656
M18	14	25,83[3]	20	210	192	410	576	691
M20	17	28,87	22	274	245	400	563	676
M22	17	31,61[3]	24	332	303	409	575	690
M24	19	34,81	26	421	353	376	528	634
M27	19	38,61[3]	30	464	459	443	623	748
M30	22	43,61	33	638	561	394	554	665

[1] In accordance with DIN EN 20273.

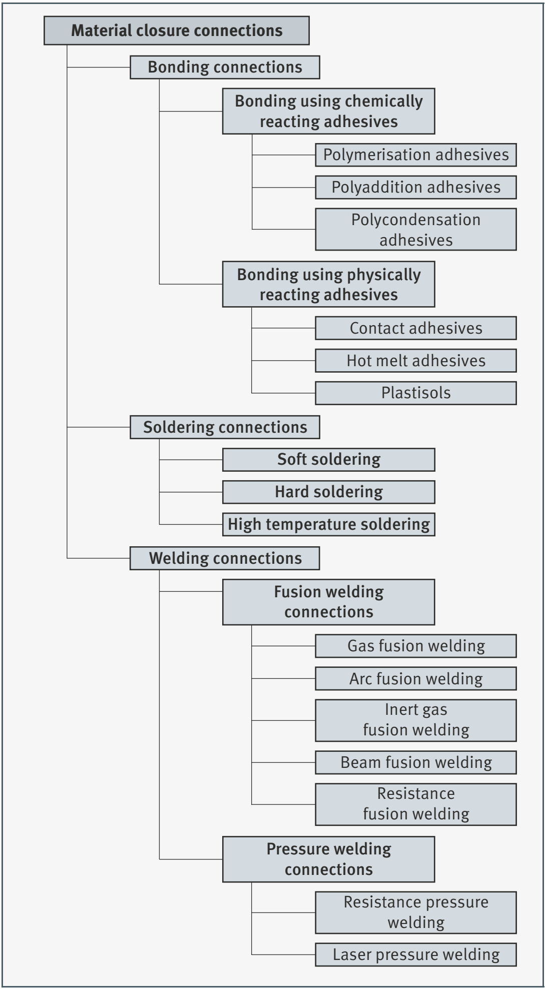
[2] In accordance with DIN EN ISO 4762.

[3] In accordance with DIN 6912.

Material closure connections

In the case of material closure connections, the parts are connected by joining using either an additional material of a characteristic type (welding) or an additional material of an uncharacteristic type (soldering, adhesive bonding). These belong to the class of non-separable connections, in other words, they cannot be separated without the occurrence of damage.

Figure 24
Material closure connections – overview



Sealing

Overview

The principal function of seals is the separation of two functionally different areas under equal or differing pressure such that there is no exchange of solid, liquid or gaseous media between these or at least that such exchange is within permissible limits (permissible leakage quantity).

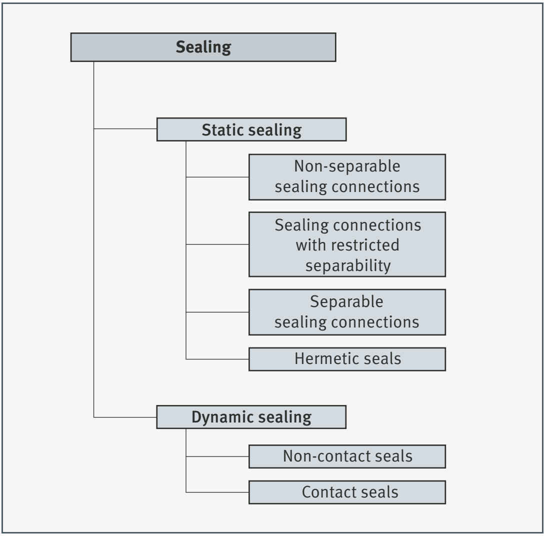


Figure 25
Sealing – overview

Examples:

- Prevention of the loss of operating materials (for example the egress of oil from bearings or of air from pneumatic lines)
- Prevention of the ingress of foreign bodies (for example, into bearings)
- Separation of different operating materials (for example, of bearing grease and lye in washing machines)

Some specific examples of sealing elements can be found in the section on rolling bearings.

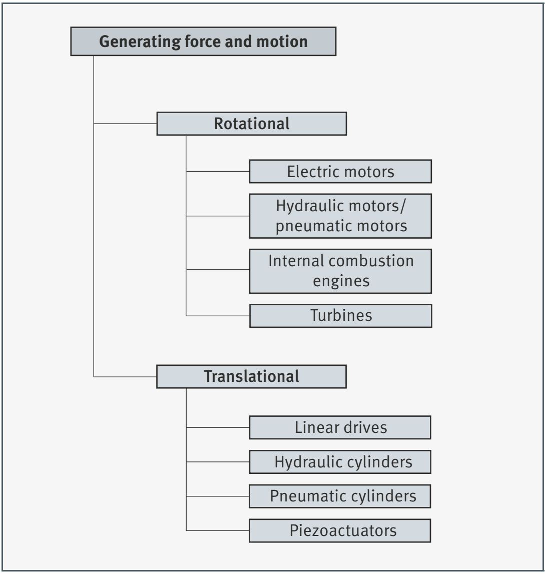
Generating force and motion

Overview

In technical systems such as machinery, plant and vehicles, it is always necessary to consider suitable drive systems as well as the actuation and adjustment functions involved. For this purpose, there is a wide spectrum available of different subsystems with a very wide range of functionality, complexity and performance capability.

Elements for generating force and motion convert particular initial forms of energy (such as flow energy of fluids, chemically bound energy in internal combustion engines, electrical energy in electric motors) into mechanical, kinetic energy (of rotational or translational character). This is always associated with conversion losses.

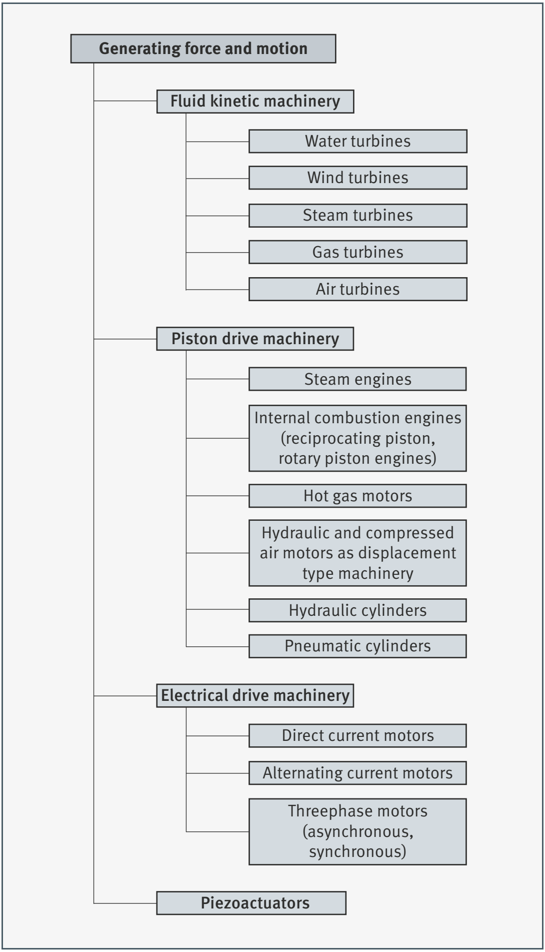
Figure 26
Generating force and motion (subdivision by type of motion) – overview



Subdivision by operating principle

In technical systems, prime movers provide the necessary drive for a wide range of driven machines. For the wide range of possible requirements, a wide spectrum of drive machines is available. These can be subdivided according to their operating principle.

Figure 27
Generating force and motion (subdivision by operating principle) – overview



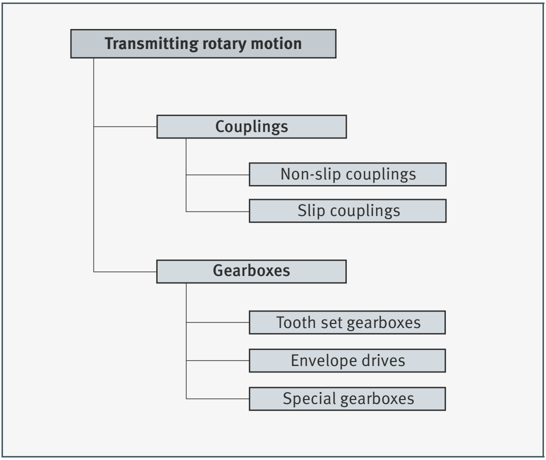
Transmitting rotary motion

Overview

In technical systems (machines, plant, vehicles) it is always necessary to transmit mainly rotary motion between the drive machine and the driven machine. Couplings and gearboxes are essential components of drive systems (comprising the three subsystems: drive machine or prime mover, drive train, driven machine).

In the drive train of such a drive system, couplings and gearboxes with their various characteristics and features play a decisive role alongside the shafts, bearings and seals. In the transmission of power in the drive system, the couplings mainly take up the function of routing power, while the gearboxes are able to convert torque and speed.

Figure 28
Transmitting rotary motion – overview



Couplings

Couplings are design elements based on form fit or force locking for the connection of rotating shafts abutting each other (with or without alignment) or rotating bodies as well as the transmission of power (torque and speed). They thus have a routing function.

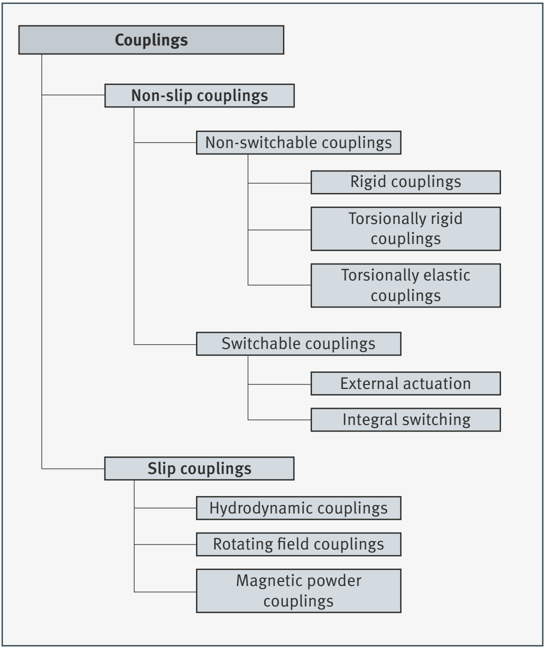
In addition to this principal function, they can take up additional functions, such as: compensation of offset (axial, radial, angular) or torsion, disengagement (closure or interruption as necessary of the power flow) as well as influencing the dynamic characteristics of a drive system (for example, reducing or damping of torque shocks and the displacement of natural frequencies).

Brakes can be defined as special examples of switchable couplings in which one half of the coupling is stationary.

Characteristics:
The wide range of functions leads to a large number of different types and designs that are available in the market in various sizes depending on the torque to be transmitted. A distinction must be made between couplings based on form fit, force locking and material closure.

Properties:
Couplings are characterised by the fact that, in stationary operation, the input torque of the coupling is always equal to the output torque. In contrast, the output speed can lie between zero and the drive speed depending on the design.

Figure 29
Couplings – overview



Torsionally rigid couplings transmit not only the necessary torque but also – without change – fluctuations in torque, shocks and vibrations that may arise from the drive machine or driven machine. In contrast, torsionally elastic couplings transmit torque by means of elastic metal or rubber spring elements, in which case they act as torsion springs and can thus reduce or damp shocks.

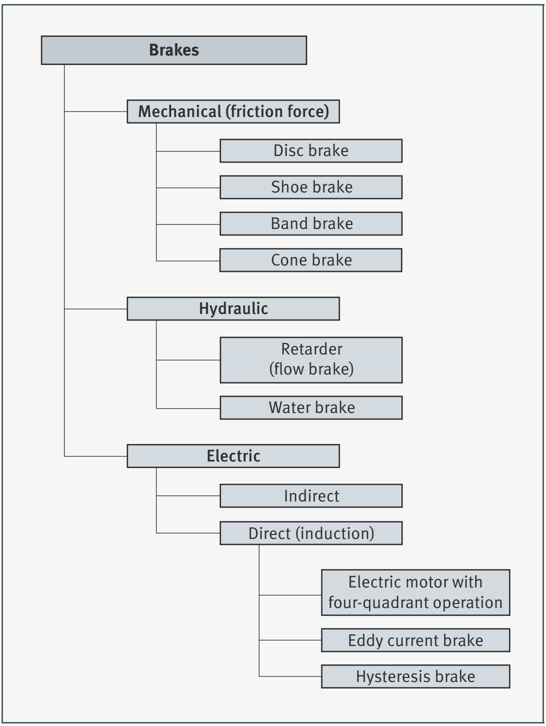
Switchable couplings allow the interruption and restoration of the connection between drive components as required in operational behaviour. Depending on the type of actuation, a distinction is made between switchable couplings with external actuation (by mechanical, electromagnetic, hydraulic or pneumatic means) or couplings with integral switching (actuated by speed, torque or direction).

In the case of slip couplings, there is always a difference between the input speed and output speed.

Brakes Brakes are couplings with a stationary output component. Their functions can include the following:

- locking (in one direction)
- holding (in both directions)
- stopping (halting motion)
- controlling (velocity)
- loading of prime movers (power brake)

Figure 30
Brakes – overview



Gearboxes Gearboxes fulfil the function of speed and torque conversion.

Systematic subdivision and overview of the wide range of gearboxes is possible and advisable on the basis of the following criteria[1]:

- Kinematics
  - uniform
  - non-uniform
- Physical principle
  - mechanical
  - hydraulic/pneumatic
  - electrical
- Operating principle
  - form fit
  - force locking
- Type of transmission
  - constant
  - stepped
  - stepless

In practice, gearboxes fulfil different adjustment tasks, such as:

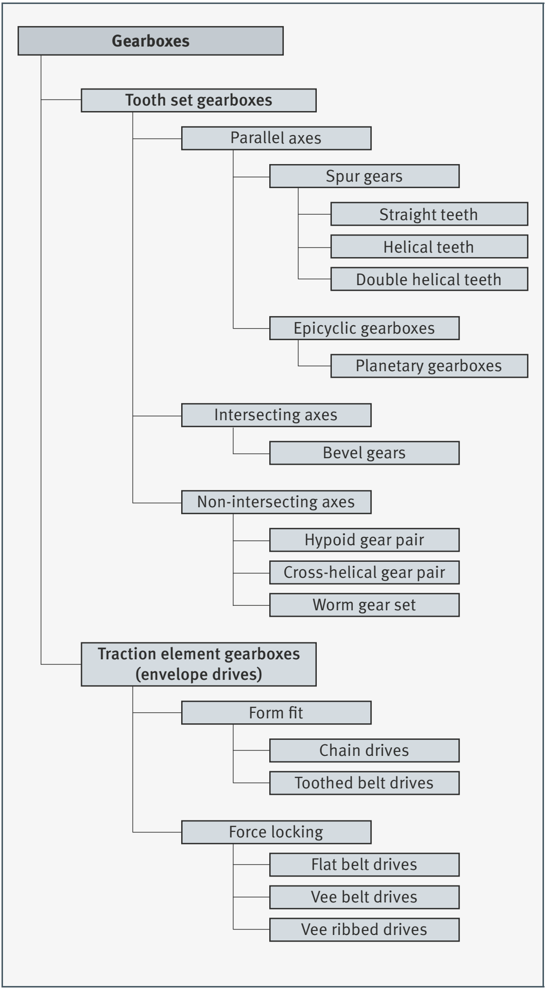
- Kinematic adjustment
  - speed
  - velocity
  - type of motion
- Geometrical adjustment
  - centre distance
  - angular position
- Adjustment of characteristics
  - operating point
  - characteristic pattern
- Adjustment of power flow
  - addition
  - division
  - conversion

Depending on the intended purpose, a distinction can also be made between gearboxes in mobile power transmission (vehicle engineering) and in stationary power transmission (machine tools, industrial plant, wind turbines).

[1] W. Steinhilper; B. Sauer: Konstruktionselemente des Maschinenbaus Band 2, Springer Verlag 2005.

In the case of gearboxes with a uniform transmission ratio, a distinction must be made between tooth set gearboxes and traction element gearboxes. Tooth set gearboxes can be subdivided in accordance with the arrangement of axes and the type of tooth set. Traction element gearboxes can be realised by means of form fit or force locking.

Figure 31
Gearboxes – overview



Tooth set gearboxes

Characteristics:
Toothed gears are drive elements that transmit power in the form of rotary motion with form fit from one shaft to the other. Toothed gears are defined by the form of the base elements, the characteristics of the flank lines and the profile form[1].

Depending on the type of tooth set and the gear form or arrangement of the axes relative to each other, a distinction is made between the following:

- spur gearbox
- bevel gearbox
- helical gearbox
- hypoid gearbox
- worm gearbox

The material predominantly used for the toothed gears is steel, which can be subjected to heat treatment in order to increase the load carrying capacity of the flanks. In pairs of toothed gears with a high sliding motion component, the mating gear is normally made from bronze in order to reduce friction. Where small loads are present, plastics can be used.

Properties:
Tooth set gearboxes are suitable for the transmission of power at a consistently high level of efficiency, while it is only in such gearboxes with a high sliding motion component that efficiency remains low. Through the combination of toothed gears with different numbers of teeth, it is possible to achieve different transmission ratios (i = ratio between input speed and output speed). Depending on the ratio between the input speed and output speed, it is possible to achieve transmission ratios to a slower or faster speed. As a result, tooth set gearboxes are highly suitable for the conversion of speeds and torques.

Envelope drives

Characteristics:
Envelope drives (also known as traction element gearboxes) comprise two or more discs or wheels that are not in contact with each other but are wrapped by means of a traction element (belt or chain)[1].

[1] W. Steinhilper; B. Sauer: Konstruktionselemente des Maschinenbaus Band 2, Springer Verlag 2005.

A distinction must be made between traction element gearboxes based on force (frictional) locking and based on form fit:

- Force locking
  - flat belt
  - vee belt
  - vee-ribbed belt
  - round belt
- Form fit
  - toothed belt
  - roller chain, pin chain, bush chain
  - toothed chain

Properties:
Envelope drives are suitable for the conversion of torques and speeds as well as changes in direction of rotation and also for the spanning of large centre distances.

Special gearboxes
Special gearboxes are characterised by the implementation of very high transmission ratios in a small design envelope.

Examples of special gearboxes include:

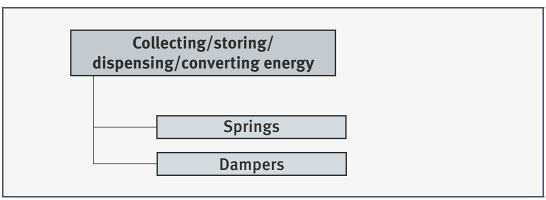
- cycloid gearboxes, such as Cyclo
- wave gearboxes, such as Harmonic Drive

Collecting/storing/dispensing/converting energy

Overview

This category is defined as comprising elements that, due to their elastic material behaviour, appropriate form or by using the compressibility of fluids, are able to collect and store energy, dispense energy or convert energy.

Figure 32
Collecting/storing/dispensing/converting energy – overview



Springs Springs are elastic elements that are characterised by their ability to store potential energy through contraction, expansion or torsion of the spring body and provide the energy at a later point in time in the form of work (minus the associated friction losses).

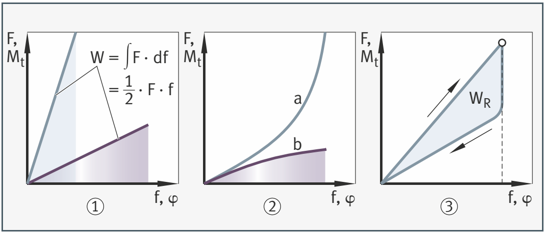
Accordingly, the use of springs covers the following functions:

- Work storage:
  the storage of potential energy (e.g. the tension spring in an air gun, the springs in mechanical timepieces)
- Force-travel converter:
  the conversion of force into travel (e.g. spring scales, expansion screws, springs in switchable and slip couplings)
- Energy converter:
  the damping of impacts and vibrations, the conversion of impact energy into thermal energy (e.g. shock absorber, buffer, rubber-bonded-to-metal elements)

The following section considers mechanical springs only. The behaviour of a spring is described by the spring diagram or the spring characteristic. This is defined as the dependence of the spring force F (or the spring torque M_t) on the deformation (change in length f or torsion angle φ).

Figure 33
Spring characteristics

① Linear characteristic line
② Curved characteristic line
a = progressive
b = degressive
③ Characteristic pattern of damping springs



For linear spring characteristics, as displayed by most metal springs, Hooke's law applies:

Equation 26

$$F = c \cdot f \quad \text{or} \quad M_t = c_t \cdot \varphi$$

Spring rate:

Equation 27

$$c = \frac{F}{f} \quad \text{or} \quad c_t = \frac{M_t}{\varphi}$$

In the case of non-linear springs, a spring rate c (spring stiffness) can be specified for the operating point by means of the tangent gradient:

Equation 28

$$c = \frac{dF}{df} \qquad c_t = \frac{dM_t}{d\varphi}$$

The elastic spring work W is the energy that is stored in a spring as potential energy when under an external load. It is given by the content of the area underneath the spring characteristic line:

Equation 29

$$W = \int_0^f F\,df \qquad \text{or} \qquad W = \int_0^{\varphi} M_t\,d\varphi$$

In the case of springs with a linear characteristic line, the following elastic spring work is stored between the unloaded and the loaded state:

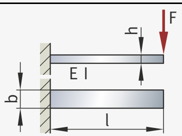
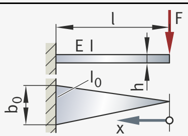
Equation 30

$$W = \frac{1}{2}\cdot c\cdot f^2 = \frac{1}{2}F\cdot f \qquad \text{or} \qquad W = \frac{1}{2}\cdot c\cdot \varphi^2 = \frac{1}{2}M_t\cdot\varphi$$

If a spring is repeatedly subjected to load and then relieved, and if its damping capacity is sufficient (material damping or external friction), the characteristic line for loading will differ from that for relief. The area enclosed by these two characteristic lines is a measure of the damping work W_R, see Figure 33 ③, Page 543.

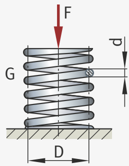
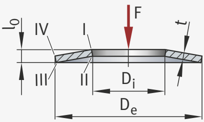
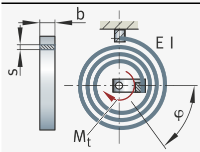
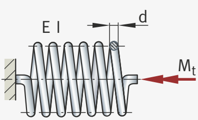
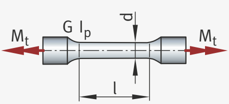
Spring rates for metallic springs

The following table shows spring rates, deformations and loads for metallic springs.

Spring type	Spring rate c, c_t Deformation f, φ	Principal load	Degree of utilisation
Rectangular spring			
	$c = \frac{3EI}{l^3} = \frac{bh^3E}{4l^3}$ $f = \frac{Fl^3}{3EI} = \frac{4Fl^3}{bh^3E}$	$\sigma_b = \frac{M_b}{W} = \frac{6Fl}{bh^2}$	$\eta_A = \frac{1}{9}$
Triangular spring			
	$c = \frac{2EI_0}{l^3} = \frac{b_0 h^3E}{6l^3}$ $f = \frac{Fl^3}{2EI_0} = \frac{6Fl^3}{b_0 h^3E}$	$\sigma_b = \frac{M_b}{W_0} = \frac{6Fl}{b_0h^2}$ $b(x) = \frac{b_0}{l}\cdot x$	Identical load $\eta_A = \frac{1}{3}$

Continuation of table, see Page 545.

Continuation of table, Spring rates for metallic springs, from Page 544.

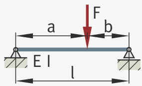
Spring type	Spring rate c, c_t Deformation f, φ	Principal load	Degree of utilisation
Cylindrical coil spring			
	$c = \dfrac{Gd^4}{8iD^3}$ $f = \dfrac{8FiD^3}{Gd^4}$ i = number of spring windings	$\tau = \dfrac{M_t}{W_p} = \dfrac{8FD}{\pi d^3}$	$\eta_A = \dfrac{1}{2}$
Disc spring			
	$c \approx \dfrac{4E}{1-\nu^2} \dfrac{t^3}{K_1 D_e^{\,2}}$ [1)] for $(l_0 - t)/t \leqq 0{,}4$ $D_e/D_i = 2$; $K_1 = 0{,}69$	$\sigma_{I,II} \approx \pm F \dfrac{K_3}{t^2}$ $\sigma_{III,IV} \approx \pm F \dfrac{K_3}{t^2} \dfrac{D_i}{D_e}$ $K_3 = 1{,}38$	$\eta_A < \dfrac{1}{3}$
Spiral spring			
	$c_t = \dfrac{EI}{l} = \dfrac{Ebs^3}{12 \cdot l}$ $\varphi = \dfrac{M_t\, l}{EI} = \dfrac{12 M_t l}{bs^3 E}$ l = length of spring	$\sigma_b = \dfrac{M_t}{W} = \dfrac{6M_t}{bs^2}$ $M_b = M_t$ = const.	Rectangular cross-section b, s $\eta_A = \dfrac{1}{3}$
Cylindrical helical torsion spring (leg spring)			
	$c_t = \dfrac{EI}{l} = \dfrac{E\pi d^4}{64 \cdot l}$ $\varphi = \dfrac{M_t l}{EI} = \dfrac{64 M_t l}{\pi d^4 E}$ l = straightened length of turns	$\sigma_b = \dfrac{M_t}{W} = \dfrac{32 M_t}{\pi d^3}$ $M_b = M_t$ = const.	Circular cross-section d $\eta_A = \dfrac{1}{4}$
Torsion bar spring			
	$c_t = \dfrac{GI_p}{l} = \dfrac{G\pi d^4}{32 \cdot l}$ $\varphi = \dfrac{M_t l}{GI_p} = \dfrac{32 M_t l}{\pi d^4 G}$	$\tau = \dfrac{M_t}{W_p} = \dfrac{16 M_t}{\pi d^3}$	$\eta_A = \dfrac{1}{2}$

[1)] The equations provided are only an approximation.
For more precise calculation, see DIN EN 16984.

Spring rates for some elastic systems

The spring rates for some elastic systems are shown below.

Bending bar supported on both sides

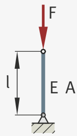


$$c = \frac{3EIl}{a^2 b^2}$$

Special case: a = b = l/2

$$c = \frac{48EI}{l^3}$$

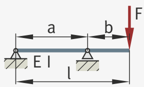
Pin support



$$c = \frac{EA}{l}$$

E·A = longitudinal stiffness

Torsion bar with protruding end

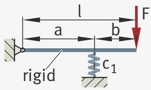


$$c = \frac{3EI}{(a+b)\,b^2}$$

Special case: a = b = l/2

$$c = \frac{12EI}{l^3}$$

Rotatable bar, spring-supported

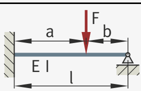


$$c = c_1 \left(\frac{a}{l}\right)^2$$

Special case = a = b = l/2

$$c = \frac{c_1}{4}$$

Torsion bar clamped and supported

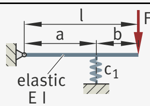


$$c = \frac{12EIl^3}{a^3 b^2 (3l+b)}$$

Special case: a = b = l/2

$$c = \frac{768EI}{7 \cdot l^3}$$

Rotatable bar, spring-supported

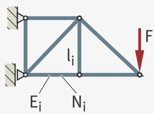


$$\frac{1}{c_{tot}} = \frac{1}{c_1}\left(\frac{l}{a}\right)^2 + \frac{(a+b)\,b^2}{3EI}$$

Torsion bar clamped and guided

$$c = \frac{12EI}{l^3}$$

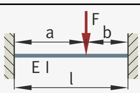
Latticework



$$c = 1 \quad F \sum_{i=1}^{n} \frac{E_i A_i}{N_i \overline{N}_i l_i}$$

$\overline{N}_i$ = normal forces as a result of “1”

Torsion bar clamped on both sides

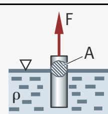


$$c = \frac{3EIl^3}{a^3 b^3}$$

Special case: a = b = l/2

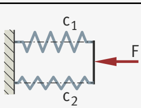
$$c = \frac{192EI}{l^3}$$

Lift



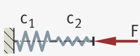
$$c = A \cdot \rho \cdot g$$

Springs in parallel



$$c_{tot} = c_1 + c_2$$

Springs in series

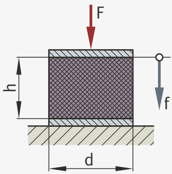
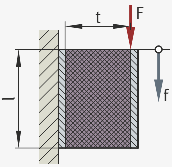
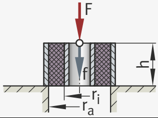
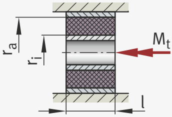
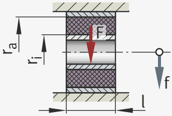


$$\frac{1}{c_{tot}} = \frac{1}{c_1} + \frac{1}{c_2}$$

$$c_{tot} = \frac{c_1 c_2}{c_1 + c_2}$$

Spring rate, deformation and loading of rubber springs

The following table gives spring rates, deformations and loads for rubber springs.

Spring type	Deformation f/ spring rate c	Load
Woodruff key (compression)		
	$f = \frac{Fh}{EA} = \frac{4Fh}{E\pi d^2}$ $c = \frac{F}{f} = \frac{EA}{h} = \frac{E\pi d^2}{4h}$ Shape factor $k = \frac{d}{4h}$	$\sigma_d = E\varepsilon = \frac{F}{A}$ $F_{per} = \frac{\pi d^2}{4}\sigma_{per}$ $\varepsilon = \frac{f}{h}$
Rectangular spring (thrust)		
	$f = \frac{Ft}{GA} = \frac{Ft}{Gbl}$ $c = \frac{F}{f} = \frac{Gbl}{t}$ Width b	$\tau = G\gamma = \frac{F}{A}$ $F_{per} = G\,b\,l\,\gamma_{per}$ $\gamma = \frac{f}{t}$
Sleeve type spring (thrust)		
	$f = \frac{F}{2\pi hG}\ln\frac{r_a}{r_i}$ $c = \frac{F}{f}$	$\tau_i = \frac{F}{A_i} = \frac{F}{2\pi r_i h}$ $F_{per} = 2\pi\,r_i\,h\,G\,\gamma_{per}$
Sleeve type spring (rotational thrust)		
	$\varphi = \frac{M_t}{4\pi lG}\left(\frac{1}{r_i^2} - \frac{1}{r_a^2}\right)$ $c_t = \frac{M_t}{\varphi}$	$\tau_i = \frac{F}{A_i} = \frac{M_t}{2\pi r_i^2 l}$ $M_{t\,per} = 2\pi\,G\,r_i^2\,l\,\gamma_{per}$
Sleeve type spring (compression, thrust)		
	$f = \frac{F}{\pi l(E+G)}\ln\frac{r_a}{r_i}$ $c = \frac{F}{f}$	$\sigma_{dmax} = \frac{2}{\pi}\frac{F_{max}}{l r_i}$ $F_{per} = \frac{\pi l r_i}{2}\sigma_{dper}$

Calculation of rubber springs

The following table shows reference values for the approximate calculation of rubber springs.

Shore hardness Sh (A)	Modulus of elasticity E_{st} N/mm² Compression k = 1/4	Modulus of elasticity E_{st} N/mm² Compression k = 1,0	Shear modulus G_{st} N/mm²	Permissible static deformation under permanent load % Compression	Permissible static deformation under permanent load % Thrust Tension	Permissible static stress under permanent load N/mm² Compression k = 1/4	Permissible static stress under permanent load N/mm² Compression k = 1,0	Permissible static stress under permanent load N/mm² Thrust Tension
30	1,1	4,5	0,3	10 ... 15	50 ... 75	0,18	0,7	0,20
40	1,6	6,5	0,4		45 ... 70	0,25	1,0	0,28
50	2,2	9,0	0,55		40 ... 60	0,36	1,4	0,33
60	3,3	13,0	0,8		30 ... 45	0,50	2,0	0,36
70	5,2	20,0	1,3		20 ... 30	0,80	3,2	0,38

Permissible alternating stresses 1/3 to 1/2 of the permissible static stresses.

Properties of elastomers for rubber springs

The following table gives some properties of elastomers for rubber springs.

Elastomers	Styrene butadiene rubber	Natural rubber (polyisoprene)	Butyl rubber	Ethylene propylene diene rubber	Chloro butadiene rubber	Acrylonitrile butadiene rubber	Polyurethane rubber	Silicone rubber	Polyacrylate rubber (PA)	Fluorocarbon rubber
Desig-nation	SBR	NR	JIR	EPDM	CR	NBR	AU, EU	VMQ	ACM	FKM
Example trade name	Buna, Hüls	Rubber	Butyl	Buna AP	Neoprene	Perbunan	Vulkollan	Silopren	Cyanacryl	Viton
Density g/cm³	0,92	0,95	0,93	–	1,23	0,98	1,26	1,19	–	–
Tensile strength (DIN 53504) N/mm²	≤ 24	≤ 28	≤ 15	18	20 ... 27	22 ... 27	30 ... 32	≤ 10	15	15
Elongation at fracture Maximum value (DIN 53504) %	700	1000	900	800	800	800	600	500	–	–
Shore hardness Sh (DIN 53505) (A)	40 ... 95	30 ... 98	40 ... 90	40 ... 90	40 ... 95	40 ... 95	65 ... 95	40 ... 90	55 ... 85	60 ... 90
Operating temperature range °C	–30 ... +90	–40 ... +70	–25 ... +110	–35 ... +130	–25 ... +100	–25 ... +100	–15 ... +80	–60 ... +200	–15 ... +150	–20 ... +220

Continuation of table, see Page 549.

Continuation of table, Properties of elastomers for rubber springs, from Page 548.

Elastomers	Styrene butadiene rubber	Natural rubber (polyisoprene)	Butyl rubber	Ethylene propylene diene rubber	Chloro butadiene rubber	Acrylonitrile butadiene rubber	Polyurethane rubber	Silicone rubber	Polyacrylate rubber (PA)	Fluorocarbon rubber
Desig-nation	SBR	NR	JIR	EPDM	CR	NBR	AU, EU	VMQ	ACM	FKM
Example trade name	Buna, Hüls	Rubber	Butyl	Buna AP	Neoprene	Perbunan	Vulkollan	Silopren	Cyanacryl	Viton
Oil resistance	Slight	Slight	Slight	Mod-erate	Mod-erate	Good	Very good	Good	Very good	Very good
Petrol resistance	–	–	–	–	–	Good	Good	Mod-erate	Very good	Very good
Ozone resistance	Slight	Slight	Very good	Excel-lent	Good	Slight	Very good	Very good	Very good	Very good
Creep resistance	Very good	Ex-cellent	Me-dium	Good	Good	Very good	Good	Good	Good	Good
Rebound elasticity	Good	Very good	Slight	Good	Good	Good	Very good	Good	Slight	Slight
Damping	Good	Mode-rate	Excel-lent	Good	Good	Very good	Good	Good	Very good	Ex-cellent
Abrasion resistance	Very good	Very good	–	–	Good	–	Very good	–	–	–
Adherence to metal	Good	Ex-cellent	Mod-erate	Mod-erate	Good	Very good	Very good	Me-dium	Medium	Good
Special properties	–	3)	1) 2)	–	–	–	5)	4)	5)	2)
Processibility	–	–	–	–	–	–	–	–	Bright pro-duction possible	Diffi-cult
Electrical insulation capability	Good	Very good	Very good	Very good	Slight	–	Slight	Good	Poor	Good
Price	Low	Low	Low	Low	Fairly low	Fairly low	Me-dium	High	High	Very high

Source: Dubbel, Taschenbuch für den Maschinenbau, 21. Auflage.

1) Gas permeability very low.

2) Acid resistance good.

3) Flammable.

4) Flame resistant.

5) Sensitive to water at 40 °C.

Dampers Dampers (vibration dampers) are elements that convert motion energy into heat energy. They are generally used in order to cause decay of the vibration of a sprung mass. In special cases, shock dampers can also be used to give gentle braking of a mass performing linear or rotary motion.

Various types of damping can be used here:

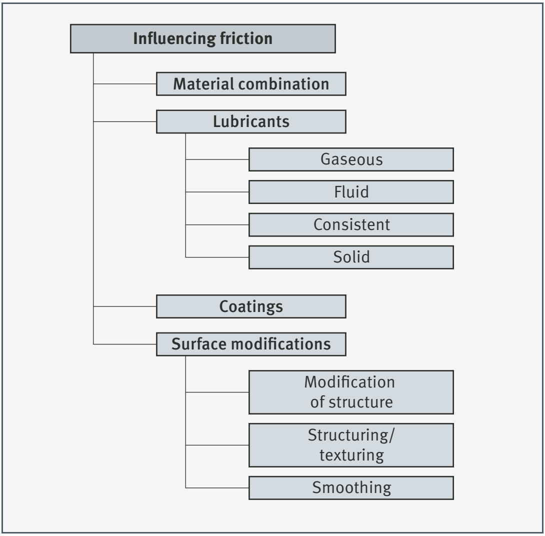
- material damping
- friction damping
- viscosity damping
- eddy current damping
- radiation damping
- turbulence damping

Influencing friction and wear

Overview

In most cases, friction and the resulting wear are undesirable. The aim is therefore to reduce friction through suitable material combinations for components and the use of lubricants, coatings or surface modifications.

Figure 34
Influencing friction and wear – overview



Lubricants

Depending on the application, lubricants in fluid, consistent, gaseous or solid form are used. These are intended principally to decrease friction and reduce wear.

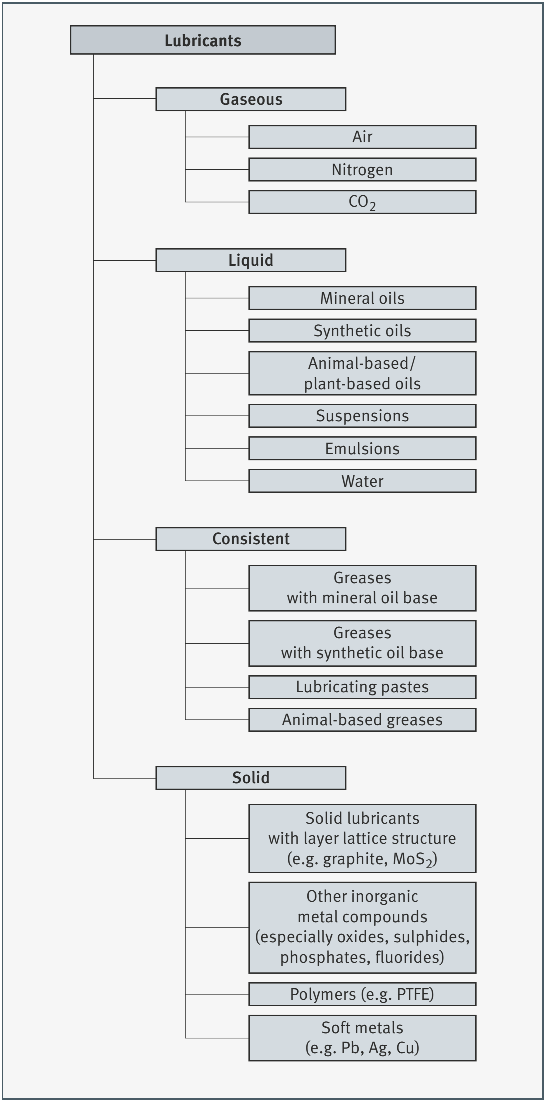
Lubricants provide complete separation (full lubrication) or partial separation (partial lubrication) of surfaces moving relative to each other and subjected to load.

Fluid lubricants are mainly mineral oils and synthetic oils as well as animal-based and plant-based oils.

Consistent lubricants are greases comprising mineral and synthetic oils (thickened using soaps).

Examples of solid lubricants include graphite, molybdenum disulphide (MoS_2) and polytetrafluorethylene (PTFE).

Figure 35
Lubricants – overview



Coatings

Introduction

An increasing number of components in technical systems are subjected to surface loading that leads to the operating limits of most metallic materials being breached.

The use of technologies from the field of industrial surface technology – in this case, mainly coating – makes it possible to refine the surfaces in such a way that the operating limits can be considerably expanded in comparison with their untreated original condition. Furthermore, it is possible to achieve functional integration or additional characteristics.

In the selection and use of surface technology, the decisive factor for success is a holistic approach that takes the entire technical system into consideration.

The component surface – an active area

Technical components are used to fulfil particular functions within a higher level machine or installation. The component comprises a particular material and has a corresponding geometry as well as a production history. The geometry can be further subdivided into the component volume and the component surface. These perform various subfunctions: while the **component surface** in the form of an **active area** supports the external loads and transmits these to the interior regions, the actual load-bearing function is performed preferably by the component volume.

The life of technical components is frequently determined not only by the strength but also by fatigue or wear of the surface. Since these phenomena take place on the surface, it is necessary for logical reasons to mainly address this area in order to solve the rating life issue.

Surface loading

The component surface generally represents the area subjected to the heaviest loading. This is where normal and frictional forces or heat flows are introduced. Electric potentials build up here or electric currents are transmitted. In many cases, wear or corrosion on the component surface determine the life of the entire component. In industrial and mobile applications, this surface loading originates essentially from the following categories:

- Tribological surface loading:
  Friction and wear (abrasive or adhesive) lead to damage in machine elements. Friction can be influenced and wear reduced by a specifically modified surface topography and coatings.

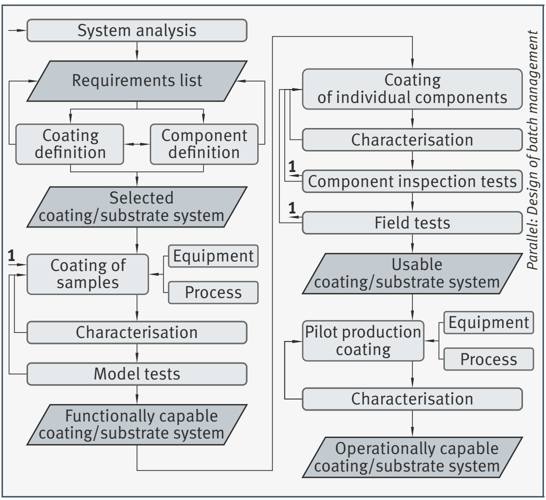
- Corrosive surface loading:
  Corrosion as a chemical or electrochemical reaction occurs in metallic materials in the presence of humidity, contact with water or aggressive media (alkalis or acids). A protective function can be provided here by suitable coatings.
- Electrical surface loading:
  As a result of current transfer or the buildup and discharge of potentials, electrical surface loading can occur. This can have effects on intervening media, which may bring about the premature ageing of lubricants. Surface engineering measures facilitate the application of insulating or current-conducting coatings.

Coatings as design elements – "Tailored Coatings"

The surfaces of components are therefore – with different functions – of enormous significance. Coatings for the refinement of surfaces must therefore be considered in terms of their different composition and application as design elements.

If this approach is followed logically, the selection and design of coatings must be carried out in the same meticulous manner as in the case of normal machine elements.

Figure 36
Systematic coatings development process



For a coatings development process, a systematic procedure is recommended in accordance with the **flow diagram** shown here.

When all the boundary conditions are taken into consideration, this gives the most suitable "tailored coating" for the application.

Available coatings For the various types of surface loading, a wide range of different coatings is available.

Surface technology offers a large quantity of coatings as well as process engineering approaches to solutions for the production of the different coatings. The selection of the coating methods to be used is influenced by the material characteristics of the substrates and coatings, the geometry of the components and economic aspects.

In essence, the coating methods that are used for the production of coatings against tribological, corrosive or electrical surface loading can be subdivided into two groups:

1. Methods for the modification and transformation of the surface zone of the substrate
2. Methods for the creation of overlay coatings.

From an industrial perspective, the following coating methods are relevant especially for the production of large quantities in volume processes:

- thermochemical diffusion methods
- conversion methods
- chemical/electrochemical methods
- PVD method (Physical Vapor Deposition) or PACVD method (Plasma Assisted Chemical Vapor Deposition), also known as: PECVD method (Plasma Enhanced Chemical Vapor Deposition)
- thermal spraying
- painting

The coating materials that can be produced for the aforenamed areas of application by means of the above methods are explained, in greater detail and in conjunction with the applications, from Page 560.

Tribological coatings Tribological coatings can reduce fatigue close to the surface and wear. They can be used to specifically influence friction and thus make a contribution to energy efficiency and CO_2 reductions.

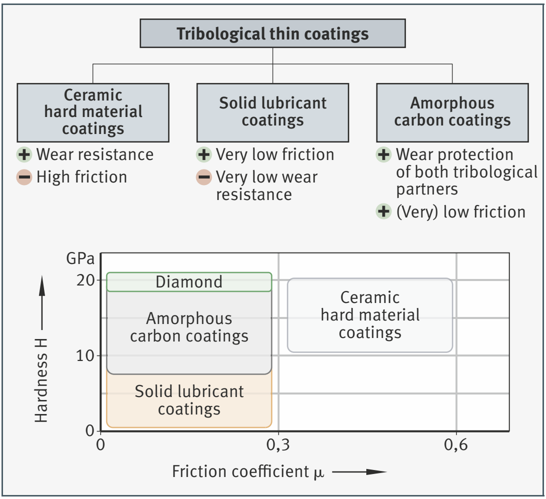
In order to prevent destruction of the surface, good surface quality (small roughness peaks, proportionally large load-bearing area) is advantageous. High friction can be reduced by means of friction-reducing coatings such as DLC (Diamond Like Carbon) or PTFE (polytetrafluoroethylene). Protection against **abrasive wear** requires a high surface hardness. The contact partners can be protected here by particularly hard coatings. The PVD and PACVD methods can be used to deposit coatings with hardnesses $>$2 000 HV. Furthermore, electroplated coatings such as chromium or NiP can prevent abrasive wear, since their hardness is greater than that of the base material.

Adhesive wear occurs principally in contact partners with similar bonding characteristics, such as metal/metal. In order to prevent this wear, it is sufficient to change the type of bonding close to the surface by the coating of one contact partner. A typical example of adhesive wear is slippage damage. This wear can be reduced by, for example, the targeted oxidation of the metal surface by means of black oxide coating. In this case, a metallic surface is converted into a surface (metal oxide) with heteropolar bonding. Through coating with an amorphous carbon layer, a covalent bonding character can be achieved on the surface.

In order to prevent wear by means of tribochemistry, solutions can be used that are similar to those for the prevention of adhesive wear. The tribochemical reactions can be suppressed by means of a suitable coating. An example of this is the phosphating of a surface.

Due to the increasing requirements in relation to performance capability and resource efficiency as well as the ever smaller availability of space, increasing importance is being attached to thin layers produced using highly eco-friendly vacuum plasma techniques. A general classification of these coatings is shown in Figure 37.

Figure 37
Classification of tribological thin film coating systems



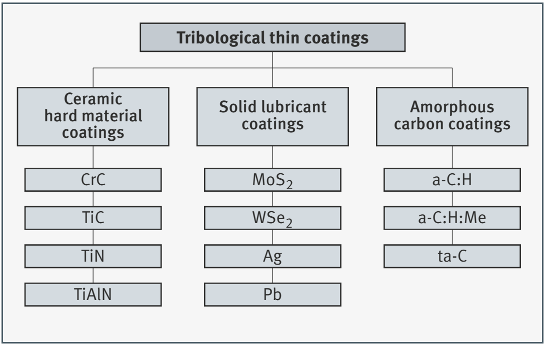


Figure 38
Examples of tribological thin film coating systems – overview

Anti-corrosion coatings

In contact with water or humidity, steels tend to undergo corrosion. In many cases, corrosion-resistant high grade steels cannot be quenched and tempered to the requisite hardnesses or lose their corrosion-inhibiting properties during the hardening process. In this case, assistance is possible using anti-corrosion coatings, mainly based on zinc and zinc alloys.

Figure 39
Bearing rings after 24 h on a test rig in the salt spray test

① Coated with zinc alloy coating
② Uncoated

Insulation coatings

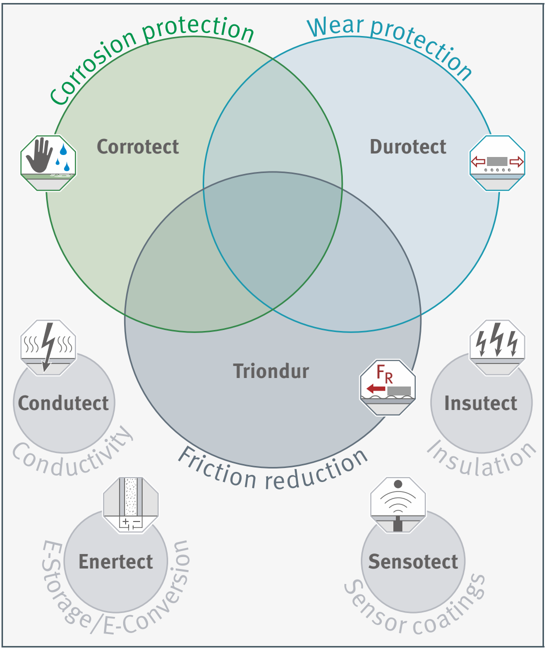
Rolling bearings can be damaged by current transmission. The damage resulting from current passage can lead to failure of the bearing. In order to give protection against electric current, there are various established solutions that are used according to the size and type of bearing. Solutions in the form of coatings or even glass fibre reinforced plastic housings are used.

Modular coating concept

For the selection of coatings for different problem areas, Schaeffler has developed a modular coating concept.

The modular coating concept shown is intended to give easier selection of suitable coatings.

Figure 40
Coating systems and their areas of application



Corrotect
Corrotect covers all coating systems that are used primarily to give protection against corrosion (film and base metal corrosion). Depending on the coating system, they are applied by means of electrochemical methods (electroplating), as a paint or by thermal spraying.

Durotect
These coating systems are used primarily in applications that require protection against wear, reduction in friction or both. Depending on the coating system, they are applied by means of chemical or electrochemical methods, as a paint or by thermal spraying.

Triondur
Triondur coating systems offer the best combination of wear protection and friction reduction for components subjected to very high tribomechanical stress. They are applied under vacuum by means of PVD or plasma-assisted CVD methods.

Insutect
Insutect covers coating systems that are used primarily to achieve insulation against current. They are applied by means of thermal spraying.

Sensotect
Sensotect is a sensory coating that facilitates expansion in the functions of components. This is of particular significance in conjunction with the subjects of Industry 4.0 and digitalization. This coating system is used for the continuous measurement of force and torque on two-dimensional and three-dimensional component geometries. The particular feature in this case is that the sensor technology can be applied by means of PVD technology and subsequent laser structuring directly to the component surface.

Enertect
Coating systems for the conversion and storage of energy

Condutect
Coating systems for electrical and thermal conductivity and for EMC shielding

Application examples

Schaeffler has developed suitable coatings for various applications. The resulting recommendations are presented below.

Recommendations for tribological coatings

Friction reduction and wear protection

The quality of a rolling bearing is determined to a significant extent by its smooth running and wear resistance. A low friction coefficient reduces not only energy consumption but also the requirement for lubricant. This is associated with lower mechanical wear, while the operating life of the bearing increases. The different wear types (abrasive, adhesive, tribochemical) require different measures.

Protection against **abrasive** wear:

- High surface quality (high hardness, small roughness peaks) necessary
- Protection of contact partners by particularly hard coatings (hardness in excess of 2 000 HV) that are applied by means of PVD (Physical Vapor Deposition) or PACVD (Plasma Assisted Chemical Vapor Deposition)
- High friction can be reduced by means of friction-reducing coatings such as DLC (Diamond Like Carbon) or PTFE (polytetrafluoroethylene).
- This is also possible using electroplated coatings such as chromium or NiP, since their hardness is greater than that of the base material

Protection against **adhesive** wear:

- Occurs principally in contact partners with similar bonding characteristics, such as metal/metal.
- In order to prevent this wear (typically: slippage damage), it is sufficient to change the type of bonding close to the surface by the coating of one contact partner.
- Remedy by targeted oxidation of the metal surface by means of black oxide coating
- Through coating with an amorphous carbon layer, a covalent bonding character can be achieved on the surface.

Protection against wear by means of **tribochemistry**:

- Solutions similar to those for the prevention of adhesive wear
- Tribochemical reactions can be suppressed by means of suitable coatings. For example, phosphating of a surface.
- In order to improve the sliding friction contacts, bearing cages are coated by electroplating means with silver or copper. This also makes it possible to prevent fretting corrosion.

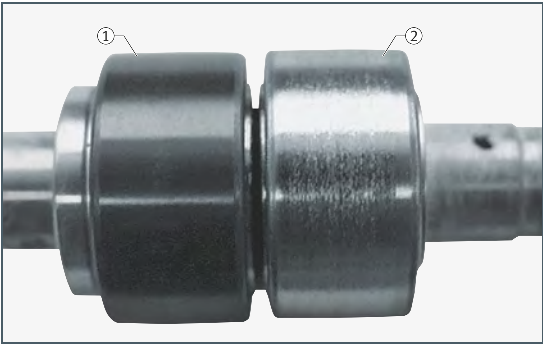


Figure 41
Comparison of coated and uncoated stud type track rollers

① Coated with Triondur C: no wear of any type
② Uncoated: adhesive wear on raceway and mating body

Recommendations for corrosion-inhibiting coatings

Bearing parts with corrosion – as a result of contact with water or humidity – can in the case of standard bearings lead to malfunctions, lower efficiency and premature failure. Corrosion-resistant rolling bearing steels provide a remedy here but are expensive. The most economical variant under moderate corrosion conditions is therefore the combination of a standard rolling bearing steel with an appropriate coating.
The following coatings have proved effective:

- zinc phosphating with application of oil (for low requirements)
- extremely thin zinc alloy coatings, applied by electroplating
- columnar thin dense chromium coating as an anti-corrosion coating resistant to wear and overrolling
- nickel-phosphorus coatings (deposited by electroless methods) for highly corrosive media such as acids and alkalis

Principal function: Corrosion protection

The following table shows Corrotect coating systems for corrosion protection.

Coating system	Composition	Principal function			Additional function	Main area of application, special feature
		Corrosion protection	Wear protection	Friction reduction		
Corrotect A*	Zinc-iron Thin film	X			–	Belt drives, selector shafts, bearings, bearing components
Corrotect N*	Zinc-iron Thin film	X			–	Belt drives, detent systems
Corrotect ZI	Zinc-iron	X			–	Belt drives, bearing components, screws with moderate corrosion protection requirements
Corrotect ZN	Zinc-nickel	X			–	Belt drives, bearing components, screws with high corrosion protection requirements
Corrotect ZK	Zinc	X			–	Simple corrosion protection applications
Corrotect ZF	Zinc flakes	X			–	Chassis engineering, components, screws and safety components with high tensile strength
Corrotect P	Paint systems	X			Current-insulating according to coating variant	Housings, flanges, slewing rings, connectors, main bearings
Corrotect CTN	Copper-tin-nickel combination	X			Wear protection based on hardness of coating	Corrosion protection in maritime applications
Corrotect H	Zinc or zinc-aluminium	X			–	Corrosion protection for inner and outer rings of large size bearings, slewing rings, main bearings, generator bearings
Corrotect HP	Zinc or zinc-aluminium with topcoat	X			–	Corrosion protection for inner and outer rings of large size bearings, slewing rings, main bearings, generator bearings

Principal function: Wear protection and friction reduction

The following table shows Durotect coating systems for wear protection and friction reduction.

Coating system	Composition	Principal function			Additional function	Main area of application, special feature
		Corrosion protection	Wear protection	Friction reduction		
Durotect B	Mixed iron oxide			X	■ Improved running-in behaviour ■ Reduced slippage damage ■ Short-term corrosion protection (for example for transport) ■ Reduced failures as a result of WEC	Belt drives, selector shafts, bearings, bearing components
Durotect Z	Zinc phosphate			X	■ Short-term corrosion protection (for example for transport) ■ Protection against fretting corrosion ■ Suitable for sliding seats ■ Bonding layer for paint, soaps, oils, vulcanisation	Aerospace, linear guidance systems, bearings, bearing components
Durotect M	Manganese phosphate			X	■ Improved running-in behaviour ■ Short-term corrosion protection (for example for transport) ■ Emergency running lubrication ■ Retention layer for dry lubricants	Aerospace, bearing components
Durotect CK	Columnar thin dense chromium coating		X		■ Corrosion protection possible, depending on the application ■ Slightly reduced friction ■ Reduced fretting corrosion	Linear technology, aerospace, vibratory screen bearings, spindle bearings
Durotect CK$^+$	Columnar thin dense chromium coating and mixed chromium oxide	X	X	X	■ Reduced fretting corrosion	Bearing components, linear technology
Durotect CM, Durotect CMT	Microcracked thin dense chromium coating		X		■ Corrosion protection possible, depending on the application ■ Slightly reduced friction	Needle roller bearings, bearing and engine components

Continuation of table, see Page 564.

Continuation of table, Principal function: Wear protection and friction reduction, from Page 563.

Coating system	Composition	Principal function			Additional function	Main area of application, special feature
		Corrosion protection	Wear protection	Friction reduction		
Durotect NP	Nickel-phosphorus	X	X		■ Friction reduction by means of PTFE additives	Drawn cups, guide ring segments
Durotect C	Copper			X	■ Emergency running lubrication ■ Dissipation of frictional heat	Cages in bearings running at high speeds
Durotect S	Silver			X	■ Emergency running lubrication ■ Dissipation of frictional heat	Aerospace, linear guidance systems, bearing components, cages in bearings running at high speeds
Durotect H	Chromium steel or manganese steel		X		–	For dimensional correction of rolling bearing rings
Durotect HT	Range of variants		X		■ Increase in adhesive friction (static or dynamic)	Synchronizer rings, inner rings, intermediate rings
Durotect HA	Hard anodising (Al)	X	X		■ Current insulation	Sliding sleeves, bearing cages, housing components
Durotect CT	Copper-tin(-bronze) combination	X	X		■ Corrosion protection ■ Security against smearing and fretting ■ Emergency running characteristics under inadequate lubrication	Linear guidance systems
Durotect P	Polymer-based coating			X	■ Protection against fretting corrosion ■ Current insulation	Bearing rings, guide sleeves, cages

Principal function: Surfaces subjected to high tribomechanical loading

The following table shows Triondur coating systems for surfaces subjected to high tribomechanical loading.

Coating system	Composition	Principal function			Additional function	Main area of application, special feature
		Corrosion protection	Wear protection	Friction reduction		
Triondur C	a-C:H:Me (metal-containing hydrogenated amorphous carbon coating)		X	X	Reduction in slippage damage	Bearing components, engine components
Triondur C^+	a-C:H (hydrogenated amorphous carbon coating)		X	X	–	Engine components, bearing components
Triondur CX^+	a-C:H:X (modified hydrogenated amorphous carbon coating)		X	X	–	Engine components, bearing components, nanostructured, ideal combination of friction reduction and wear protection
Triondur CH	ta-C (tetrahedral hydrogen-free amorphous carbon coating)		X	X	–	Engine components, friction reduction with appropriate lubricant, highest wear resistance of all coating systems
Triondur CN	Cr_xN		X	X	–	Valve train components
Triondur TN	Titanium nitride TiN		X		–	Bearing components, rib surfaces
Triondur MN	CuMoN (nitridic hard material layer)		X	X	Increase in temperature resistance	Engine components, bearing components, nanostructured, wear protection and friction reduction under tribomechanical load

Recommendations for electrically insulating coatings

In order to prevent rolling bearing failures as a result of current passage, the cylindrical surfaces and end faces of the bearing rings can be provided with ceramic insulating coatings, see Figure 42.

Current insulation is achieved by means of plasma spray coating of the outside diameter and the lateral faces on the outer ring or the bore and lateral faces of the inner ring. The insulation coating comprises aluminium oxide, in which the pores are sealed with resin to give protection against the ingress of moisture.

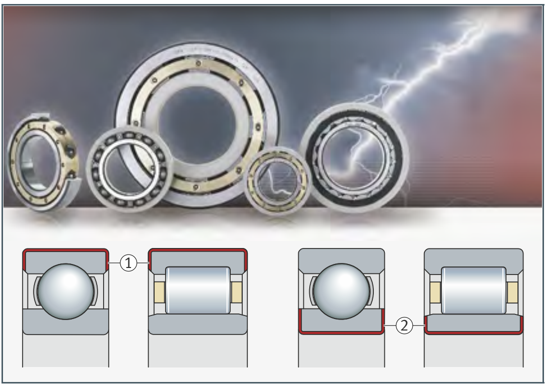
Principal function: Current insulation

The following table shows the features of the Insutect coating system for current insulation.

Coating system	Composition	Principal function	Main area of application, special feature
Insutect A	Aluminium oxide	Current insulation	Rail vehicles, electric motors, generators

Figure 42
Current-insulating bearings

① Coating of outer ring
② Coating of inner ring



Advantages of coated bearings:

- High level of insulation, even in a damp environment, due to a special sealing process
- The external dimensions of the bearing correspond to the dimensions in accordance with DIN 616 and are thus interchangeable with standard bearings.
- The puncture strength of thin layers is up to 500 VDC, while the puncture strength of thick layers is guaranteed to be at least 1 000 VDC.

Recommendations for sensory coatings

The innovative thin film sensor technology Sensotect allows, with neutral effect on design envelope and in real time, measurement of the load condition at locations where classic sensors such as adhesive bonded strain gauges cannot be used.

Sensotect is a strain-sensitive metal coating with a thickness measured in the submicrometre range that is structured by microprocessing. This measurement structures allows the continuous measurement of force and torque during operation.

Principal function: Sensor technology

The following table shows the features of the Sensotec coating system for measuring force and torque.

Coating system	Composition	Principal function	Main area of application, special feature
Sensotect	Multi-layer system comprising insulation coating and strain-sensitive PVD coating	Measurement of force and torque	Rolling bearings, bottom bracket bearings, wheel bearings, shafts, bending beams

Figure 43
Wheel bearing with Sensotect coating

Advantages of Sensotect:

- Very precise measurement of force and torque on functional components where the possibilities associated with conventional methods are limited.
- Sensor layer is deposited directly on the substrate surface.
- Measurement possible on 2D and 3D geometries
- Sensor technology with neutral effect on design envelope
- No use of adhesives or transfer polymers
- Continuous measurement of force and torque during operation
- High sensitivity with very little deviation in hysteresis and linearity
- No temperature deviations
- No ageing effects
- Wireless transfer of data and energy (telemetry)

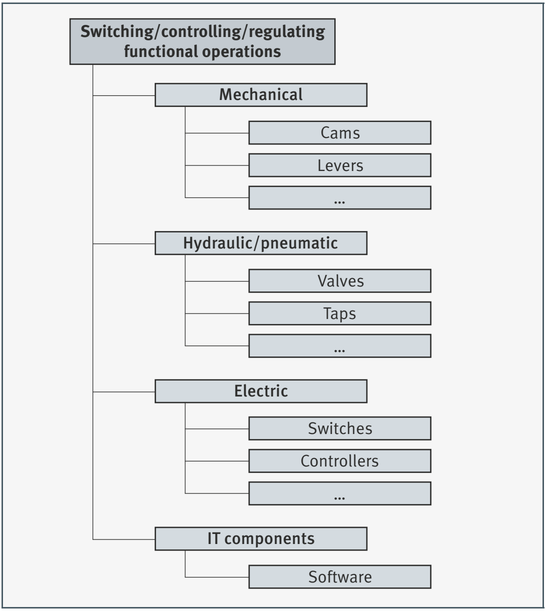
Switching/controlling/regulating functional operations

Overview

Technical systems must implement various functional operations in order to be able to achieve the required operational states. For this purpose, subsystems and the complete system must be switched, controlled and regulated in a suitable form.

In addition to the classical mechanical and hydraulic/pneumatic systems, a decisive role is now being played by electronic and IT-based systems and components. The latter are now represented, in modern mechatronic systems, by the information processing component of the complete system.

Figure 44
Switching/controlling/regulating functional operations – overview



Guiding elements in a rotary direction – rolling bearings

Rotary bearings allow relative rotary motions. These can involve rotation in one direction or oscillation.

Support and guidance function

The task (function) of rotary rolling bearings is to guide parts that are movable in relation to each other and support them relative to the adjacent structure. They support forces and transmit these into the adjacent construction. In this way, they perform support and guidance tasks and thus form the connection between stationary and moving machine elements.

Support as a function

The function “Support” comprises the transmission of forces and moments between parts moving relative to each other.

Guidance as a function

The function “Guidance” principally comprises defining to an appropriate (normally high) accuracy the position of parts moving relative to each other.

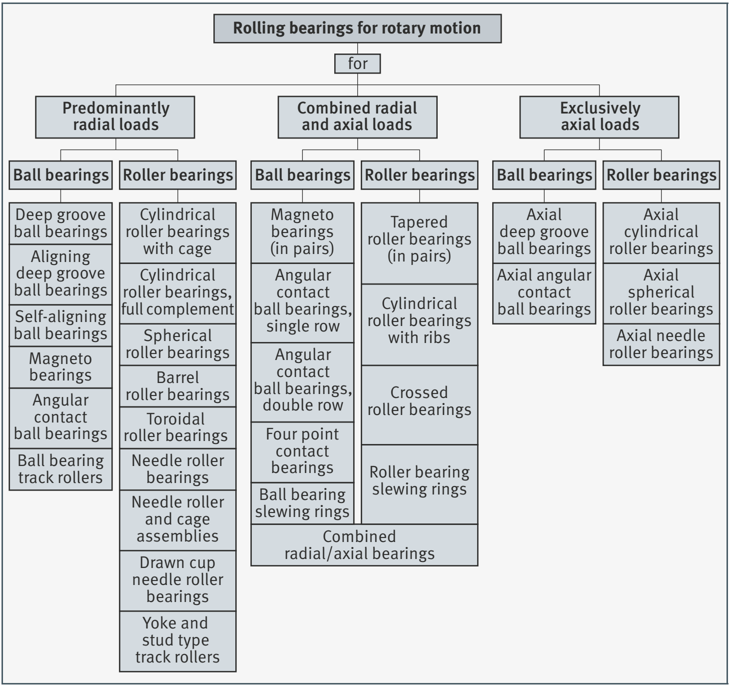
Principal requirements placed on bearings

Technical implementation is oriented to the two principal requirements:

- Function must be ensured and fulfilled for as along as possible.
- The resistance to motion (bearing friction) should be as low as possible in order to reduce the energy required for motion (energy efficiency).

Overview of common rolling bearings

The following diagram shows an overview of common bearing types for rotary motion, which are then described in detail starting on Page 650.



Dimensioning and design of rolling bearing arrangements

The design of rolling bearing arrangements requires consideration of a large number of factors. This includes the following steps:

- selection of the bearing type and bearing arrangement
- calculation of the bearing size
- design of the bearing position
- definition of the lubrication
- considerations relating to mounting and dismounting

Objectives and influencing factors

Long operating life, high reliability and cost-efficiency are essential objectives in the design of a rolling bearing arrangement. In order to achieve this, the designers must record in a design brief all the conditions and requirements that are an influence on the bearing arrangement.

At the draft stage, it is important to select not only the correct bearing type, bearing design and bearing arrangement, but also to ensure that the adjacent parts, namely the shaft, housing and fasteners, as well as the sealing and, in particular, the lubrication, are matched to the influences recorded in the design brief.

Design data

The following data should be available:

- machine, device and mounting positions of the bearings (diagram)
- **Operating conditions**
  (load, speed, design envelope, temperature, ambient conditions, shaft arrangement, rigidity of the adjacent components)
- **Requirements**
  (rating life, accuracy, noise, friction and operating temperature, lubrication and maintenance, mounting and dismounting)
- **Commercial data**
  (delivery dates, quantities, costs)

Technical principles

The following chapters give an overview of the technical principles that must be applied in the design of a bearing arrangement:

- Dimensioning – load carrying capacity and life, Page 571
- Speeds, Page 599
- Noise, Page 604
- Lubrication, Page 606
- Bearing data, Page 614
- Design of bearing arrangements, Page 625

In addition to the approximate calculation specifications in the printed catalogues, online software programs from the Schaeffler Group such as ➔ Bearinx and ➔ **medias** are available for more precise calculations of the bearing arrangement.

At the end of the chapter, various applications are presented to give examples of the design of bearing arrangements.

Dimensioning – load carrying capacity and life

The rating life calculation standardised in ISO 281 is based on Lundberg and Palmgren's fatigue theory which always gives a final rating life. However, modern, high quality bearings can exceed by a considerable margin the values calculated for the basic rating life under favourable operating conditions. Ioannides and Harris have developed a further model of fatigue in rolling contact that expands on the Lundberg and Palmgren theory and gives a better description of the performance capability of modern bearings.

The method "Expanded calculation of the adjusted rating life" takes account of the following influences:

- the bearing load
- the fatigue limit of the material
- the extent to which the surfaces are separated by the lubricant
- the cleanliness in the lubrication gap
- the additives in the lubricant
- the internal load distribution and friction conditions in the bearing

The influencing factors, particularly those relating to contamination, are very complex. A great deal of experience is essential for an accurate assessment. The tables and diagrams can give only guide values.

Calculation of the bearing size

The required size of a rolling bearing is dependent on the demands made on its:

- rating life
- load carrying capacity
- operational reliability

Dynamic load carrying capacity and operating life

The dynamic load carrying capacity of the rolling bearing is determined by the fatigue behaviour of the material. The dynamic load carrying capacity is described in terms of the basic dynamic load ratings, which are based on DIN ISO 281.

The dynamic load carrying capacity is described in terms of the basic dynamic load rating C and the basic rating life.

The fatigue life is dependent on:

- the load
- the operating speed
- the statistical probability of the initial appearance of failure

The basic dynamic load rating C applies to rotating rolling bearing. It is:

- a constant radial load C_r for radial bearings
- a constant, concentrically acting axial load C_a for axial bearings

The basic dynamic load rating C is that load of constant magnitude and direction which a sufficiently large number of apparently identical bearings can endure for a basic rating life of one million revolutions.

Calculation of the rating life

The methods for calculating the rating life are:

- the basic rating life L_{10} and L_{10h} in accordance with ISO 281
- the adjusted rating life L_{na} in accordance with DIN ISO 281:1990 (no longer a constituent part of ISO 281:2007)
- the expanded adjusted rating life L_{nm} in accordance with ISO 281, see Page 573

Basic rating life

The basic rating life L_{10} and L_{10h} is determined as follows:

Equation 31

$$L_{10} = \left(\frac{C}{P}\right)^p$$

Equation 32

$$L_{10h} = \frac{16\,666}{n} \cdot \left(\frac{C}{P}\right)^p$$

Legend

L_{10} — 10^6 revolutions
The basic rating life in millions of revolutions is the life reached or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue develops

L_{10h} — h
The basic rating life in operating hours according to the definition for L_{10}

C — N
Basic dynamic load rating

P — N
Equivalent dynamic bearing load for radial and axial bearings

p — –
Life exponent;
for roller bearings: $p = 10/3$
for ball bearings: $p = 3$

n — min^{-1}
Operating speed (nominal speed).

Equivalent dynamic bearing load

The equivalent dynamic bearing load P is a calculated value. This value is constant in magnitude and direction; it is a radial load for radial bearings and an axial load for axial bearings. A load corresponding to P will give the same rating life as the combined load occurring in practice.

Equation 33

$$P = X \cdot F_r + Y \cdot F_a$$

Legend

P — N
Equivalent dynamic bearing load

F_r — N
Radial bearing load

F_a — N
Axial bearing load

X — –
Radial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

Y — –
Axial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings.

This calculation cannot be applied to radial needle roller bearings, axial needle roller bearings and axial cylindrical roller bearings. Combined loads are not permissible with these bearings.

Expanded adjusted rating life

Calculation of the expanded adjusted rating life L_{nm} was standardised in DIN ISO 281 Appendix 1. Since 2007, it has been standardised in the worldwide standard ISO 281. Computer-aided calculation to DIN ISO 281 Appendix 4 has been specified since 2008 in ISO/TS 16281 and standardised since 2010 in DIN 26281.

L_{nm} is calculated as follows:

Equation 34

$$L_{nm} = a_1 \cdot a_{ISO} \cdot L_{10}$$

Legend

L_{nm} — 10^6 revolutions
Expanded adjusted rating life in millions of revolutions in accordance with ISO 281:2007

a_1 — –
Life adjustment factor for a requisite reliability other than 90%, see following table

a_{ISO} — –
Life adjustment factor for operating conditions

L_{10} — 10^6 revolutions
Basic rating life, see Page 572.

Life adjustment factor a_1

The values for the life adjustment factor a_1 were redefined in ISO 281:2007 and differ from the previous data.

Requisite reliability %	Expanded adjusted rating life L_{nm}	Life adjustment factor a_1
90	L_{10m}	1
95	L_{5m}	0,64
96	L_{4m}	0,55
97	L_{3m}	0,47
98	L_{2m}	0,37
99	L_{1m}	0,25
99,2	$L_{0,8m}$	0,22
99,4	$L_{0,6m}$	0,19
99,6	$L_{0,4m}$	0,16
99,8	$L_{0,2m}$	0,12
99,9	$L_{0,1m}$	0,093
99,92	$L_{0,08m}$	0,087
99,94	$L_{0,06m}$	0,08
99,95	$L_{0,05m}$	0,077

Life adjustment factor a_{ISO}

The standardised method for calculating the life adjustment factor a_{ISO} essentially takes account of the following influences:

- the load on the bearing
- the lubrication conditions (viscosity and type of lubricant, speed, bearing size, additives)
- the fatigue limit of the material
- the type of bearing
- the residual stress in the material
- the ambient conditions
- contamination of the lubricant

Equation 35

$$a_{ISO} = f\left[\frac{e_C \cdot C_u}{P}, \kappa\right]$$

Legend

a_{ISO} –
Life adjustment factor for operating conditions, see Figure 45, Page 575 to Figure 48, Page 576

e_C –
Life adjustment factor for contamination, see table Contamination factor e_C, Page 579

C_u N
Fatigue limit load

P N
Equivalent dynamic bearing load

κ –
Viscosity ratio, see Page 577
For $\kappa > 4$, calculation should be carried out using $\kappa = 4$.
This calculation method cannot be used for $\kappa < 0,1$.

In accordance with ISO 281, EP additives in the lubricant can be taken into consideration as follows:

- For a viscosity ratio $\kappa < 1$ and a contamination factor $e_C \geqq 0,2$, calculation of lubricants with EP additives that have been proven effective can be carried out using the value $\kappa = 1$. Under severe contamination (contamination factor $e_C < 0,2$), evidence must be obtained of the effectiveness of the additives under these contamination conditions. The effectiveness of the EP additives can be demonstrated in the actual application or on a rolling bearing test rig FE8 in accordance with DIN 51819-1.
- If calculation in the case of EP additives that have been proven effective is carried out using the value $\kappa = 1$, the life adjustment factor must be restricted to $a_{ISO} \leqq 3$. If the value a_{ISO} calculated for the actual κ is greater than 3, this value can be used in calculation.
- For practical purposes, the life adjustment factor should be restricted to $a_{ISO} \leqq 50$. This limit value also applies if $e_C \cdot C_u/P > 5$.

The following diagrams show the life adjustment factor a_{ISO} for various bearings.

Figure 45
Life adjustment factor a_{ISO} for radial roller bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio

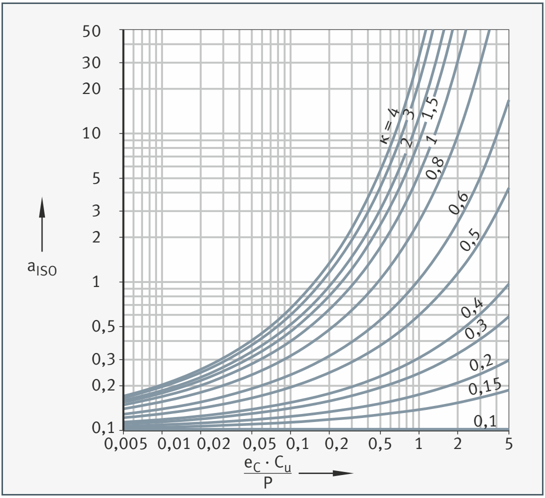
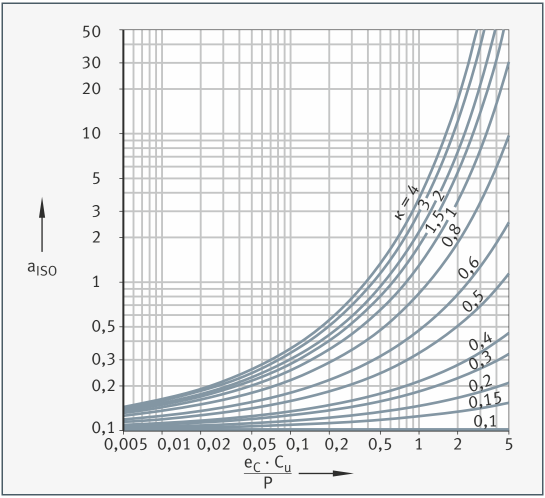


Figure 46
Life adjustment factor a_{ISO} for axial roller bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio



Continuation of diagrams showing the life adjustment factor a_{ISO} for various bearings.

Figure 47
Life adjustment factor a_{ISO} for radial ball bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio

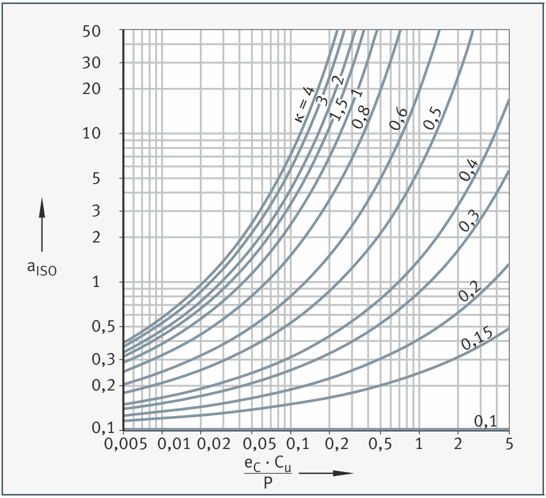
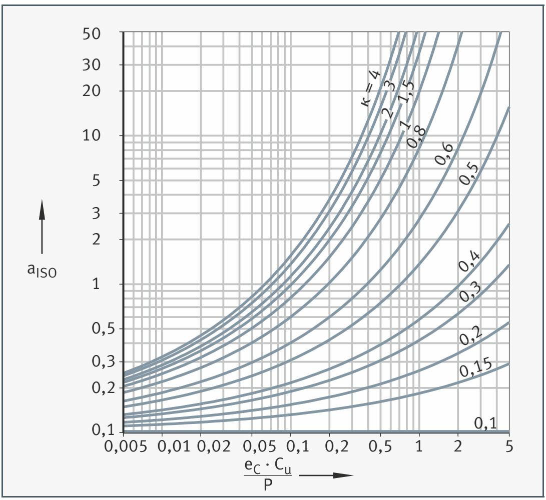


Figure 48
Life adjustment factor a_{ISO} for axial ball bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio



Fatigue limit load

The fatigue limit load C_u in accordance with ISO 281 is defined as the load below which, under laboratory conditions, no fatigue occurs in the material.

Viscosity ratio

The viscosity ratio κ is an indication of the quality of lubricant film formation:

Equation 36

$$\kappa = \frac{\nu}{\nu_1}$$

Legend

ν	mm^2/s	ν_1	mm^2/s
Kinematic viscosity of the lubricant at operating temperature		Reference viscosity of the lubricant at operating temperature.	

The reference viscosity ν_1 is determined from the mean bearing diameter $d_M = (D+d)/2$ and the operating speed n, see Figure 49, Page 578.

The nominal viscosity of the oil at +40 °C is determined from the required operating viscosity ν and the operating temperature ϑ, see Figure 50, Page 578. In the case of greases, the operating viscosity ν of the base oil is the decisive factor.

In the case of heavily loaded bearings with a high proportion of sliding contact, the temperature in the contact area of the rolling elements may be up to 20 K higher than the temperature measured on the stationary ring (without the influence of any external heat sources).

For information on taking account of EP additives in calculation of the expanded adjusted rating life L_{nm}, see Page 574.

The following diagrams show methods for determining the reference viscosity ν_1 and the operating viscosity ν.

Figure 49
Determining the reference viscosity ν_1

ν_1 = reference viscosity
d_M = mean bearing diameter (d + D)/2
n = operating speed

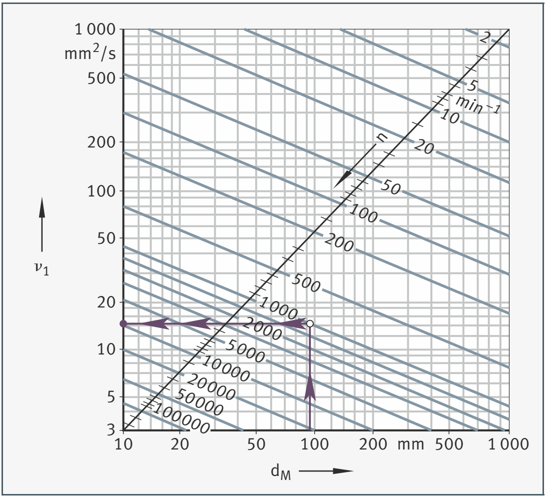
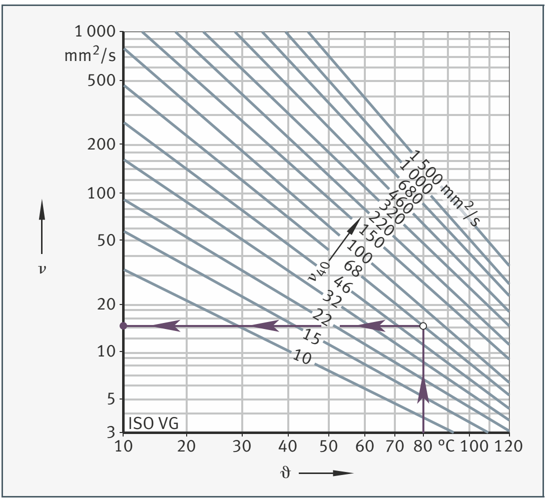


Figure 50
Determining the operating viscosity ν

ν = operating viscosity
ϑ = operating temperature
ν_{40} = viscosity at +40 °C



Life adjustment factor for contamination e_C
The life adjustment factor for contamination e_C takes account of the influence of contamination in the lubrication gap on the rating life.

The rating life is reduced by solid particles in the lubrication gap and is dependent on:

- the type, size, hardness and number of particles
- the relative lubricant film thickness
- the size of the bearing

Due to the complex interactions between these influencing factors, it is only possible to give approximate guide values. The values in the tables are valid for contamination by solid particles (factor e_C). No account is taken of other contamination such as that caused by water or other fluids. Under severe contamination ($e_C \rightarrow 0$), the bearings may fail due to wear. In this case, the operating life is substantially less than the calculated life.

For contamination of various degrees of severity, the contamination factors e_C are in accordance with the following table:

Contamination	**Contamination factor e_C**	
	$d_M < 100$ mm[1]	**$d_M \geqq 100$ mm[1]**
Extreme cleanliness ■ Particle size within the order of magnitude of the lubricant film thickness ■ Laboratory conditions	1	1
High cleanliness ■ Oil filtered through extremely fine filter ■ Sealed, greased bearings	0,8 to 0,6	0,9 to 0,8
Normal cleanliness ■ Oil filtered through fine filter	0,6 to 0,5	0,8 to 0,6
Slight contamination ■ Slight contamination of oil	0,5 to 0,3	0,6 to 0,4
Typical contamination ■ Bearing contaminated by wear debris from other machine elements	0,3 to 0,1	0,4 to 0,2
Heavy contamination ■ Bearing environment heavily contaminated ■ Bearing arrangement inadequately sealed	0,1 to 0	0,1 to 0
Very heavy contamination	0	0

[1] d_M = mean bearing diameter $(d + D)/2$.

Equivalent operating values

The rating life equations are based on the assumption that the bearing load P and bearing speed n are constant. If the load and speed are not constant, equivalent operating values can be determined that induce the same fatigue as the actual loading conditions.

The equivalent operating values calculated here already take account of the life adjustment factors a_{ISO}. This factor a_{ISO} must not be applied again in calculation of the expanded adjusted rating life. If only a basic rating life is to be calculated, the terms $1/a_{ISO}$ can be omitted from the equations.

Variable load and speed

If the load and speed vary over a time period T, the speed n and the equivalent bearing load P are calculated as follows (for an explanation of the designations used, see Page 582):

Equation 37

$$n = \frac{1}{T}\int_0^T n(t) \cdot dt$$

Equation 38

$$P = \sqrt[p]{\frac{\int_0^T \frac{1}{a_{ISO}(t)} \cdot n(t) \cdot F^p(t) \cdot dt}{\int_0^T n(t) \cdot dt}}$$

Variation in steps

If the load and speed vary in steps over a time period T, n and P are calculated as follows:

Equation 39

$$n = \frac{q_1 \cdot n_1 + q_2 \cdot n_2 + \ldots + q_z \cdot n_z}{100}$$

Equation 40

$$P = \sqrt[p]{\frac{\frac{1}{a_{ISO\,i}} \cdot q_i \cdot n_i \cdot F_i^p + \ldots + \frac{1}{a_{ISO\,z}} \cdot q_z \cdot n_z \cdot F_z^p}{q_i \cdot n_i + \ldots + q_z \cdot n_z}}$$

Variable load at constant speed
If the function F describes the variation in the load over a time period T and the speed is constant, P is calculated as follows:

Equation 41

$$P = \sqrt[p]{\frac{1}{T}\int_0^T \frac{1}{a_{ISO}(t)} \cdot F^p(t) \cdot dt}$$

Load varying in steps at constant speed
If the load varies in steps over a time period T and the speed is constant, P is calculated as follows:

Equation 42

$$P = \sqrt[p]{\frac{\frac{1}{a_{ISO\,i}} \cdot q_i \cdot F_i^p + \ldots + \frac{1}{a_{ISO\,z}} \cdot q_z \cdot F_z^p}{100}}$$

Constant load at variable speed
If the speed varies but the load remains constant, this gives:

Equation 43

$$n = \frac{1}{T}\int_0^T \frac{1}{a_{ISO}(t)} \cdot n(t) \cdot dt$$

Constant load at speed varying in steps
If the speed varies in steps, this gives:

Equation 44

$$n = \frac{\frac{1}{a_{ISO\,i}} \cdot q_i \cdot n_i + \ldots + \frac{1}{a_{ISO\,z}} \cdot q_z \cdot n_z}{100}$$

For oscillating bearing motion
The equivalent speed is calculated as follows:

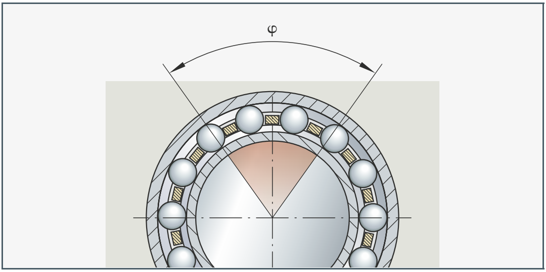
Equation 45

$$n = n_{osc} \cdot \frac{\varphi}{180°}$$

The equation is valid only if the swivel angle is larger than twice the pitch angle of the rolling elements. If the swivel angle is smaller, there is a risk of false brinelling.

The following diagram shows the swivel angle φ.

Figure 51
Swivel angle φ



Symbols, units and definitions
The following values are used in calculation of the equivalent operating values:

Legend

n	min^{-1}	n_i, $n(t)$	min^{-1}
Mean speed		Bearing speed for a particular operating condition	
T	min	q_i	%
Time period under consideration		Duration of an operating condition as a proportion of the total operating period; $q_i = (\Delta t_i/T) \cdot 100$	
P	N	F_i, $F(t)$	N
Equivalent bearing load		Bearing load for a particular operating condition	
p	–	n_{osc}	min^{-1}
Life exponent; for roller bearings: $p = 10/3$ for ball bearings: $p = 3$		Frequency of swivel motion	
$a_{ISO\,i}$, $a_{ISO}(t)$	–	φ	°
Life adjustment factor a_{ISO} for current operating condition, see Page 574		Swivel angle, see Figure 51.	

Requisite rating life

If no information is available on the rating life, guide values can be taken from tables or diagrams, such as Figure 52 and the following table. Guide values for dimensioning: see also Schaeffler catalogue HR 1, Rolling Bearings.

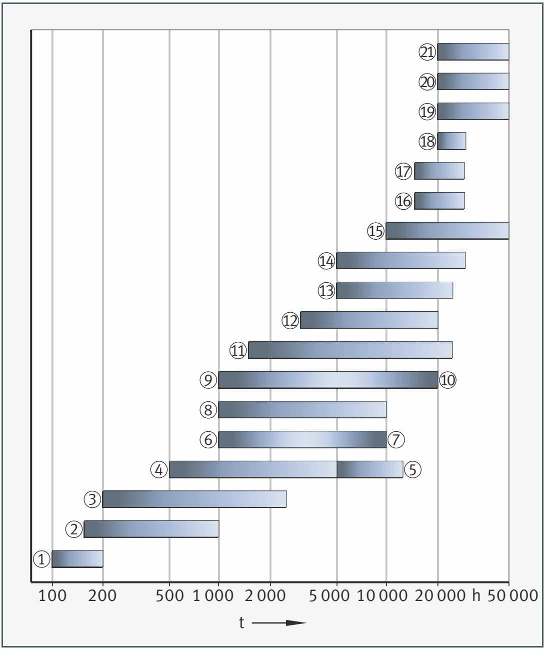
Bearings should not be overspecified, for the precise rating life see Schaeffler catalogue HR 1, Rolling Bearings.

Pay attention to the minimum load for the bearings, see Schaeffler catalogue HR 1, Rolling Bearings.

Figure 52
Guide values for rating life

t = operating hours

① DIY tools
② Power tools
③ Household appliances
④ Agricultural machinery
⑤ Tractors
⑥ Passenger cars
⑦ Commercial vehicles
⑧ Construction machinery
⑨ Hydraulic units, mobile
⑩ Hydraulic units, stationary
⑪ Office and computer equipment
⑫ Compressors
⑬ Handling equipment
⑭ Industrial gearboxes
⑮ Construction materials machinery
⑯ Crushers
⑰ Machine tools
⑱ Extruders
⑲ Rolling mills
⑳ Textile machinery
㉑ Printing machinery



Operating mode	Operating hours/year
Intermittent operation	≈ 500 to 1 000 h
Single shift operation	≈ 2 000 h
Double shift operation	≈ 4 000 h
Triple shift operation	≈ 6 000 h
Continuous operation	≈ 8 000 h

Static load carrying capacity

If high, static or shock loads occur, the raceways and rolling elements may undergo plastic deformation. These deformations limit the static load carrying capacity of the rolling bearing with respect to the permissible noise level during operation of the bearing.

If a rolling bearing operates without rotary motion or with only infrequent rotary motion, its size is determined in accordance with the basic static load rating C_0.

In accordance with DIN ISO 76, this is:

- a constant radial load C_{0r} for radial bearings
- a constant, concentrically acting axial load C_{0a} for axial bearings

The basic static load rating C_0 is that load under which the Hertzian pressure at the most heavily loaded point between the rolling elements and raceways reaches the following values:

- for roller bearings, 4 000 N/mm^2
- for ball bearings, 4 200 N/mm^2
- for self-aligning ball bearings, 4 600 N/mm^2

Under normal contact conditions, this load causes a permanent deformation at the contact points of approximately one tenth of a thousandth of the rolling element diameter.

Static load safety factor

In addition to dimensioning on the basis of the fatigue life, it is advisable to check the static load safety factor. The guide values for shock loads occurring in operation as given in the following table should be taken into consideration.

The static load safety factor S_0 is the ratio between the basic static load rating C_0 and the equivalent static bearing load P_0:

Equation 46

$$S_0 = \frac{C_0}{P_0}$$

Legend

S_0 – Static load safety factor	P_0 (P_{0r}, P_{0a}) N Equivalent static load on the radial or axial bearing, see Page 585.
C_0 (C_{0r}, C_{0a}) N Basic static load rating	

Guide values for static load safety factor

Guide values for axial spherical roller bearings and high precision bearings: see Schaeffler catalogue HR 1, Rolling Bearings. For drawn cup needle roller bearings, the value must be $S_0 \geqq 3$.

For the static load safety factor, the following guide values can be used:

Operating conditions and application	Static load safety factor S_0 min.	
	Ball bearings	Roller bearings
Low-noise, smooth running, free from vibrations, high rotational accuracy	2	3
Normal, smooth running, free from vibrations, normal rotational accuracy	1	1,5
Pronounced shock loading[1)]	1,5	3

1) If the order of magnitude of the shock loading is not known, the values used for S_0 should be at least 1,5. If the order of magnitude of the shock loading is known precisely, lower values are possible.

Equivalent static bearing load

The equivalent static bearing load P_0 is a calculated value. It corresponds to a radial load in radial bearings and a concentric axial load in axial bearings.

P_0 induces the same load at the centre point of the most heavily loaded contact point between the rolling element and raceway as the combined load occurring in practice.

The relationship is as follows:

Equation 47

$$P_0 = X_0 \cdot F_r + Y_0 \cdot F_a$$

Legend

P_0 N
Equivalent static bearing load

F_r N
Largest radial load present

F_a N
Largest axial load present

X_0 –
Radial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

Y_0 –
Axial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings.

This calculation cannot be applied to radial needle roller bearings, axial needle roller bearings and axial cylindrical roller bearings. Combined loads are not permissible with these bearings.

In the case of radial needle roller bearings and all radial cylindrical roller bearings, $P_0 = F_{0r}$. For axial needle roller bearings and axial cylindrical roller bearings, $P_0 = F_{0a}$.

Operating life

The operating life is defined as the life actually achieved by the bearing. It may differ significantly from the calculated value.

This may be due to wear or fatigue as a result of:

- deviations in the operating data
- misalignment between the shaft and housing
- insufficient or excessive operating clearance
- contamination
- inadequate lubrication
- excessive operating temperature
- oscillating bearing motion with very small swivel angles (false brinelling)
- high vibration loads and false brinelling
- very high shock loads (static overloading)
- prior damage during installation

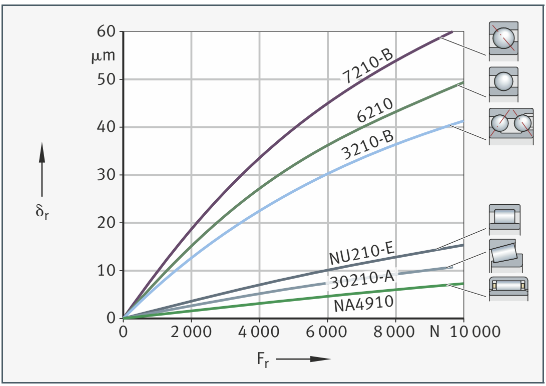
Due to the wide variety of possible mounting and operating conditions, it is not possible to precisely predetermine the operating life. The most reliable way of arriving at a close estimate is by comparison with similar applications.

Rigidity

The rigidity is determined by the type, size and operating clearance of the bearing. It increases with the number of rolling elements supporting the load. Due to the line contact between the rolling elements and raceways, it is higher in roller bearings than in ball bearings. Figure 53 shows typical characteristic curves for the radial deflection of various bearings with the same bore diameter.

Figure 53
Radial deflection of various radial bearings of bore diameter d = 50 mm

δ_r = radial deflection
F_r = radial bearing load



Calculation of the radial or axial displacement

Rolling bearings have a progressive deflection rate. The displacement values for needle and cylindrical roller bearings can be determined using approximation equations. This simplified calculation cannot be applied to other bearing types. The displacement and rigidity at the operating point can be determined using the calculation program ➔ Bearinx-online.

The equations are valid for bearings without misalignment and with a rigid surrounding structure. In axial bearings, a concentrically acting load is assumed.

Equation 48

$$\delta_r = \frac{1}{c_s} \cdot F_r^{0,84} + \frac{s}{2}$$

Equation 49

$$\delta_a = \frac{1}{c_s} \cdot \left[\left(F_{aV} + F_a \right)^{0,84} - F_{aV}{}^{0,84} \right]$$

Equation 50

$$c_s = K_c \cdot d^{0,65}$$

Legend

c_s — $N^{0,84}$ / µm
Rigidity parameter

d — mm
Bearing bore diameter

δ_r — µm
Radial displacement between shaft axis and bore centre, Figure 54, Page 588

δ_a — µm
Axial displacement between shaft locating washer and housing locating washer, Figure 55, Page 588

s — µm
Radial operating clearance of fitted, unloaded bearing

F_r — N
Radial bearing load

F_a — N
Axial bearing load

F_{aV} — N
Axial preload force

K_c — –
Factor for determining the rigidity parameter, see table Page 587.

Factor K_c

The factor K_c is given in the following table.

Bearing series	Factor K_c	Bearing series	Factor K_c	Bearing series	Factor K_c
NA48	24,9	NJ2..-E	11,1	SL1818	12,8
NA49	23,5	NJ3..-E	11,3	SL1829, SL1830, SL1923	16
NA69	37,3	NJ22..-E	15,4	SL1850, SL0148, SL0248, SL0249	29,2
NKIS	21,3	NJ23..-E	16,9	K811, 811, K812, 812	36,7
NKI	$4,4 \cdot B^{0,8}/d^{0,2}$	NU10	9,5		
HK, BK	$4,2 \cdot C^{0,8}/d^{0,2}$	NU19	11,3	K893, 893, K894, 894	59,7
		NN30..-AS-K	18,6		

The following diagrams show the radial and axial displacement.

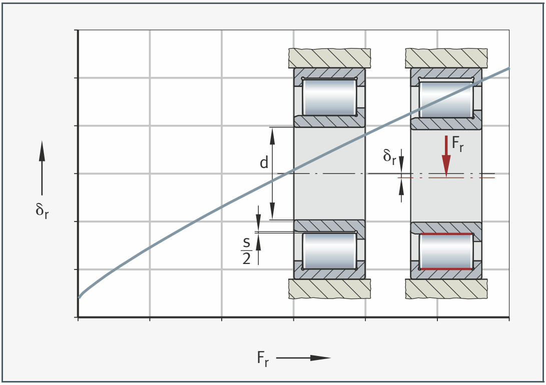


Figure 54
Radial displacement – radial cylindrical roller bearings

δ_r = radial displacement
F_r = radial bearing load

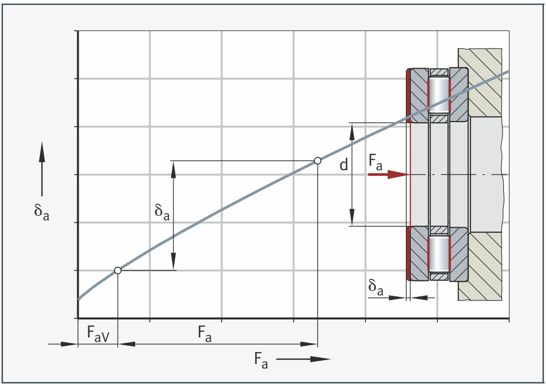


Figure 55
Axial displacement – axial cylindrical roller bearings

δ_a = axial displacement
F_a = axial bearing load
F_{aV} = axial preload force

Friction and increases in temperature

The friction in a rolling bearing is made up of several components. Due to the large number of influencing factors, such as dynamics in speed and load, tilting and skewing as a result of mounting, actual frictional torques and frictional power may deviate significantly from the calculated values.

If the frictional torque is an important design criterion, please consult Schaeffler. The calculation module ➔ Bearinx-online Easy Friction, which is available from Schaeffler free of charge, can be used to calculate and analyse the frictional torque.

The frictional component and the influencing factor are shown in the following table.

Frictional component	Influencing factor
Rolling friction	Magnitude of load
Sliding friction of rolling elements Sliding friction of cage	Magnitude and direction of load Speed and lubrication conditions, running-in condition
Fluid friction (flow resistance)	Type and speed Type, quantity and operating viscosity of lubricant
Seal friction	Type and preload of seal

The idling friction is dependent on the lubricant quantity, speed, operating viscosity of the lubricant, seals and the running-in condition of the bearing.

Heat dissipation

Friction is converted into heat. This must be dissipated from the bearing. The equilibrium between the frictional power and heat dissipation allows calculation of the thermally safe operating speed n_ϑ, see Page 600.

Heat dissipation via the lubricant

Lubricating oil dissipates a portion of the heat. Recirculating oil lubrication with additional cooling is particularly effective. Grease does not give dissipation of heat.

Heat dissipation via the shaft and housing

Heat dissipation via the shaft and housing is dependent on the temperature difference between the bearing and the surrounding structure.

Any additional adjacent sources of heat or thermal radiation must be taken into consideration.

Determining the friction values

For this process, the speed and load must be known. The type of lubrication, lubrication method and viscosity of the lubricant at operating temperature are further important factors in calculation.

Total frictional torque M_R:
(calculation of axially loaded cylindrical roller bearings, see Page 597):

Equation 51

$$M_R = M_0 + M_1$$

Frictional power N_R:

Equation 52

$$N_R = M_R \cdot \frac{n}{9\,550}$$

Frictional torque as a function of speed for $\nu \cdot n \geqq 2\,000$:

Equation 53

$$M_0 = f_0 \cdot (\nu \cdot n)^{2/3} \cdot d_M{}^3 \cdot 10^{-7}$$

Frictional torque as a function of speed for $\nu \cdot n < 2\,000$:

Equation 54

$$M_0 = f_0 \cdot 160 \cdot d_M{}^3 \cdot 10^{-7}$$

Frictional torque as a function of load for needle roller and cylindrical roller bearings:

Equation 55

$$M_1 = f_1 \cdot F \cdot d_M$$

Frictional torque as a function of load for ball bearings, tapered roller bearings and spherical roller bearings:

Equation 56

$$M_1 = f_1 \cdot P_1 \cdot d_M$$

Legend

M_R Nmm
Total frictional torque

M_0 Nmm
Frictional torque as a function of speed

M_1 Nmm
Frictional torque as a function of load

N_R W
Frictional power

n min^{-1}
Operating speed

f_0 –
Bearing factor for frictional torque as a function of speed, see Figure 56, Page 591 and tables from Page 591 to Page 596

f_1 –
Bearing factor for frictional torque as a function of load, see tables from Page 591 to Page 596

ν mm^2/s
Kinematic viscosity of lubricant at operating temperature. In the case of grease, the decisive factor is the viscosity of the base oil at operating temperature

F_r, F_a N
Radial load for radial bearings, axial load for axial bearings

P_1 N
Decisive load for frictional torque. For ball bearings, tapered roller bearings and spherical roller bearings, see Page 596

d_M mm
Mean bearing diameter (d + D)/2.

Bearing factors

The bearing factors f_0 and f_1 are mean values derived from series of tests and correspond to the data according to ISO 15312.

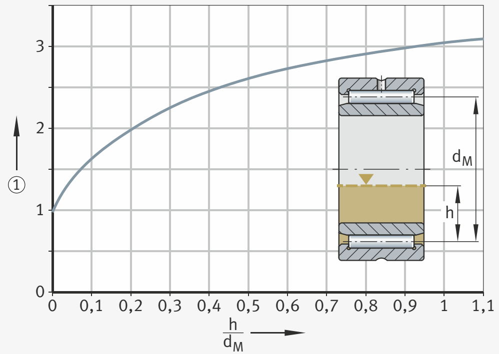
They are valid for bearings after running-in and with uniform distribution of lubricant. In the freshly greased state, the bearing factor f_0 can be two to five times higher.

If oil bath lubrication is used, the oil level must reach the centre of the lowest rolling element. If the oil level is higher, f_0 may be up to 3 times the value given in the table, Figure 56.

Figure 56
Increase in the bearing factor, as a function of the oil level

h = oil level
d_M = mean bearing diameter (d + D)/2

① Increase in the bearing factor f_0



The bearing factors for various rolling bearings are given in the following tables.

Bearing factors for needle roller bearings, drawn cup needle roller bearings, needle roller and cage assemblies:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
NA48	3	5	0,000 5
NA49	4	5,5	
RNA48	3	5	
RNA49	4	5,5	
NA69, RNA69	7	10	
NKI, NK, NKIS, NKS, NAO, RNO, RNAO, K	(12 · B)/(33 + d)	(18 · B)/(33 + d)	
NK..-TW, NKI..-TW, NK..-D	(10 · B)/(33 + d)	(15 · B)/(33 + d)	
HK, BK	(24 · B)/(33 + d)	(36 · B)/(33 + d)	
HN	(30 · B)/(33 + d)	(45 · B)/(33 + d)	

Bearing factors for cylindrical roller bearings, full complement:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
SL1818	3	5	0,000 55
SL1829	4	6	
SL1830	5	7	
SL1822	5	8	
SL0148, SL0248	6	9	
SL0149, SL0249	7	11	
SL1923	8	12	
SL1850	9	13	

Bearing factors for cylindrical roller bearings with cage:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
LSL1923	1	3,7	0,000 20
ZSL1923	1	3,8	0,000 25
NU2..-E, NNU41	1,3	2	0,000 30
NU3..-E			0,000 35
NU4			0,000 40
NU10, NU19			0,000 20
NU22..-E	2	3	0,000 40
NU23..-E	2,7	4	0,000 40
NU30..-E, NN30..-E	1,7	2,5	0,000 40

Bearing factors for axial roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
AXK, AXW	3	4	0,001 5
810, K810, 811, K811, 812, K812, 893, K893, 894, K894	2	3	

Bearing factors for combined bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
ZARN, ZARF	3	4	0,001 5
NKXR	2	3	
NX, NKX	2	3	$0,001 \cdot (P_0/C_0)^{0,33}$
ZKLN, ZKLF	4	6	0,000 5
NKIA, NKIB	3	5	

Bearing factors for tapered roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
302, 303, 329 320, 330, JK0S, T4CB, T4DB, T7FC	2	3	0,000 4
313, 322, 323, 331, 332, T2EE, T2ED, T5ED	3	4,5	

Bearing factors for axial and radial spherical roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
213..-E1	2,3	3,5	$0,000\,5 \cdot (P_0/C_0)^{0,33}$
222..-E1	2,7	4	
223	3	4,5	$0,000\,8 \cdot (P_0/C_0)^{0,33}$
238, 239, 230			$0,000\,75 \cdot (P_0/C_0)^{0,5}$
231	3,7	5,5	$0,001\,2 \cdot (P_0/C_0)^{0,5}$
232	4	6	$0,001\,6 \cdot (P_0/C_0)^{0,5}$
240	4,3	6,5	$0,001\,2 \cdot (P_0/C_0)^{0,5}$
248, 249, 241	4,7	7	$0,002\,2 \cdot (P_0/C_0)^{0,5}$
292..-E	1,7	2,5	0,000 23
293..-E	2	3	0,000 30
294..-E	2,2	3,3	0,000 33

Bearing factors for toroidal roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
C22..-K	3,7	5,5	$0{,}0012 \cdot (P_0/C_0)^{0,5}$
C22..-V	4	6	
C23..-K	3,8	5,7	$0{,}0014 \cdot (P_0/C_0)^{0,5}$
C23..-V	4,3	6,5	
C30..-K	3,3	5	
C30..-V, C31..-V	4	6	
C31..-K	3,7	5,5	
C32..-K	3,8	5,7	$0{,}0016 \cdot (P_0/C_0)^{0,5}$
C39..-K	3,3	5	$0{,}0014 \cdot (P_0/C_0)^{0,5}$
C40..-K, C41..-K	5	7,5	$0{,}0018 \cdot (P_0/C_0)^{0,5}$
C40..-V, C41..-V	6	9	

Bearing factors for deep groove ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
618	1,1	1,7	$0{,}0005 \cdot (P_0/C_0)^{0,5}$
160, 60, 619	1,1	1,7	$0{,}0007 \cdot (P_0/C_0)^{0,5}$
622, 623	1,1	1,7	$0{,}0009 \cdot (P_0/C_0)^{0,5}$
62	1,3	2	
63, 630, 64	1,5	2,3	
60..-C	1,1	1,5	$0{,}0006 \cdot (P_0/C_0)^{0,5}$
62..-C	1,3	1,7	$0{,}0007 \cdot (P_0/C_0)^{0,5}$
63..-C	1,5	2	
42..-B	2,3	3,5	$0{,}0010 \cdot (P_0/C_0)^{0,5}$
43..-B	4	6	

Bearing factors for angular contact ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
708, 719, 718..-B, 70..-B, 72..-B	1,3	2	$0{,}001 \cdot (P_0/C_0)^{0{,}33}$
73..-B	2	3	
74..-B	2,5	4	
30..-B, 32..-B, 38..-B	2,3	3,5	
33..-B	4	6	
32..-BD	2	3	
33..-BD	3,5	5	

Bearing factors for self-aligning ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
10, 112	1,7	2	$0{,}000\,3 \cdot (P_0/C_0)^{0{,}4}$
12	1,7	2,5	
13	2,3	3,5	
22	2	3	
23	2,7	4	

Bearing factors for four point contact bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
QJ2	1,3	2	$0{,}001 \cdot (P_0/C_0)^{0{,}33}$
QJ3	2	3	
QJ10	1,3	2	

Bearing factors for axial deep groove ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
511, 512, 513, 514, 532, 533, 534	1	1,5	$0{,}0012 \cdot (F_a/C_0)^{0{,}33}$
522, 523, 524, 542, 543, 544	1,3	2	

The following table shows the decisive load P_1 for the frictional torque as a function of load M_1 for ball bearings, tapered roller bearings and spherical roller bearings:

Bearing type	Decisive load P_1	
	Single bearing	Bearing pair
Deep groove ball bearings	$3{,}3 \cdot F_a - 0{,}1 \cdot F_r$	–
Angular contact ball bearings, single row	$F_a - 0{,}1 \cdot F_r$	$1{,}4 \cdot F_a - 0{,}1 \cdot F_r$
Angular contact ball bearings, double row	$1{,}4 \cdot F_a - 0{,}1 \cdot F_r$	–
Four point contact bearings	$1{,}5 \cdot F_a + 3{,}6 \cdot F_r$	–
Tapered roller bearings	$2 \cdot Y \cdot F_a$ or F_r, use the greater value	$1{,}21 \cdot Y \cdot F_a$ or F_r, use the greater value
Spherical roller bearings	$1{,}6 \cdot F_a/e$, if $F_a/F_r > e$ $F_r \cdot \{1 + 0{,}6 \cdot [F_a/(e \cdot F_r)]^3\}$, if $F_a/F_r \leqq e$	
Cylindrical roller bearings	F_r, the friction component of the axial load F_a must be taken into consideration by means of M_2	

If $P_1 \leqq F_r$, then $P_1 = F_r$.

Cylindrical roller bearings under axial load

In radial cylindrical roller bearings under axial load, sliding friction between the end faces of the rolling elements and the ribs on the rings leads to an additional frictional torque M_2.

The total frictional torque M_R is therefore calculated as follows:

Equation 57

$$M_R = M_0 + M_1 + M_2$$

Equation 58

$$M_2 = f_2 \cdot F_a \cdot d_M$$

Equation 59

$$A = k_B \cdot 10^{-3} \cdot d_M{}^{2,1}$$

Legend

M_R Nmm
Total frictional torque

M_0 Nmm
Frictional torque as a function of speed

M_1 Nmm
Frictional torque as a function of radial load

M_2 Nmm
Frictional torque as a function of axial load

f_2 –
Factor as a function of the bearing series, Figure 57 and Figure 58, Page 598

A –
Bearing parameter

F_a N
Axial dynamic bearing load

k_B –
Bearing factor as a function of the bearing series, see table, Page 599

d_M mm
Mean bearing diameter (d + D)/2.

The bearing factors f_2 are subject to wide scatter. They are valid for recirculating oil lubrication with an adequate quantity of oil. The curves must not be extrapolated, Figure 57 and Figure 58, Page 598.

Bearings of TB design

In the case of bearings of TB design (bearings with a toroidal roller end), the axial load carrying capacity has been significantly improved through the use of new calculation and manufacturing methods.

Optimum contact conditions between the roller and rib are ensured by means of a special curvature of the roller end faces. As a result, axial surface pressures on the rib are significantly reduced and a lubricant film with improved load carrying capacity is achieved. Under normal operating conditions, wear and fatigue at the rib contact running and roller end faces is completely eliminated.

In addition, axial frictional torque is reduced by up to 50%. The bearing temperature during operation is therefore significantly lower.

Bearing factor k_B

The bearing factor k_B in the equations takes into consideration the size and thus the load carrying capacity of the hydrodynamic contacts at the bearing ribs; see table Page 599.

The following diagrams show the bearing factors for cylindrical roller bearings.

Figure 57
Bearing factor f_2, as a function of operating parameter

Cylindrical roller bearings of standard design

f_2 = bearing factor
ν = operating viscosity
n = operating speed
d_M = mean bearing diameter
$\nu \cdot n \cdot d_M$ = operating parameter
F_a = axial dynamic bearing load
A = bearing parameter

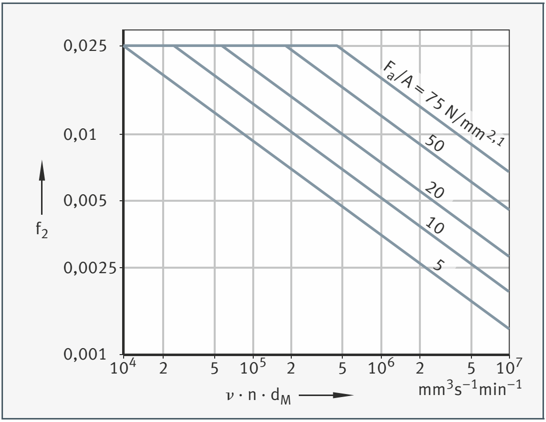
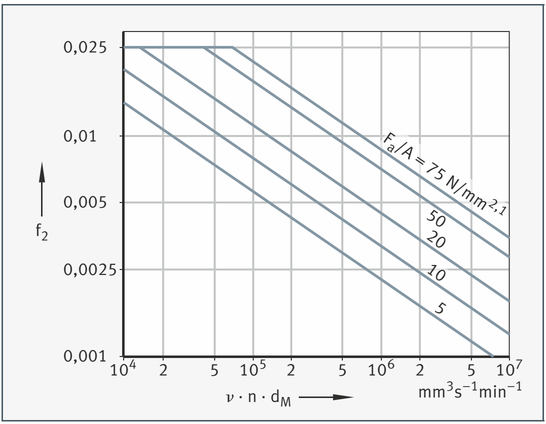


Figure 58
Bearing factor f_2, as a function of operating parameter

Cylindrical roller bearings of TB design

f_2 = bearing factor
ν = operating viscosity
n = operating speed
d_M = mean bearing diameter
$\nu \cdot n \cdot d_M$ = operating parameter
F_a = axial dynamic bearing load
A = bearing parameter



Bearing factor k_B

Bearing series	Bearing factor k_B	Bearing series	Bearing factor k_B
SL1818, SL0148	4,5	SL1923	30
SL1829, SL0149	11	NJ2..-E, NJ22..-E, NUP2..-E, NUP22..-E	15
SL1830, SL1850	17		
SL1822	20	NJ3..-E, NJ23..-E, NUP3..-E, NUP23..-E	20
LSL1923, ZSL1923	28	NJ4	22

Speeds Calculation of the thermal speed rating $n_{\vartheta r}$ is standardised in ISO 15312.

Limiting speed The limiting speed n_G is based on practical experience and takes account of additional criteria such as smooth running, sealing function and centrifugal forces. The limiting speed must not be exceeded even under favourable operating and cooling conditions.

Thermal speed rating The thermal speed rating $n_{\vartheta r}$ is used as an ancillary value in calculating the thermally safe operating speed n_{ϑ}. This is the speed at which, under defined reference conditions, a bearing operating temperature of +70 °C is achieved.
The thermal speed rating is not a speed limit for the application of a bearing. It is primarily for the purpose of comparing the speed suitability of different bearing types under defined reference conditions.

A speed limit taking account of the thermal balance can be calculated using the thermally safe operating speed.

Reference conditions The reference conditions are based on the normal operating conditions of the most significant bearing types and sizes and are defined in ISO 15312 as follows:

- mean ambient temperature ϑ_{Ar} = +20 °C
- mean bearing temperature at the outer ring ϑ_r = +70 °C
- load on radial bearings (pure, constant radial load) $P_1 = 0{,}05 \cdot C_{0r}$
- load on axial bearings (concentrically acting, constant axial load) $P_1 = 0{,}02 \cdot C_{0a}$
- oil bath lubrication with an oil level up to the centre of the lowest rolling element
- lubricant with a kinematic viscosity ν_r at ϑ_r = +70 °C
  - radial bearings: $\nu_r = 12\ mm^2/s$ (ISO VG 32)
  - axial bearings: $\nu_r = 24\ mm^2/s$ (ISO VG 68)
- heat dissipation via the bearing seating surfaces. As a simplification, calculation is based on the bearing seating surfaces dissipating heat under the reference conditions A_r. For the calculation of the bearing seating surfaces dissipating heat under the reference conditions, see Legend, Page 603. The relationships described below apply in this case.

For radial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r \leqq 50\,000$ mm^2:

Equation 60

$$q_r = 0{,}016\ \mathrm{W/mm^2}$$

For radial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r > 50\,000$ mm^2:

Equation 61

$$q_r = 0{,}016 \cdot \left(\frac{A_r}{50\,000}\right)^{-0{,}34} \mathrm{W/mm^2}$$

For axial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r \leqq 50\,000$ mm^2:

Equation 62

$$q_r = 0{,}020\ \mathrm{W\,/\,mm^2}$$

For axial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r > 50\,000$ mm^2:

Equation 63

$$q_r = 0{,}020 \cdot \left(\frac{A_r}{50\,000}\right)^{-0{,}16} \mathrm{W/mm^2}$$

Thermally safe operating speed

The thermally safe operating speed n_ϑ is calculated in accordance with DIN 732:2010. The basis for the calculation is the heat balance in the bearing, the equilibrium between the frictional energy as a function of speed and the heat dissipation as a function of temperature. When conditions are in equilibrium, the bearing temperature is constant.

The permissible operating temperature determines the thermally safe operating speed n_ϑ of the bearing. The preconditions for calculation are correct mounting, normal operating clearance and constant operating conditions.

The calculation method is not valid for:

- sealed bearings with contact seals, since the maximum speed is restricted by the permissible sliding velocity at the seal lip
- yoke and stud type track rollers
- aligning needle roller bearings
- axial deep groove ball bearings and axial angular contact ball bearings

The limiting speed n_G must always be observed.

Calculation of the thermally safe operating speed

The thermally safe operating speed n_ϑ is a product of the thermal speed rating $n_{\vartheta r}$ and the speed ratio f_n:

Equation 64

$$n_\vartheta = n_{\vartheta r} \cdot f_n$$

The equation derived from the equilibrium of the frictional power and the heat dissipation can be expressed, through the introduction of a lubricant parameter K_L, a load parameter K_P and the speed ratio f_n in the following form:

Equation 65

$$K_L \cdot f_n^{5/3} + K_P \cdot f_n = 1$$

The speed ratio f_n can be determined, in the normal value range of $0{,}01 \leqq K_L \leqq 10$ and $0{,}01 \leqq K_P \leqq 10$, using an approximation equation, see also Figure 59, Page 602:

Equation 66

$$f_n = 490{,}77 \cdot \left(1 + 498{,}78 \cdot K_L^{0{,}599} + 852{,}88 \cdot K_P^{0{,}963} - 504{,}5 \cdot K_L^{0{,}055} \cdot K_P^{0{,}832}\right)^{-1}$$

Heat dissipation via the bearing seating surfaces $\dot{Q}_S$, Figure 60, Page 602:

Equation 67

$$\dot{Q}_S = k_q \cdot A_S \cdot \Delta\vartheta_A$$

Heat dissipation via the lubricant $\dot{Q}_L$:

Equation 68

$$\dot{Q}_L = 0{,}028\,6 \frac{\text{kW}}{\text{l/min} \cdot \text{K}} \cdot \dot{V}_L \cdot \Delta\vartheta_L$$

Total dissipated heat flow $\dot{Q}$:

Equation 69

$$\dot{Q} = \dot{Q}_S + \dot{Q}_L + \dot{Q}_E$$

Lubricant parameter K_L:

Equation 70

$$K_L = 10^{-6} \cdot \frac{\pi}{30} \cdot n_{\vartheta r} \cdot \frac{10^{-7} \cdot f_0 \cdot \left(\nu \cdot n_{\vartheta r}\right)^{\frac{2}{3}} \cdot d_M^{\,3}}{\dot{Q}}$$

Load parameter K_P:

Equation 71

$$K_P = 10^{-6} \cdot \frac{\pi}{30} \cdot n_{\vartheta r} \cdot \frac{f_1 \cdot P_1 \cdot d_M}{\dot{Q}}$$

The following diagrams show the speed ratio and the heat transition coefficient.

Figure 59
Speed ratio

f_n = speed ratio
K_L = lubricant parameter
K_P = load parameter

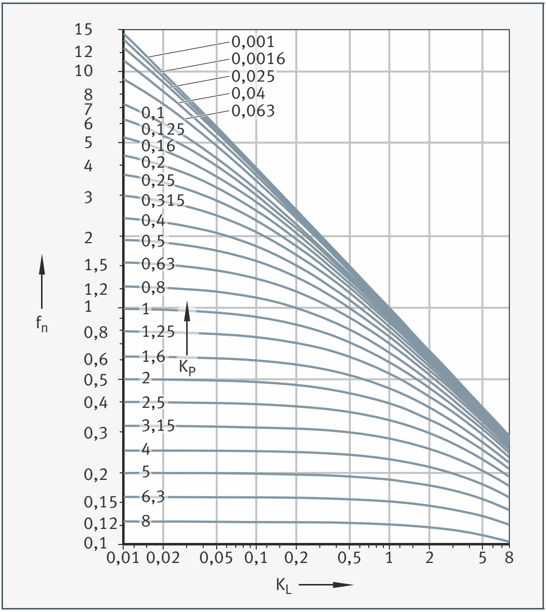
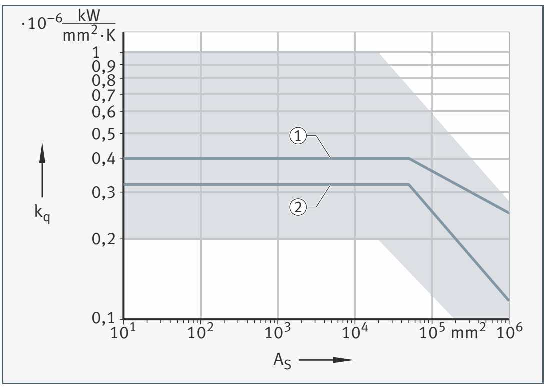


Figure 60
Heat transition coefficient, as a function of the bearing seating surface

k_q = heat transition coefficient
A_S = heat-dissipating bearing seating surface

① Reference condition for radial bearings
② Reference condition for axial bearings



Symbols, units and definitions

The following values are used in calculation of the thermally safe operating speed n_ϑ:

Legend

A_S — mm^2
Heat-dissipating bearing seating surface: In general, $A_S = A_r$

A_r — mm^2
Heat-dissipating bearing seating surface under reference conditions:
radial bearings: $A_r = \pi \cdot B \cdot (D + d)$
axial bearings: $A_r = (\pi/2) \cdot (D^2 - d^2)$
tapered roller bearings: $A_r = \pi \cdot T \cdot (D + d)$
axial spherical roller bearings:
$A_r = (\pi/4) \cdot (D^2 + d_1{}^2 - D_1{}^2 - d^2)$

B — mm
Bearing width

d — mm
Bearing bore diameter

D — mm
Bearing outside diameter

d_1 — mm
Outside diameter of shaft locating washer

D_1 — mm
Inside diameter of housing locating washer

d_M — mm
Mean bearing diameter $(D + d)/2$

f_0 — –
Bearing factor for frictional torque as a function of speed, see section Friction and increases in temperature, Page 588

f_1 — –
Bearing factor for frictional torque as a function of load, see section Friction and increases in temperature, Page 588

f_n — –
Speed ratio, Figure 59, Page 602

K_L — –
Lubricant parameter

K_P — –
Load parameter

n_ϑ — min^{-1}
Thermally safe operating speed

$n_{\vartheta r}$ — min^{-1}
Thermal speed rating
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

k_q — 10^{-6} $kW/(mm^2 \cdot K)$
Heat transition coefficient of bearing seating surface, Figure 60, Page 602. This is dependent on the housing design and size, the housing material and the mounting situation. For normal applications, the heat transition coefficient for bearing seating surfaces up to 25,000 mm^2 is between 0,2 and $1{,}0 \cdot 10^{-6}$ $kW/(mm^2 \cdot K)$

P_1 — N
Radial load for radial bearings, axial load for axial bearings

q_r — W/mm^2
Heat flow density

$\dot{Q}$ — kW
Total dissipated heat flow

$\dot{Q}_E$ — kW
Heat flow due to heating by external source

$\dot{Q}_L$ — kW
Heat flow dissipated by the lubricant

$\dot{Q}_S$ — kW
Heat flow dissipated via the bearing seating surfaces

T — mm
Total width of tapered roller bearing

$\dot{V}_L$ — l/min
Oil flow

$\Delta\vartheta_A$ — K
Differential between mean bearing temperature and ambient temperature

$\Delta\vartheta_L$ — K
Differential between oil inlet temperature and oil outlet temperature

ν — mm^2/s
Kinematic viscosity of lubricant at operating temperature.

Noise Schaeffler Noise Index

The Schaeffler Noise Index (SGI) has been developed as a new feature for comparing the noise level of different bearing types. The SGI value is based on the maximum permissible noise level of a bearing in accordance with internal standards, which is calculated on the basis of ISO 15242.
In order that different bearing types and series can be compared, the SGI value is plotted against the basic static load rating C_0, allowing the direct comparison of bearings of the same load carrying capacity. The upper limit value is given in the diagrams. This means that the average noise level of the bearings is lower than illustrated in the diagram.

Until now, the Noise Index has only been available for the main series of radial deep groove ball bearings, radial angular contact ball bearings, tapered roller bearings and cylindrical roller bearings. Additional bearing types and series will be updated and introduced in subsequent publications.

The SGI is an additional performance characteristic in the selection of bearings for noise-sensitive applications. The specific suitability of a bearing for an application in terms of installation space, load carrying capacity or speed limit, for example, must be checked independently of this.

The following section presents the diagrams for radial deep groove ball bearings and tapered roller bearings and an example assessment is shown using the radial deep groove ball bearings. Diagrams for other types of bearings can be found in the Schaeffler catalogue HR 1, Rolling Bearings, or in the electronic product catalogue **medias***:*
https://medias.schaeffler.com.

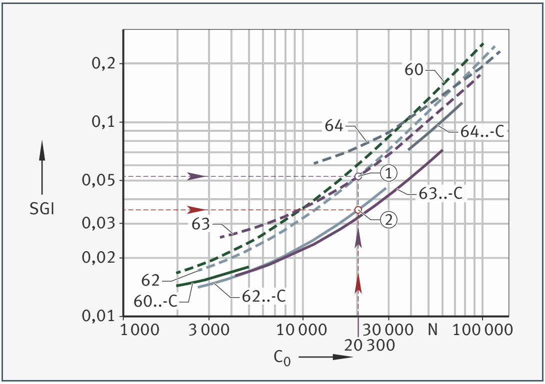


Figure 61
Example: Comparison of deep groove ball bearings using the Schaeffler Noise Index

SGI = Schaeffler Noise Index
C_0 = basic static load rating
① = standard series 62
② = series 62..-C (Generation C)

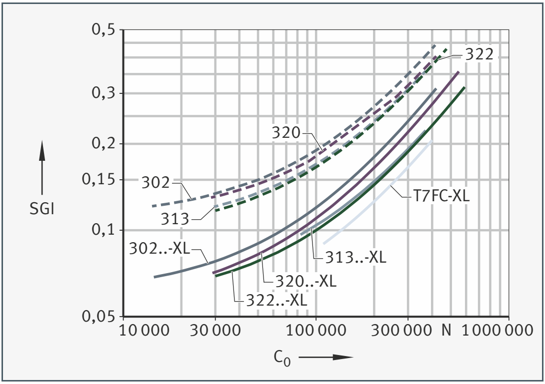


Figure 62
Schaeffler Noise Index for tapered roller bearings

SGI = Schaeffler Noise Index
C_0 = basic static load rating

Example of Noise Index calculation

If the requisite basic load rating is known for an application, the bearing arrangement can also be designed using the Noise Index as an additional performance characteristic.

If the requisite basic static load rating is C_0 = 20 300 N, for example, various ball bearings are available with a different SGI value, see Figure 61, Page 605. As a result, the calculation can be carried out for the bearing application using the smallest SGI value. In particular, bearings of Generation C offer an advantage in this respect.

Lubrication

Lubrication and maintenance are important for the reliable operation and long operating life of rolling bearings.

Functions of the lubricant

The lubricant should:

- form a lubricant film sufficiently capable of supporting loads on the contact surfaces and thus prevent wear and premature fatigue.
- dissipate heat in the case of oil lubrication.
- provide additional sealing for the bearing against external solid and fluid contaminants in the case of grease lubrication.
- give damping of running noise and protect the bearing against corrosion.

Selection of the type of lubrication

It should be determined as early as possible in the design process whether bearings should be lubricated using grease or oil. The following factors are decisive in determining the type of lubrication and quantity of lubricant: the operating conditions, the type and size of the bearing, the adjacent construction and the type of lubricant feed.

Characteristics of grease lubrication

In the case of grease lubrication, the following advantages and disadvantages must be considered:

- very little design work required
- the sealing action and reservoir action
- long operating life with little maintenance work (lifetime lubrication possible in certain circumstances)
- in the case of relubrication, the provision of collection areas for old grease and feed ducts
- no heat dissipation by the lubricant
- no rinsing out of wear debris and other particles

Characteristics of oil lubrication

In the case of oil lubrication, the following criteria must be considered:

- good lubricant distribution and supply to contact areas
- the possibility for dissipation of heat from the bearing (significant principally at high speeds and loads)
- rinsing out of wear debris
- very low friction losses with minimal quantity lubrication
- more demanding requirements in terms of feed and sealing

Grease lubrication For grease lubrication, rolling bearing greases K in accordance with DIN 51825 are suitable.

Greases should be selected in accordance with the operating conditions of the bearing: temperature, pressure capacity, speed, water and moisture.

Pressure capacity
The viscosity at operating temperature must be sufficiently high for the formation of a lubricant film capable of supporting loads. At high loads, greases with EP (extreme pressure) characteristics and high base oil viscosity should be used (KP grease to DIN 51825). Such greases should also be used for bearings with a substantial sliding component and with line contact.

Silicone greases can only be used at low loads ($P \leqq 0{,}03 \cdot C$).

Greases with solid lubricants should preferably be used for applications with mixed or boundary friction conditions. The solid lubricant particle size should not exceed 5 μm.

Speed
Greases should be selected in accordance with the speed parameter $n \cdot d_M$ for grease, see table Lubricating greases, Page 608:
- For rolling bearings running at high speeds or with a low starting torque, greases with a high speed parameter should be selected.
- For bearings running at low speeds, greases with a low speed parameter should be used.

Under centrifugal accelerations $> 500 \cdot g$, separation (of the thickener and base oil) may occur. In this case, please consult the lubricant manufacturer.

The consistency of polycarbamide greases can be altered by shear stresses to a greater extent than that of metal soap greases.

Water and moisture
Water in the grease has a highly detrimental effect on the operating life of bearings:
- The static behaviour of greases in the presence of water is assessed in accordance with DIN 51807.
- The anti-corrosion characteristics can be tested in accordance with DIN 51802 (Emcor test) (information in the datasheets of grease manufacturers).

Lubricating greases The following greases have proved particularly suitable:

Designation[1]	Classification	Type of grease
GA01	Ball bearing grease for $\vartheta < +180$ °C	Polycarbamide Ester oil
GA02	Ball bearing grease for $\vartheta < +160$ °C	Polycarbamide SHC
GA13	Standard ball bearing and insert bearing grease for $D > 62$ mm	Lithium soap Mineral oil
GA14	Low noise ball bearing grease for $D \leqq 62$ mm	Lithium soap Mineral oil
GA15	Low noise ball bearing grease for high speeds	Lithium soap Ester oil/SHC
GA22	Free-running grease with low frictional torque	Lithium soap Ester oil, mineral oil
L069	Insert bearing grease for wide temperature range	Polycarbamide Ester oil
GA08	Grease for line contact	Lithium complex soap Mineral oil
GA26	Standard grease for drawn cup roller clutches	Calcium/lithium soap Mineral oil
GA28	Screw drive bearing grease	Lithium soap Synthetic oil/mineral oil
GA11	Rolling bearing grease resistant to media for temperatures up to +250 °C	PTFE Alkoxyfluoroether
GA47	Rolling bearing grease resistant to media for temperatures up to +140 °C	Barium complex soap Mineral oil

[1] GA.. stands for **G**rease **A**pplication Group.., based on Grease Spec 00.
[2] The upper continuous limit temperature $\vartheta_{upperlimit}$ must not be exceeded if a reduction in the grease operating life due to temperature is to be avoided.
[3] Dependent on bearing type.

Operating temperature range °C	Upper continuous limit temperature $\vartheta_{upperlimit}$ [2)] °C	NLGI class	Speed parameter $n \cdot d_M$ $min^{-1} \cdot mm$	ISO VG class (base oil)[3)]	Designation[1)]	Recommended Arcanol grease for relubrication
−30 to +180	+125	2 to 3	600 000	68 to 220	**GA01**	–
−40 to +160	+90	2 to 3	500 000	68 to 220	**GA02**	–
−20 to +120	+75	3	500 000	68 to 150	**GA13**	**MULTI3**
−30 to +120	+75	2	500 000	68 to 150	**GA14**	**MULTI2**
−40 to +120	+75	2 to 3	1 000 000	22 to 32	**GA15**	–
−50 to +120	+70	2	1 500 000	10 to 22	**GA22**	–
−40 to +180	+120	2	700 000	68 to 220	**L069**	–
−20 to +140	+95	2 to 3	500 000	150 to 320	**GA08**	**LOAD150**
−20 to +80	+60	2	500 000	10 to 22	**GA26**	–
−30 to +140	+80	2	800 000	15 to 100	**GA28**	**MULTITOP**
−30 to +260	+200	2	300 000	460 to 680	**GA11**	**TEMP200**
−20 to +130	+70	1 to 2	350 000	150 to 320	**GA47**	–

Oil lubrication

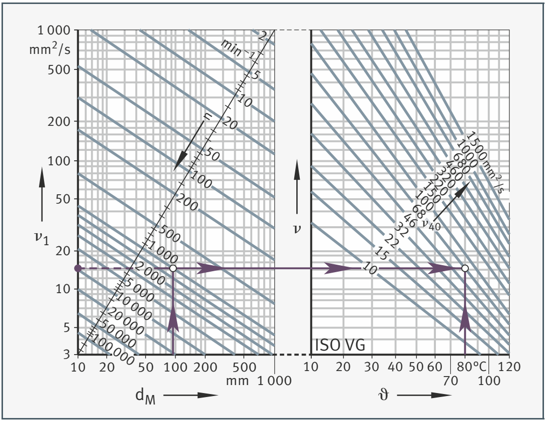
For the lubrication of rolling bearings, mineral oils and synthetic oils are essentially suitable. Oils with a mineral oil base are used most frequently. They must, as a minimum, fulfil the requirements in accordance with DIN 51517 or DIN 51524.

Reference viscosity for mineral oils

The guide value for ν_1 is dependent on the mean bearing diameter d_M and the speed n. It takes account of the EHD theory of lubricant film formation and practical experience. Depending on the operating speed, the oil at operating temperature must as a minimum have the reference viscosity ν_1, see Figure 63.

Figure 63
Reference viscosity and ν/ϑ diagram for mineral oils

n = operating speed
ν_1 = reference viscosity
ν = operating viscosity
d_M = mean bearing diameter (d + D)/2
ϑ = operating temperature
ν_{40} = viscosity at +40 °C



Calculation of reference viscosity

The reference viscosity ν_1 is determined as follows:

- Allocate ν_1 to an ISO VG nominal viscosity between 10 and 1500 (mid-point viscosity in accordance with ISO 3448).
- Round intermediate values to the nearest ISO VG (due to the steps between groups).

This method cannot be used for synthetic oils, since these have different V/P (viscosity/pressure) and V/T (viscosity/temperature) characteristics.

Influence of temperature on viscosity

As the temperature increases, the viscosity of the oil decreases. This temperature-dependent change in the viscosity is described using the viscosity index VI. In the case of mineral oils, the viscosity index should be at least 95.

When selecting the viscosity, the lower operating temperature must be taken into consideration, since the increasing viscosity will reduce the flowability of the lubricant. As a result, the level of power losses may increase.

A very long life can be achieved with a viscosity ratio $\kappa = \nu/\nu_1 = 3$ to 4. Highly viscous oils do not, however, bring only advantages. In addition to the power losses arising from lubricant friction, there may be problems with the feed and removal of oil at low or even at normal temperatures.

The oil selected must be sufficiently viscous that it gives the highest possible fatigue life. At the same time, it must be ensured that the bearings are always supplied with adequate quantities of oil.

Pressure capacity and anti-wear additives

If the bearings are subjected to high loads or the operating viscosity ν is less than the reference viscosity ν_1, oils with anti-wear additives (type P in accordance with DIN 51502) should be used. Such oils are also necessary for rolling bearings with a substantial proportion of sliding contact (for example, bearings with line contact).

These additives form boundary layers to reduce the harmful effects of metallic contact occurring at various points (wear). The suitability of these additives varies and is normally heavily dependent on temperature. Their effectiveness can only be assessed by means of testing in the rolling bearing (for example, on a test rig FE8 in accordance with DIN 51819).

Silicone oils should only be used for low loads ($P \leqq 0{,}03 \cdot C$).

Lubrication methods

The essential lubrication methods are:

- drip feed oil lubrication
- pneumatic oil lubrication (also used, in order to protect the environment, as a substitute for oil mist lubrication)
- oil bath lubrication (immersion or sump lubrication)
- recirculating oil lubrication

Drip feed oil lubrication

This is suitable for bearings running at high speeds. The oil quantity required is dependent on the type and size of bearing, the operating speed and the load. The guide value is between 3 drops/min and 50 drops/min for each rolling element raceway (one drop weighs approx. 0,025 g). Excess oil must be allowed to flow out of the bearing arrangement.

Pneumatic oil lubrication

This method is particularly suitable for radial bearings running at high speeds and under low loads ($n \cdot d_M$ = 800 000 to 3 000 000 $min^{-1} \cdot mm$). Oil is fed to the bearing by means of clean, compressed air that is free from water. As a result, an excess pressure is generated. This prevents contaminants from entering the bearing.

With a pneumatic oil lubrication system designed for minimal quantity lubrication, low frictional torque and a low operating temperature can be achieved.

Oil bath lubrication

The oil level should reach the centre line of the lowest rolling element. If the oil level is higher than this, the bearing temperature may increase at high circumferential velocities (with losses due to splashing). Furthermore, foaming of the oil may occur.

The speed capacity is generally up to $n \cdot d_M = 300\,000\ min^{-1} \cdot mm$. At $n \cdot d_M < 150\,000\ min^{-1} \cdot mm$, the bearing may be completely immersed.

In bearings with an asymmetrical cross-section (such as angular contact ball bearings), oil return ducts must be provided due to the pumping effect so that recirculation can be achieved.

In axial bearings, the oil level must cover the inside diameter of the axial cage.

The oil quantity in the housing must be adequately proportioned, otherwise very short oil change intervals will be necessary.

Recirculating oil lubrication

During recirculating oil lubrication, the oil is cooled. The oil can therefore dissipate heat from the bearing. The quantity of oil required for heat dissipation is dependent on the cooling conditions, see section Speeds, Page 599.

For bearings with an asymmetrical cross-section (such as angular contact ball bearings, tapered roller bearings, axial spherical roller bearings), larger throughput quantities are permissible due to the pumping effect than for bearings with a symmetrical cross-section. Large quantities can be used to dissipate wear debris or heat.

Design of adjacent construction for oil lubrication
The lubrication holes in the housing and shaft must align with those in the rolling bearings.
Adequate cross-sections must be provided for annular slots, pockets, etc.
The oil must be able to flow out without pressure (this prevents oil build-up and additional heating of the oil).
In axial bearings, the oil must always be fed from the inside to the outside.

Oil injection lubrication
In bearings running at high speeds, the oil is injected into the gap between the cage and bearing ring. Injection lubrication using large recirculation quantities is associated with high power loss.

Heating of the bearings can only be held within limits with a considerable amount of effort. The appropriate upper limit for the speed parameter $n \cdot d_M = 1\,000\,000 \text{ min}^{-1} \cdot \text{mm}$ can, in the case of suitable bearings (for example, spindle bearings), be exceeded to a considerable degree when using injection lubrication.

Heat dissipation via the lubricant
Oil can dissipate frictional heat from the bearing. It is possible to calculate the heat flow $\dot{Q}_L$ that is dissipated by the lubricant and the necessary lubricant volume flow $\dot{V}_L$, see Schaeffler catalogue HR 1, Rolling Bearings.

Oil change

At temperatures in the bearing of less than +50 °C and with only slight contamination, an oil change once per year is generally sufficient.
The precise oil change intervals should be agreed in consultation with the oil manufacturer.

Severe operating conditions
Under more severe conditions, the oil should be changed more frequently. This applies, for example, in the case of higher temperatures and low oil quantities with a high circulation index.

The circulation index indicates how often the entire oil volume available is circulated or pumped around the system per hour, see Schaeffler catalogue HR 1, Rolling Bearings.

Equation 72

$$\text{Circulation index} = \frac{\text{Pump displacement m}^3/\text{h}}{\text{Container volume m}^3}$$

Bearing data

The difference between internal clearance and operating clearance is explained below. In addition, bearing parts such as bearing materials, cages, guides, etc. are described.

Radial internal clearance

The radial internal clearance applies to bearings with an inner ring and is determined on the unmounted bearing. It is defined as the amount by which the inner ring can be moved relative to the outer ring in a radial direction from one extreme position to the other, see Figure 65, Page 615.

The following table gives the radial internal clearance groups in accordance with DIN 620-4 and ISO 5753-1 respectively.

Internal clearance group in accordance with DIN 620-4	Internal clearance group in accordance with ISO 5753-1	Description	Area of application
C2	Group 2	Internal clearance $<$ CN	For heavy alternating loads combined with swivel motion
CN	Group N	Normal internal clearance, CN is not normally included in bearing designations	For normal operating conditions with shaft and housing tolerances, see section Operating clearance, Page 615, and Design of bearing arrangements, Page 625
C3	Group 3	Internal clearance $>$ CN	For bearing rings with press fits and large temperature differential between the inner and outer ring
C4	Group 4	Internal clearance $>$ C3	
C5	Group 5	Internal clearance $>$ C4	

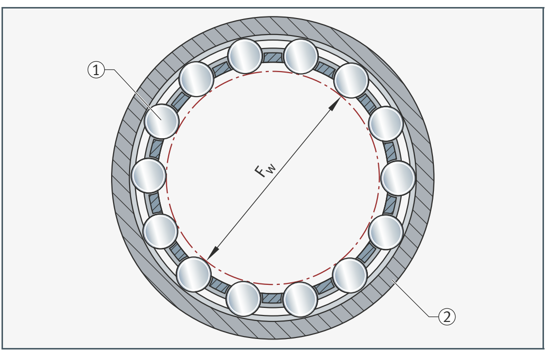
Enveloping circle

For bearings without an inner ring, the enveloping circle F_W is used. This is the inner inscribed circle of the rolling elements in clearance-free contact with the outer raceway, see Figure 64.

Figure 64
Enveloping circle

F_W = enveloping circle diameter

1 Rolling element
2 Outer raceway



Operating clearance

The operating clearance is determined on a mounted bearing still warm from operation. It is defined as the amount by which the shaft can be moved in a radial direction from one extreme position to the other.
The operating clearance is derived from the radial internal clearance and the change in the radial internal clearance as a result of interference fit and thermal influences in the mounted condition.

Operating clearance value

The operating clearance value is dependent on the operating and mounting conditions of the bearing, see also section Design of bearing arrangements, Page 625.

A larger operating clearance is, for example, necessary if heat is transferred via the shaft, the shaft undergoes deflection or if misalignment occurs.
An operating clearance smaller than CN should only be used in special cases, for example in high precision bearing arrangements.
The normal operating clearance is achieved with an internal clearance of CN or, in the case of larger bearings, predominantly with C3 if the recommended shaft and housing tolerances are maintained, see section Design of bearing arrangements, Page 625.

The operating clearance can be calculated as follows:

Equation 73

$$s = s_r - \Delta s_P - \Delta s_T$$

Legend

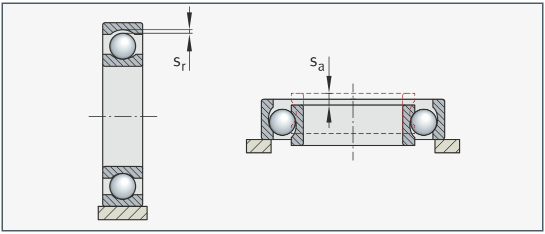
s	μm	Δs_P	μm
Radial operating clearance of mounted bearing warm from operation		Reduction in radial internal clearance due to fit	
s_r	μm	Δs_T	μm
Radial internal clearance		Reduction in radial internal clearance due to temperature.	

Axial internal clearance

The axial internal clearance s_a is defined as the amount by which one bearing ring can be moved relative to the other, without load, along the bearing axis, see Figure 65.

Figure 65
Axial and radial internal clearance

s_a = axial internal clearance
s_r = radial internal clearance



With various bearing types, the radial internal clearance s_r and the axial internal clearance s_a are dependent on each other. The following table shows guide values for the relationship between the radial and axial internal clearance for some bearing types.

Bearing type		Ratio between axial and radial internal clearance s_a/s_r
Self-aligning ball bearings		$2,3 \cdot Y_0$[1)]
Spherical roller bearings		$2,3 \cdot Y_0$[1)]
Tapered roller bearings	Single row, arranged in pairs	$4,6 \cdot Y_0$[1)]
	Matched pairs (N11CA)	$2,3 \cdot Y_0$[1)]
Angular contact ball bearings	Double row, series 32 and 33	1,4
	Double row, series 32..-B and 33..-B	2
Four point contact bearings		1,4

[1)] Y_0 = axial load factor in accordance with product table.

Bearing materials

Schaeffler rolling bearings fulfil the requirements for fatigue strength, wear resistance, hardness, toughness and structural stability.

The material used for the rings and rolling elements is generally a low-alloy, through hardening chromium steel of high purity. For bearings subjected to considerable shock loads and reversed bending stresses, case hardening steel is also used.

In recent years, the improved quality of rolling bearing steels has been the principal factor in achieving considerable increases in basic load ratings.

The results of research as well as practical experience confirm that bearings made from the steel currently used as standard can achieve their endurance limit if loads are not excessively high and the lubrication and cleanliness conditions are favourable.

High Nitrogen Steel

Through the use of special bearings made from HNS (High Nitrogen Steel), it is possible to achieve adequate service life even under the most challenging conditions (high temperatures, moisture, contamination).

High performance steels Cronidur and Cronitect

For increased performance requirements, highly corrosion-resistant, nitrogen-alloyed martensitic HNS steels are available such as Cronidur and Cronitect.

In contrast to Cronidur, the more economical alternative Cronitect has nitrogen introduced into the structure by means of a surface layer hardening process.

Both steels are clearly superior to conventional corrosion-resistant steels for rolling bearings in terms of corrosion resistance and fatigue strength.

Ceramic materials

Ceramic hybrid spindle bearings contain balls made from silicon nitride. These ceramic balls are significantly lighter than steel balls. The centrifugal forces and friction are considerably lower.

Hybrid bearings allow very high speeds, even with grease lubrication, as well as long operating life and low operating temperatures.

Materials for bearing components

The following table shows suitable materials and their applications in bearing technology.

Material	Bearing components (example)
Through hardening steel – rolling bearing steel in accordance with ISO 683-17	Outer and inner ring, axial washer, balls, rollers
HNS – High Nitrogen Steel	Outer and inner ring
Corrosion-resistant steel – rolling bearing steel in accordance with ISO 683-17	Outer and inner ring
Case hardening steel	For example, outer ring of yoke type track rollers
Flame or induction hardening steel	Roller stud of stud type track rollers
Steel strip in accordance with EN 10139, SAE J403	Outer ring for drawn cup needle roller bearings
Silicon nitride	Ceramic balls
Brass alloy	Cage
Aluminium alloy	Cage
Polyamide (thermoplastic)	Cage
NBR, FPM, TPU	Sealing ring

Cages The most important functions of the cage are:

- to separate the rolling elements from each other, in order to minimise friction and heat generation.
- to maintain the rolling elements at the same distance from each other, in order to ensure uniform load distribution.
- to prevent the rolling elements from falling out in bearings that can be dismantled or swivelled out.
- to guide the rolling elements in the unloaded zone of the bearing.

Rolling bearing cages are subdivided into sheet metal and solid section cages.

Sheet metal cages These cages are predominantly made from steel and for some bearings from brass, see Figure 66, Page 619. In comparison with solid section cages made from metal, sheet metal cages are of lower mass.

Since a sheet metal cage only fills a small proportion of the gap between the inner and outer ring, lubricant can easily reach the interior of the bearing and is held on the cage.

In general, a sheet steel cage is only included in the bearing designation if it is not defined as a standard version of the bearing.

Solid section cages These cages are made from metal, laminated fabric or plastic, see Figure 67 and Figure 68, Page 619. They can be identified from the bearing designation.

Solid section cages made from metal or laminated fabric
Solid section cages made from metal are used where there are requirements for high cage strength and at high temperatures.

Solid section cages are also used if the cage must be guided on ribs. Rib-guided cages for bearings running at high speeds are made in many cases from light materials, such as light metal or laminated fabric, in order to achieve low inertia forces.

Solid section cages made from polyamide PA66
Solid section cages made from polyamide PA66 are produced using injection moulding, see Figure 68, Page 619. As a result, cage types can generally be realised that allow designs with particularly high load carrying capacity. The elasticity and low mass of polyamide are favourable under shock type bearing loads, high accelerations and decelerations and tilting of the bearing rings in relation to each other. Polyamide cages have very good sliding and emergency running characteristics.

Cages made from glass fibre reinforced polyamide PA66 are suitable for long term temperatures up to +120 °C. For higher operating temperatures, plastics such as PA46 or PEEK can be used.

When using oil lubrication, additives in the oil can impair the cage operating life. Aged oil can also impair the cage operating life at high temperatures, so attention must be paid to compliance with the oil change intervals.

Cage designs

The following images show some typical cage designs.

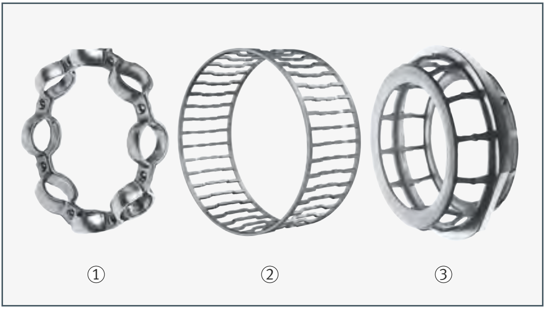


Figure 66
Sheet steel cages

① Riveted cage for deep groove ball bearings
② Window cage for needle roller bearings
③ Window cage for spherical roller bearings

Figure 67
Solid brass cages

① Riveted solid section cage for deep groove ball bearings
② Window cage for angular contact ball bearings
③ Riveted cage with crosspiece rivets for cylindrical roller bearings

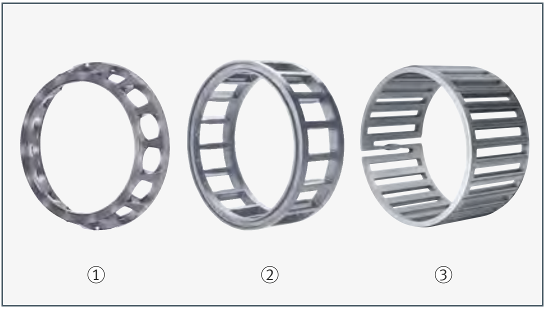


Figure 68
Solid section cages made from glass fibre reinforced polyamide

① Window cage for single row angular contact ball bearings
② Window cage for cylindrical roller bearings
③ Window cage for needle roller bearings

ISO dimension plans for bearing types

German and international standards (DIN, ISO) contain the external dimensions of common rolling bearings defined in the form of dimension plans. The principal basis for the formulation of dimension plans is several series of rolling bearings that were manufactured to coincident external dimensions as early as the start of the 20th Century. These series have been systematically expanded and additional series have been added. In order to prevent dimensions from multiplying and spreading in an uncontrolled manner, attention was paid not simply to the range to be considered at present but also to defining the dimensions of bearings that may be designed and manufactured in future.

ISO describes the dimension plans for the various types in separate documents as follows:

- radial bearings (excluding single row needle roller bearings, radial insert ball bearings and tapered roller bearings) in ISO 15
- tapered roller bearings in ISO 355
- axial bearings in ISO 104

DIN 616 describes dimension plans for radial and axial bearings. An overview of ISO and DIN rolling bearing standards is given in DIN 611:2010.

Advantages of dimension plans

The dimension plans are valid for different bearing types. Standard rolling bearings of different types can be manufactured to the same external dimensions. As a result, a designer working on the same design envelope can make a selection between bearings of several types with the same external dimensions.

In the dimension plans, one bearing bore is allocated several outside diameters and width dimensions, see Figure 71, Page 622. In this way, it is possible to design several bearings of the same type that, for the same bore, exhibit different load carrying capacities. The development of new bearing series and individual new rolling bearings in accordance with the dimension plans has considerable advantages for users and manufacturers. The dimension plans should always be used as a basis for all future developments.

Width and diameter series

Width and diameter series are described using numbers. In the case of radial bearings in accordance with DIN 616 and ISO 15, these are as follows:

- for identification of the width series, the numbers 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, see Figure 69, Page 621
- for identification of the diameter series, the numbers 7, 8, 9, 0, 1, 2, 3, 4, 5, see Figure 70, Page 621

The dimension series, part series and diameter series are included in numerous standardised rolling bearing designations.

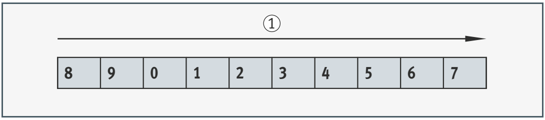


Figure 69
Identifying the width series

① Width series

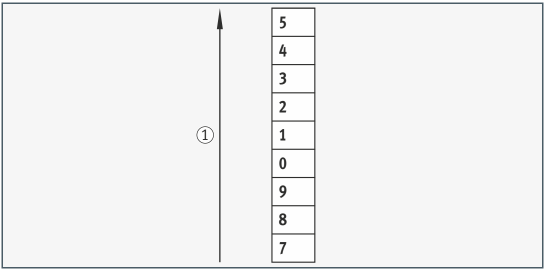


Figure 70
Identifying the diameter series

① Diameter series

Dimension series
The specific number of the width and diameter series, when combined, identifies the dimension series, see following table. When this table is used, for example, for a radial bearing of the width series 2 and the diameter series 3, the result is the dimension series 23, see Figure 71, Page 622. If the bearing bore code is then added, the bearing size is completely defined.

Width series – increase in cross-sectional width										
Diameter series – increase in cross-sectional height	8	9	0	1	2	3	4	5	6	7
5	–	–	–	–	–	–	–	–	–	–
4	–	–	04	–	24	–	–	–	–	–
3	83	–	03	12	23	33	–	–	–	–
2	82	–	02	12	22	32	42	52	62	–
1	–	–	01	11	21	31	41	51	61	–
0	–	–	00	10	20	30	40	50	60	–
9	–	–	09	19	29	39	49	59	69	–
8	–	–	08	18	28	38	48	58	68	–
7	–	–	–	17	27	37	47	–	–	–

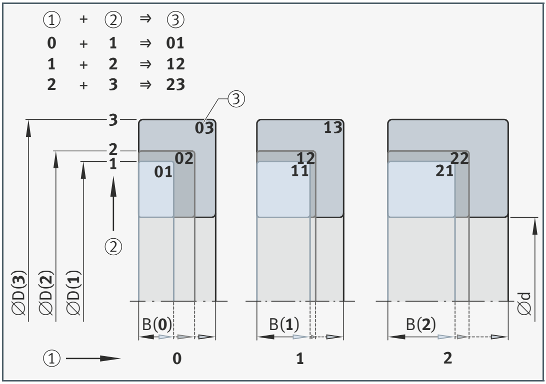


Figure 71
Generation of the dimension series

① Width series
② Diameter series
③ Dimension series

Specification of the bearing bore

For certain bearing types, the bearing bores are stated directly or in an encoded form in accordance with DIN 623-1. Up to $d < 10$ mm, the bearing bore diameter is specified in the dimension-specific part of the designation (basic designation) directly as a number indicating the diameter.
Example: Deep groove ball bearing 623, bore diameter = 3 mm.

Bore code
For nominal dimensions $d \geqq 10$ mm to $d < 500$ mm, the diameter is described by means of a bore code.

For bores from 10 mm to 17 mm, this is as follows:
d = 10 mm, bore code 00

d = 12 mm, bore code 01

d = 15 mm, bore code 02

d = 17 mm, bore code 03.

For all rolling bearings in the range from d = 20 mm to d = 480 mm (with the exception of double direction axial bearings), the bore code is generated by dividing the bearing bore dimension by 5.
Example: Bearing bore d = 360 mm divided by 5 (360 : 5), bore code = 72.

At or above $d > 480$ mm, the unencoded bore diameter is indicated with an oblique after the bearing series, for example 618/500 with bore diameter d = 500 mm. The intermediate sizes such as bore diameter d = 22, 28 and d = 32 mm are also indicated with an oblique as /22, /28 and /32.

In the case of magneto bearings, the unencoded nominal bore dimension is given.

Bearing designations

Each rolling bearing has a designation that clearly indicates the type, dimensions, tolerances and internal clearance, if necessary with other important features. Bearings that have the same standardised designation are interchangeable with each other. In the case of separable bearings, it cannot always be ensured that individual parts with the same origin can be interchanged with each other.

In Germany, the bearing designations are standardised in DIN 623-1. These designations are also used in many other countries.

Rolling bearing designations

The designation for the bearing series comprises numbers and letters or letters and numbers. This indicates the type of bearing, the diameter series and, in many cases, the width series as well, see Figure 72.

Basic designation, prefix and suffix

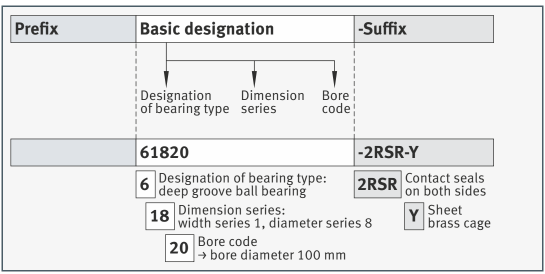
The basic designation contains the code for the bearing series and the bearing bore. Examples of basic designations: see section Bearing designations comprising the basic designations – examples, Page 624.

The prefix normally identifies individual bearing parts of complete bearings or material variants of a bearing (in certain cases, this may also be part of the basic designation).
The suffix defines special designs and features.

The prefix and suffix describe other features of the bearing but are not standardised in all cases and may vary in use depending on the manufacturer.

Figure 72 shows an example of designations on the basis of their definition.

Figure 72
Example of bearing identification data for a deep groove ball bearing



Bearing designations comprising the basic designations – examples

6203
62 = bearing series 62, deep groove ball bearing, width series 0 (not included in basic designation), diameter series 2
03 = bore code, bore d = 17 mm

2201
22 = bearing series 22, self-aligning ball bearing, width series 2, diameter series 2
01 = bore code, bore d = 12 mm

239/800
239 = bearing series 239, spherical roller bearing, width series 3, diameter series 9
800 = bore code, bore d = 800 mm

3315
33 = bearing series 33, angular contact ball bearing, double row, width series 3, diameter series 3
15 = bore code, bore d = 75 mm ($15 \cdot 5$)

NU2314
NU23 = bearing series NU23, cylindrical roller bearing with two ribs on outer ring, width series 2, diameter series 3
14 = bore code, bore d = 70 mm ($14 \cdot 5$)

51268
512 = bearing series 512, axial deep groove ball bearing, height series 1, diameter series 2
68 = bore code, bore d = 340 mm ($68 \cdot 5$).

Bearing designations that comprise only the basic designation and do not include prefixes or suffixes, identify normal bearings with normal dimensional, geometrical and running accuracy as well as with normal radial internal clearance.

Deviations from the normal design are indicated in the prefixes and suffixes.

Design of bearing arrangements

The guidance and support of a rotating machine part generally requires at least two bearings arranged at a certain distance from each other (exceptions: four point contact, crossed roller and slewing bearings). Depending on the application, a decision is made between a locating/non-locating bearing arrangement, an adjusted bearing arrangement and a floating bearing arrangement.

Locating/non-locating bearing arrangement

On a shaft supported by two radial bearings, the distances between the bearing seats on the shaft and in the housing frequently do not coincide as a result of manufacturing tolerances. The distances may also change as a result of temperature increases during operation. These differences in distance are compensated in the non-locating bearing. Examples of locating/non-locating bearing arrangements, see Figure 73, Page 626 to Figure 76, Page 627.

Non-locating bearings

Ideal non-locating bearings are cylindrical roller bearings with cage N and NU or needle roller bearings, see Figure 73 ②, ④, Page 626. In these bearings, the roller and cage assembly can be displaced on the raceway of the bearing ring without ribs.

All other bearing types, for example deep groove ball bearings and spherical roller bearings, can only act as non-locating bearings if one bearing ring has a fit that allows displacement, see Figure 74, Page 627. The bearing ring subjected to point load therefore has a loose fit; this is normally the outer ring, see table Conditions of rotation, Page 634.

Locating bearings

The locating bearing guides the shaft in an axial direction and supports external axial forces. In order to prevent axial stresses, shafts with more than two bearings have only one locating bearing. The type of bearing selected as a locating bearing depends on the magnitude of the axial forces and the accuracy with which the shafts must be axially guided.

A double row angular contact ball bearing, see Figure 75 ①, Page 627, will, for example, give closer axial guidance than a deep groove ball bearing or a spherical roller bearing. A pair of symmetrically arranged angular contact ball bearings or tapered roller bearings, see Figure 76, Page 627, used as a locating bearing will provide extremely close axial guidance.

There are particular advantages in using angular contact ball bearings of the universal design, see Figure 77, Page 628. The bearings can be fitted in pairs in any O or X arrangement without shims. Angular contact ball bearings of the universal design are matched so that, in an X or O arrangement, they have a low axial internal clearance (design UA), zero clearance (UO) or slight preload (UL).

Spindle bearings of the universal design UL, see Figure 78, Page 628, have slight preload when mounted in an X or O arrangement.

In gearboxes, a four point contact bearing is sometimes mounted directly adjacent to a cylindrical roller bearing to give a locating bearing arrangement, see Figure 75 ③, Page 627. The four point contact bearing, without radial support of the outer ring, can only support axial forces. The radial force is supported by the cylindrical roller bearing.

If a lower axial force is present, a cylindrical roller bearing with cage NUP can also be used as a locating bearing, see Figure 76 ③, Page 627.

No adjustment or setting work with matched pairs of tapered roller bearings

Mounting is also made easier with a matched pair of tapered roller bearings as a locating bearing (313..-DF), see Figure 79 ②, Page 628. They are matched by the manufacturer with appropriate axial internal clearance such that no adjustment or setting work is required.

Examples of locating/non-locating bearing arrangements

The following figures show examples of locating/non-locating bearing arrangements.

Figure 73
Locating/non-locating bearing arrangements

① Locating bearing: deep groove ball bearing
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: axial angular contact ball bearing ZKLN
④ Non-locating bearing: needle roller bearing NKIS

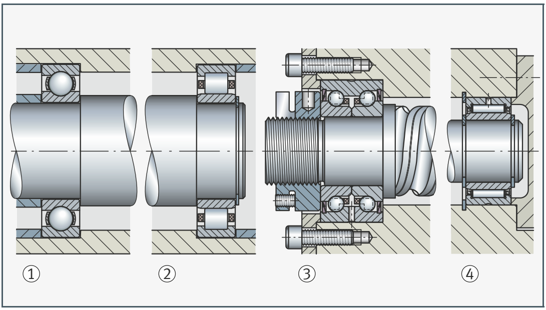


Figure 74
Locating/non-locating bearing arrangements

① Locating bearing: deep groove ball bearing
② Non-locating bearing: deep groove ball bearing
③ Locating bearing: spherical roller bearing
④ Non-locating bearing: spherical roller bearing

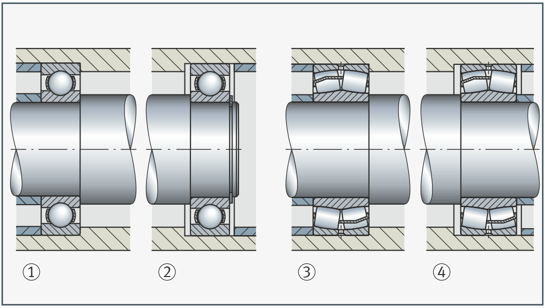


Figure 75
Locating/non-locating bearing arrangements

① Locating bearing: double row angular contact ball bearing
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: four point contact bearing and cylindrical roller bearing
④ Non-locating bearing: cylindrical roller bearing NU

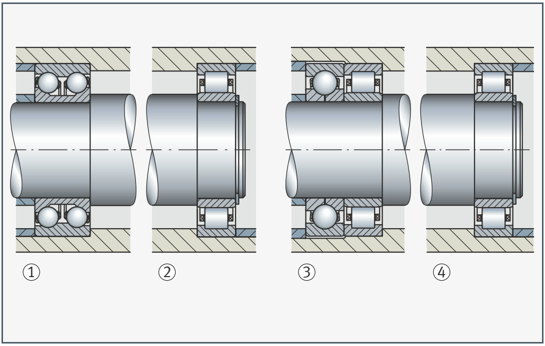


Figure 76
Locating/non-locating bearing arrangements

① Locating bearing: two tapered roller bearings
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: cylindrical roller bearing NUP
④ Non-locating bearing: cylindrical roller bearing NU

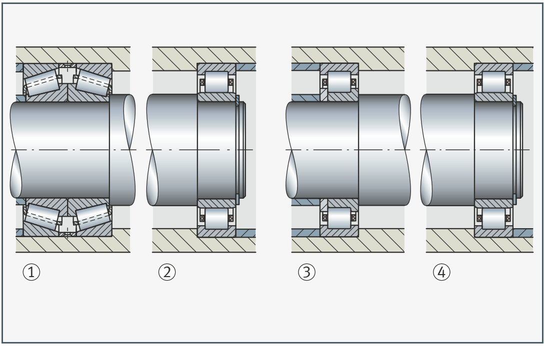


Figure 77
Locating bearing arrangements

Pair of angular contact ball bearings
of universal design

① O arrangement
② X arrangement

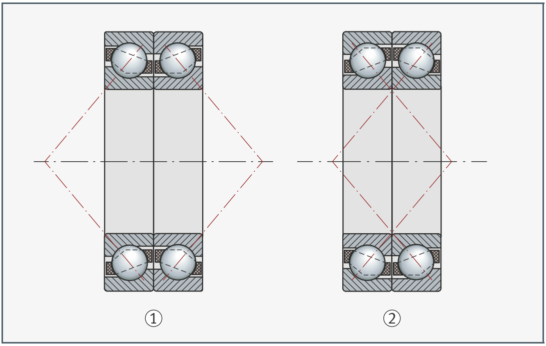


Figure 78
Locating bearing arrangements

Spindle bearings
of universal design

① O arrangement
② X arrangement
③ Tandem O arrangement

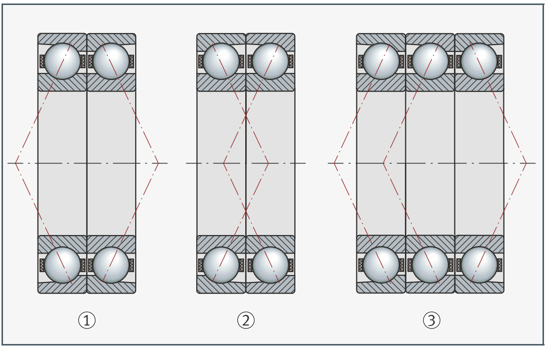
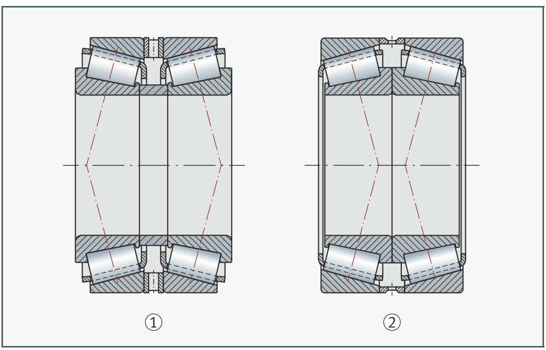


Figure 79
Locating bearing arrangements

Pair of tapered roller bearings

① O arrangement
② X arrangement



Adjusted bearing arrangement

These bearing arrangements normally consist of two symmetrically arranged angular contact bearings (angular contact ball bearings, tapered roller bearings), see Figure 80. The inner and outer rings of the bearings are displaced relative to each other until the required clearance or the required preload is achieved. This process is known as "adjustment".

Angular contact bearings and deep groove ball bearings suitable for adjusted bearing arrangements

Angular contact bearings support forces comprising a radial and an axial component. These are thus a combination of a radial and an axial bearing. Depending on the size of the nominal contact angle α, angular contact bearings are classified as radial or axial bearings. Deep groove ball bearings can also be used for an adjusted bearing arrangement; these are then angular contact ball bearings with a small nominal contact angle. Due to the possibility of regulating the clearance, adjusted bearing arrangements are particularly suitable if close guidance is necessary.

O or X arrangement

In an adjusted bearing arrangement, an O or X arrangement of the bearings is essentially possible, see Figure 80.

In the O arrangement, the cones and their apexes S formed by the contact lines point outwards; in the X arrangement, the cones point inwards.

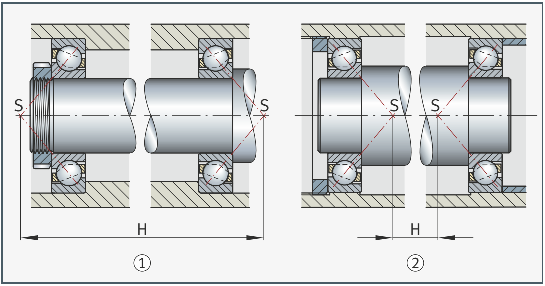
The support base H, i.e. the distance between the apexes of the contact cones, is wider in the O arrangement than in an X arrangement.

An O arrangement should be used in preference if the component with small bearing spacing must be guided with the smallest possible tilting clearance or tilting forces must be supported.

Figure 80
Adjusted bearing arrangement

S = contact cone apex
H = support distance
Angular contact ball bearings

① O arrangement
② X arrangement



Influence of thermal expansion in O and X arrangements

When deciding between an O and X arrangement, attention must also be paid to the temperature conditions and thermal expansions. This is based on the position of the roller cone apexes R. The roller cone apex R represents the intersection point of the extended inclined outer ring raceway with the bearing axis, see Figure 81, Page 630.

If the shaft is warmer than the housing ($T_W > T_G$), the shaft expands more than the housing in an axial and radial direction. As a result, the clearance set in an X arrangement decreases in every case (assuming the following precondition: shaft and housing of same material).

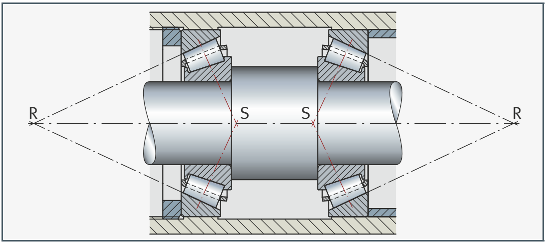


Figure 81
Adjusted bearing arrangement

S = contact cone apex
R = roller cone apex
Tapered roller bearings

X arrangement

The behaviour is different in an O arrangement. A distinction is made here between three cases:

- If the roller cone apexes R coincide at a point, the axial and radial thermal expansion cancel each other out and the clearance set is maintained, see Figure 82 ①.
- If there is a small distance between the bearings and the roller cones overlap, see Figure 82 ②, the radial expansion has a stronger effect than the axial expansion on the bearing clearance: the axial clearance is reduced. This must be taken into consideration in the adjustment of the bearings.
- If there is a large distance between the bearings and the roller cones do not overlap, see Figure 82 ③, the radial expansion has a lesser effect than the axial expansion on the bearing clearance: the axial clearance is increased.

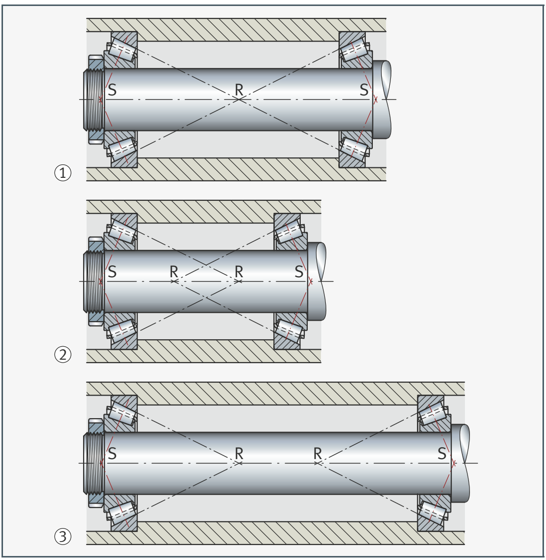


Figure 82
Tapered roller bearings, O arrangement

S = contact cone apex
R = roller cone apex

① Roller cone apexes coincide
② Roller cone apexes overlap
③ Roller cone apexes do not overlap

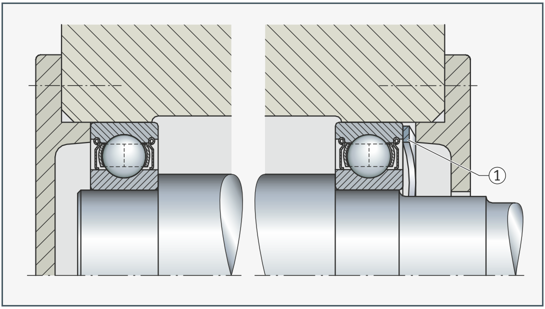
Elastic adjustment

Adjusted bearing arrangements can also be achieved by preloading using springs, see Figure 83 ①. This elastic adjustment method compensates for thermal expansion. It can also be used where bearing arrangements are at risk of vibration while stationary.

Figure 83
Adjusted bearing arrangement

Deep groove ball bearing preloaded by means of spring washer

① Spring washer



Floating bearing arrangement

The floating bearing arrangement is essentially similar in its arrangement to the adjusted bearing arrangement. While freedom from clearance or even preload is desirable when warm from operation in the latter case, floating bearing arrangements always have an axial clearance s of several tenths of a millimetre depending on the bearing size; see Figure 84, Page 632. The value s is defined as a function of the required guidance accuracy such that the bearings are not axially stressed even under unfavourable thermal conditions.

Suitable bearing types

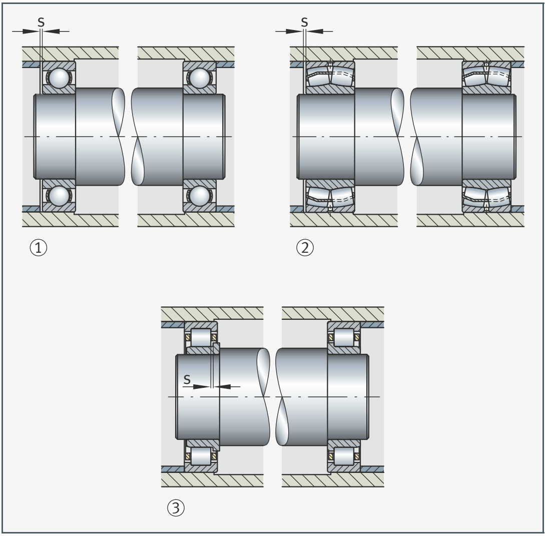
For a floating bearing arrangement, almost all bearing types that cannot be adjusted may be considered; examples: see Figure 84, Page 632. Floating arrangements are thus possible with, for example, deep groove ball bearings, self-aligning ball bearings and spherical roller bearings; one ring of each of the two bearings (usually the outer ring) then has a sliding seat. In the floating bearing arrangement with cylindrical roller bearings NJ, length compensation is possible within the bearing.

Tapered roller bearings and angular contact ball bearings are not usually suitable for a floating bearing arrangement with a large axial clearance.

Figure 84
Floating bearing arrangements

s = axial displacement (axial clearance)

① Two deep groove ball bearings
② Two spherical roller bearings
③ Two cylindrical roller bearings NJ



Fits

Rolling bearings are located on the shaft and in the housing in a radial, axial and tangential direction in accordance with their function. In a radial and tangential direction, this occurs by means of a tight fit. However, this is only possible under certain conditions in an axial direction, therefore rolling bearings are generally axially located by means of form fit.

Criteria for selection of fits

The following must be taken into consideration in determining the fit:

- The bearing rings must be well supported on their circumference in order to allow full utilisation of the load carrying capacity of the bearing.
- The rings must not creep on their mating parts, otherwise the seats will be damaged.
- The non-locating bearing must compensate changes in the length of the shaft and housing, so one ring must be axially adjustable.
- The bearings must be easy to mount and dismount.

Good support of the bearing rings on their circumference requires a tight fit. The requirement that rings must not creep on their mating parts also requires a tight fit. If non-separable bearings must be mounted and dismounted, a tight fit can only be achieved for one bearing ring.
In the case of cylindrical roller bearings N, NU and needle roller bearings, both rings can have tight fits, since the length compensation takes place within the bearing and since the rings can be fitted separately.

As a result of tight fits and a temperature differential between the inner and outer ring, the radial internal clearance of the bearing is reduced. This must be taken into consideration when selecting the radial internal clearance.

If a material other than cast iron or steel is used for the adjacent construction, the modulus of elasticity and the differing coefficients of thermal expansion of the materials must also be taken into consideration to achieve a tight fit.

For aluminium housings, thin-walled housings and hollow shafts, a closer fit should be selected if necessary in order to achieve the same force locking as with cast iron, steel or solid shafts.

Higher loads, especially shocks, require a larger interference fit and narrower geometrical tolerances.

Bearing seat for axial bearings

Axial bearings, which support axial loads only, must not be guided radially – with the exception of axial cylindrical roller bearings which have a degree of freedom in the radial direction due to flat raceways. In the case of groove-shaped raceways this is not present and must be achieved by a loose seat for the stationary washer. A rigid seat is normally selected for the rotating washer.

Where axial bearings also support radial forces, such as in axial spherical roller bearings, fits should be selected in the same way as for radial bearings.

The contact surfaces of the mating parts must be perpendicular to the axis of rotation (total axial run-out tolerance to IT5 or better), in order to ensure uniform load distribution over all the rolling elements.

Conditions of rotation

The condition of rotation indicates the motion of one bearing ring with respect to the load direction and is present as either circumferential load or point load, see table Conditions of rotation, Page 634.

Point load

If the ring remains stationary relative to the load direction, there are no forces that displace the ring relative to its seating surface. This type of loading is described as point load.

There is no risk that the seating surface will be damaged. In this case, a loose fit is possible.

Circumferential load

If forces are present that displace the ring relative to its seating surface, every point on the raceway is subjected to load over the course of one revolution of the bearing. This type of loading is described as circumferential load.

There is a risk that the seating surface will be damaged. A tight fit should therefore be provided.

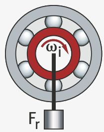
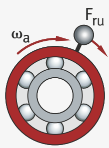
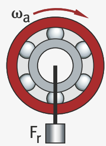
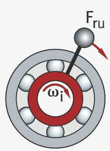
Indeterminate load direction

If the load changes in direction irregularly or by swivelling, or if shocks or vibrations occur, the raceway is also subjected to load. This type of loading is described as an indeterminate load direction.

There is a risk that the seating surface will be damaged. A tight fit should therefore be provided.

Guidelines for the selection of shaft and hole tolerances for the various bearing designs under specific mounting and loading conditions can be found in the Schaeffler catalogues for rolling bearings.

The following table shows different conditions of rotation (conditions of motion):

Conditions of rotation	Example	Schematic	Load case	Fit
Rotating inner ring Stationary outer ring Constant load direction	Shaft with weight load		Circumferential load on inner ring and Point load on outer ring	Inner ring: tight fit necessary and Outer ring: loose fit permissible
Stationary inner ring Rotating outer ring Load direction rotates with outer ring	Hub bearing arrangement with significant imbalance			
Stationary inner ring Rotating outer ring Constant load direction	Passenger car front wheel, track roller, (hub bearing arrangement)		Point load on inner ring and Circumferential load on outer ring	Inner ring: loose fit permissible and Outer ring: tight fit necessary
Rotating inner ring Stationary outer ring Load direction rotates with inner ring	Centrifuge, vibrating screen			

Shaft and housing tolerances

The fit is determined by the ISO tolerances for shafts and housings (ISO 286), in conjunction with the tolerances $t_{\Delta dmp}$ for the bore and $t_{\Delta Dmp}$ for the outside diameter of the bearings (ISO 492 for radial bearings, ISO 199 for axial bearings).

ISO tolerance classes

The shaft and housing tolerances are defined in the form of ISO tolerance classes to ISO 286-1 and ISO 286-2. The designation of the tolerance classes, for example “E8”, comprises one or two upper case letters for housings or lower case letters for shafts (= fundamental deviation identifier, which defines the tolerance position relative to the zero line, such as “E”) and the grade number of the standard tolerance grade (this defines the tolerance quality, for example “8”).

Reference to tables of shaft and housing tolerances

The tables on Page 635 to Page 638 contain recommendations for the selection of shaft and housing tolerances that are valid for normal mounting and operating conditions.

Deviations are possible if particular requirements apply, for example in relation to running accuracy, smooth running or operating temperature. Increased running accuracies thus require closer tolerances such as standard tolerance grade 5 instead of 6. If the inner ring is warmer than the shaft during operation, the seating may loosen to an impermissible extent. A tighter fit must then be selected, for example m6 instead of k6.

In such cases, the question of fits can only be resolved by a compromise. The individual requirements must be weighed against each other and those selected that give the best overall solution.

Shaft tolerances for radial bearings with cylindrical bore

For radial bearings with a cylindrical bore, the shaft tolerances are as follows:

Condition of rotation	Bearing type	Shaft diameter in mm		Displacement facility Load	Tolerance class[1]
		over	incl.		
Point load on inner ring	Ball bearings, roller bearings	All sizes		Inner ring easily displaced	g6 (g5)
				Inner ring not easily displaced Angular contact ball bearings and tapered roller bearings with adjusted inner ring	h6 (j6)
	Needle roller bearings	All sizes		Non-locating bearings	h6 (g6)[2]

Continuation of table, see Page 636.

[1] The envelope requirement Ⓔ applies.

[2] For easier mounting.

Continuation of table, Shaft tolerances for radial bearings with cylindrical bore, from Page 635.

Condition of rotation	Bearing type	Shaft diameter in mm		Displacement facility Load	Tolerance class[1]
		over	incl.		
Circumferential load on inner ring or indeterminate load direction	Ball bearings	–	50	Normal loads[2]	j6 (j5)
		50	100	Low loads[3]	j6 (j5)
				Normal and high loads[4]	k6 (k5)
		100	200	Low loads[2]	k6 (m6)
				Normal and high loads[5]	m6 (m5)
		200	–	Low loads	m6 (m5)
				Normal and high loads	n6 (n5)
	Roller bearings	–	60	Low loads	j6 (j5)
				Normal and high loads	k6 (k5)
		60	200	Low loads	k6 (k5)
				Normal loads	m6 (m5)
				High loads	n6 (n5)
		200	500	Normal loads	m6 (n6)
				High loads, shocks	p6
		500	–	Normal loads	n6 (p6)
				High loads	p6
Circumferential load on inner ring or indeterminate load direction	Needle roller bearings	–	50	Low loads	k6
				Normal and high loads	m6
		50	120	Low loads	m6
				Normal and high loads	n6
		120	250	Low loads	n6
				Normal and high loads	p6
		250	400	Low loads	p6
				Normal and high loads	r6
		400	500	Low loads	r6
				Normal and high loads	s6
		500	–	Low loads	r6
				Normal and high loads	s6

1) The envelope requirement Ⓔ applies.
2) $C_0/P_0 > 10$.
3) $C_0/P_0 > 12$.
4) $C_0/P_0 < 12$.
5) $C_0/P_0 < 10$.

Shaft tolerances for axial bearings

For axial bearings, the shaft tolerances are as follows:

Load	Bearing type	Shaft diameter in mm		Operating conditions	Tolerance class[1]
		over	incl.		
Axial load	Axial deep groove ball bearing	All sizes		–	j6
	Axial deep groove ball bearing, double direction			–	k6
	Axial cylindrical roller bearing with shaft locating washer			–	h8
	Axial cylindrical roller and cage assembly			–	h8
Combined load	Axial spherical roller bearing	All sizes		Point load on shaft locating washer	j6
		–	200	Circumferential load on shaft locating washer	j6 (k6)
		200	–		k6 (m6)

[1] The envelope requirement Ⓔ applies.

Housing tolerances for radial bearings

For radial bearings with a cylindrical bore, the housing tolerances are as follows:

Condition of rotation	Displacement facility Load	Operating conditions	Tolerance class[1]
Point load on outer ring	Outer ring easily displaced, housing unsplit	The tolerance grade is determined by the running accuracy required	H7 (H6)[2]
	Outer ring easily displaced, housing split		H8 (H7)
	Outer ring not easily displaced, housing unsplit	High running accuracy required	H6 (J6)
	Outer ring not easily displaced, angular contact ball bearings and tapered roller bearings with adjusted outer ring, housing split	Normal running accuracy	H7 (J7)
	Outer ring easily displaced	Heat input via shaft	G7[3]

Continuation of table, see Page 638.

[1] The envelope requirement Ⓔ applies.

[2] G7 for housings made from flake graphite cast iron if bearing outside diameter D > 250 mm and temperature differential between outer ring and housing > 10 K.

[3] F7 for housings made from flake graphite cast iron if bearing outside diameter D > 250 mm and temperature differential between outer ring and housing > 10 K.

Continuation of table, Housing tolerances for radial bearings, from Page 637.

Condition of rotation	Displacement facility Load	Operating conditions	Tolerance class[1]
Circumferential load on outer ring or indeterminate load direction	Low loads, outer ring cannot be displaced	For high running accuracy requirements: K6, M6, N6 and P6	K7 (K6)
	Normal loads, shocks, outer ring cannot be displaced		M7 (M6)
	High loads, shocks ($C_0/P_0 < 6$), outer ring cannot be displaced		N7 (N6)
	High loads, severe shocks, thin-walled housing, outer ring cannot be displaced		P7 (P6)

1) The envelope requirement Ⓔ applies.

Housing tolerances for axial bearings

For axial bearings with a cylindrical bore, the housing tolerances are as follows:

Load, condition of rotation	Bearing type	Operating conditions	Tolerance class[1]
Axial load	Axial deep groove ball bearing	Normal running accuracy	E8
		High running accuracy	H6
	Axial cylindrical roller bearing with housing locating washer	–	H9
	Axial cylindrical roller and cage assembly	–	H10
	Axial spherical roller bearing	Normal loads	E8
		High loads	G7
Combined load Point load on housing locating washer	Axial spherical roller bearing	–	H7
Combined load Circumferential load on housing locating washer	Axial spherical roller bearing	–	K7

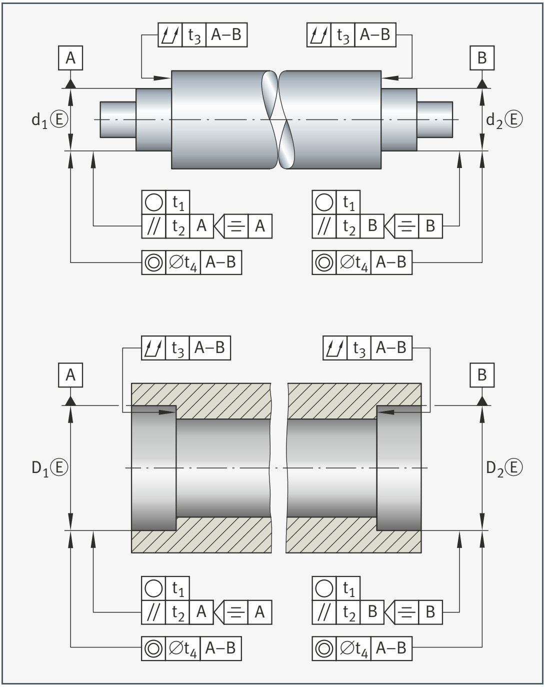
1) The envelope requirement Ⓔ applies.

Geometrical and positional tolerances of bearing seating surfaces

In order to achieve the required fit, the bearing seats and fit surfaces of the shaft and housing bore must conform to certain tolerances, see Figure 85 and table, Page 640.

Figure 85
Guide values for the geometrical and positional tolerances of bearing seating surfaces

t_1 = roundness tolerance
t_2 = parallelism tolerance
t_3 = total axial run-out tolerance of abutment shoulders
t_4 = coaxiality tolerance



For bearing seating surfaces, the geometrical and positional tolerances are as follows:

Tolerance class of bearings		Bearing seating surface	Fundamental tolerance grades[1]				
To ISO 492	To DIN 620		Diameter tolerance	Load case	Roundness tolerance t_1	Parallelism tolerance t_2	Total axial run-out tolerance of abutment shoulder t_3
Normal 6X	PN (P0) P6X	Shaft	IT6 (IT5)	Circumferential load	IT4/2	IT4/2	IT4
				Point load	IT5/2	IT5/2	
		Housing	IT7 (IT6)	Circumferential load	IT5/2	IT5/2	IT5
				Point load	IT6/2	IT6/2	
6	P6	Shaft	IT5	Circumferential load	IT3/2	IT3/2	IT3
				Point load	IT4/2	IT4/2	
		Housing	IT6	Circumferential load	IT4/2	IT4/2	IT4
				Point load	IT5/2	IT5/2	
5	P5	Shaft	IT5	Circumferential load	IT2/2	IT2/2	IT2
				Point load	IT3/2	IT3/2	
		Housing	IT6	Circumferential load	IT3/2	IT3/2	IT3
				Point load	IT4/2	IT4/2	
4	P4 P4S[2] SP[2]	Shaft	IT4	Circumferential load	IT1/2	IT1/2	IT1
				Point load	IT2/2	IT2/2	
		Housing	IT5	Circumferential load	IT2/2	IT2/2	IT2
				Point load	IT3/2	IT3/2	
	UP[2]	Shaft	IT3	Circumferential load	IT0/2	IT0/2	IT0
				Point load	IT1/2	IT1/2	
		Housing	IT4	Circumferential load	IT1/2	IT1/2	IT1
				Point load	IT2/2	IT2/2	

[1] ISO fundamental tolerances (IT grades) in accordance with DIN ISO 286.

[2] Not included in DIN 620.

Accuracy of bearing seating surfaces

The degree of accuracy of the bearing seat tolerances on the shaft and in the housing is given in the table Geometrical and positional tolerances of bearing seating surfaces, Page 640.

Second bearing seat

The positional tolerances t_4 for a second bearing seat on the shaft (d_2) or in the housing (D_2) are dependent on the types of bearings used and the operating conditions. Values for the tolerances t_4 can be requested from Schaeffler.

Housings

In split housings, the joints must be free from burrs. The accuracy of the bearing seats is determined as a function of the accuracy of the bearing selected.

Roughness of bearing seats

The roughness of the bearing seats must be matched to the tolerance class of the bearings. The mean roughness value Ra must not be too high, in order to maintain the interference loss within limits. Shafts must be ground, while bores must be precision turned.

For bearing seating surfaces, the guide values for roughness are as follows:

Nominal diameter of bearing seat d (D) mm		Recommended mean roughness value for ground bearing seats Ramax µm			
		Diameter tolerance (IT grade)			
over	incl.	IT7	IT6	IT5	IT4
–	80	1,6	0,8	0,4	0,2
80	500	1,6	1,6	0,8	0,4
500	1 250	3,2[1)]	1,6	1,6	0,8

1) For the mounting of bearings using the hydraulic method, do not exceed the value Ra = 1,6 µm.

Axial location of bearings

Axial location of the bearing rings is matched to the specific bearing arrangement (locating bearings, non-locating bearings, bearings in adjusted and floating arrangements).

Design guidelines

The bearing rings must be located by force locking or form fit in order to prevent lateral creep. The bearing rings must only be in contact with the shaft or housing shoulder, but not with the fillet.

The shoulders on the mating parts must be large enough to provide a sufficiently wide contact surface even with the largest chamfer dimension of the bearing (DIN 5418).

Locating bearings

Locating bearings support axial forces. The retaining element must be matched to these axial forces. Shoulders on the shaft and housing, retaining rings, snap rings, housing covers, shaft covers, nuts and spacer rings are suitable.

Non-locating bearings

Non-locating bearings must support low axial forces occurring in thermal expansion. The means of axial location therefore only needs to prevent creep of the rings. A tight fit is often sufficient for this purpose.

Self-retaining bearings

In non-separable bearings, one bearing ring must have a tight fit, while the other ring is retained by the rolling elements.

For further information, see the Schaeffler catalogues.

Seals

The sealing arrangement has a considerable influence on the operating life of a bearing arrangement. Its function is to retain the lubricant in the bearing and prevent the ingress of contaminants into the bearing.

Contaminants may have various effects:

- A large quantity of very small, abrasive particles causes wear in the bearing. The increase in clearance or noise brings the operating life of the bearing to an end.
- Large, overrolled hard particles reduce the fatigue life since pitting occurs at the indentation points under high bearing loads.

A basic distinction is made between contact and non-contact seals in the adjacent construction and the bearing.

Non-contact seals in the adjacent construction

With non-contact seals, only lubricant friction occurs in the lubrication gap. The seals do not undergo wear and remain capable of operation for a long period. Since they generate no heat, non-contact seals are also suitable for very high speeds.

Gap seals

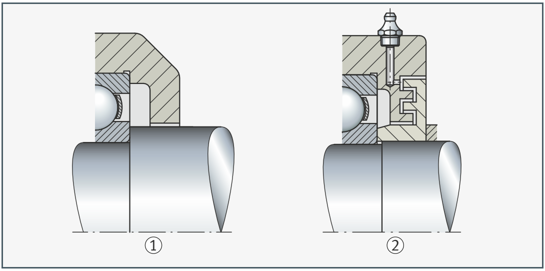
A simple design, although adequate in many cases, is a narrow seal gap between the shaft and housing.

Labyrinth seals

A considerably greater sealing action than with gap seals is achieved by labyrinths incorporating gaps filled with grease. In contaminated environments, grease should be pressed from the interior into the seal gap at short intervals.

Figure 86
Gap seals and labyrinth seals

① Simple gap seal
② Labyrinth seal



Ring with runoff edges

Where oil lubrication is used with a horizontal shaft, rings with a runoff edge are suitable for preventing the escape of oil. The oil outlet hole on the underside of the seal location must be sufficiently large that it cannot be clogged by contamination.

Baffle plates

Stationary (rigid) baffle plates ensure that grease remains in the area around the bearing. The grease collar that forms at the seal gap protects the bearing against contaminants.

Flinger shields

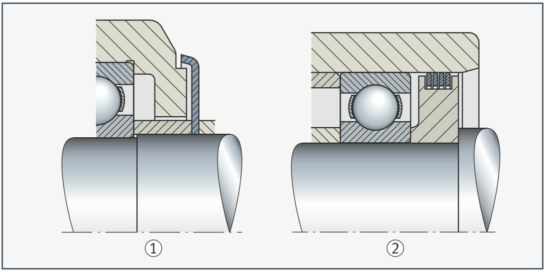
Co-rotating flinger shields have the effect of shielding the seal gap from heavy contamination.

Lamellar rings

Lamellar rings made from steel and radially sprung either outwards or inwards require little mounting space. They give protection against loss of grease and ingress of dust and are also used as an outer seal against spray water.

Figure 87
Flinger shields and lamellar rings

① Flinger shield
② Lamellar rings



Non-contact seals in the bearing

Non-contact seals can be fitted in the bearing instead of the adjacent construction.

Sealing shields

Sealing shields are compact sealing elements fitted on one or both sides of the bearing. Bearings with sealing shields on both sides are supplied with a grease filling.

BRS seals (labyrinth seals)

The friction in this case is as low as that in bearings with sealing shields. They have the advantage over these, however, that the outer rubber-elastic rim gives good sealing when fitted in the slot in the outer ring. This is important with a rotating outer ring since the base oil in the grease is separated from the soap suspension by centrifugal force and would escape through the unsealed metallic seat in the outer ring if sealing shields were fitted.

Contact seals in the adjacent construction

Contact seals are normally in contact with the running surface under radial contact force. The contact force should be kept small in order to avoid an excessive increase in frictional torque and temperature. The frictional torque and temperature as well as the wear of the seal are also affected by the lubrication condition at the running surface, the roughness of the running surface and the sliding velocity.

Felt rings

Felt rings and felt strips are sealing elements that have proved very effective with grease lubrication. They are impregnated with oil before mounting and give particularly good sealing against dust. They are suitable for circumferential velocities at the running surface of up to 4 m/s. In unfavourable environmental conditions, two felt rings are arranged adjacent to each other. Felt rings and annular slots are standardised in accordance with DIN 5419.

Rotary shaft seals

For the oil sealing of rotating shafts, rotary shaft seals (RWDR) in accordance with DIN 3760 and DIN 3761 and with spring preload are suitable. The sealing rings are designed for applications with slight pressure differentials. Depending on the seal material and the surface structure of the shaft, the geometry of the seal lips generates a pumping action in the sealing gap towards the steep flank of the seal lip. The sealing ring is therefore mounted with the steep flank facing in the direction of the medium against which sealing is required.

In the case of grease lubrication, the steep flank of the rotary shaft seal is often placed in the direction of grease egress. As a result, some grease passes under the seal lip for lubrication of the sealing edge.

Springless Schaeffler sealing rings for needle roller bearings

Low-friction sealing of bearing positions with a small radial design envelope, such as bearing positions with needle roller bearings, can be effectively achieved using sealing rings G, GR and SD, see Figure 88. These sealing rings can be used individually or in a double arrangement.

In the double arrangement, one seal lip faces inwards to seal the lubrication medium, while the second seal lip faces outwards to give protection against contamination. In order to improve the protective function, the space between the seals can be filled with grease.
With an extended inner ring, a sealing ring with the same outside diameter as the outer ring can be used, where the seal lip runs on the extended inner ring. Sealing rings give good protection against contamination and spray water as well as against the egress of oil and grease under slight pressure differentials. In order to reduce friction and protect the seal lip against damage, the sealing edge must be lubricated.

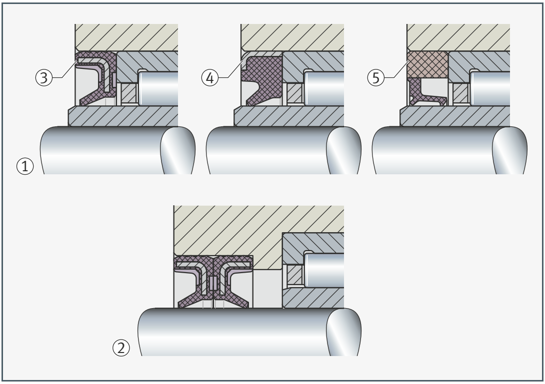


Figure 88
Schaeffler sealing rings

① Single arrangement, bearing with extended inner ring
② Double arrangement, bearing with inner ring
③ G sealing ring
④ GR sealing ring
⑤ SD sealing ring

Lip seal with axial sealing action
Lip seals are seals with one or more seal lips that give axial or radial sealing. These seals are predominantly elastomer seals.

The V-ring is a lip seal with axial sealing action. The ring is made from elastic rubber NBR. During mounting, it is stretched and slid onto the shaft so that the seal lip is in contact with the housing wall. At circumferential velocities over 12 m/s, experience shows that the V-ring must be radially located so that it does not become detached due to centrifugal force. Precise circumferential velocities for specific applications must always be agreed in consultation with the sealing ring manufacturer.

Metallic sealing washers
When using grease lubrication, effective sealing can also be achieved by means of axially sprung sealing shields. The sheet metal shields are clamped to the end face of the inner ring or outer ring and are axially sprung against the other bearing ring.

Contact seals in the bearing

Contact seals can be fitted in the bearing instead of the adjacent construction.

Sealing washers
Bearings fitted with one or two sealing washers allow simple designs. The washers are suitable for giving protection against dust, contamination, damp atmospheres and slight pressure differentials. Sealing washers are used, for example, in maintenance-free bearings with grease filling. The sealing washer design RSR made from acrylonitrile butadiene rubber (NBR), normally used in deep groove ball bearings, is located under radial contact pressure against a cylindrically ground inner ring rib.

Multi-lip seals are also used. Double lip seals made from NBR are used, for example, in plummer block housings, while single lip and multi-lip seals are used in a sandwich arrangement in radial insert ball bearings. Further information: see Schaeffler catalogue HR 1, Rolling Bearings.

Products – overview

This section shows an excerpt from the Schaeffler product range of rotary rolling bearings in the Industrial Division.

The products are also used to a significant extent – once designed and matched in accordance with the specific requirements – in applications in the Automotive Division.

The product descriptions are, in accordance with the philosophy of this publication, presented in a condensed form and are essentially intended as an overview. While they highlight important characteristics of the products and will in many cases allow preliminary assessment of a particular bearing for its suitability in a bearing arrangement, they cannot be used directly for the design of bearing arrangements. The specific application is always the decisive factor in the use of products. In addition, the information in the specific product descriptions must always be taken into consideration.

Product information documents

The comprehensive technical product information documents (catalogues, Technical Product Information, datasheets, mounting manuals, operating manuals, etc.) can be requested from Schaeffler.

Schaeffler can, upon request, provide support in the selection of bearings and the design of bearing arrangements.

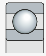
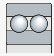
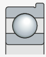
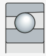
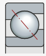
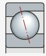
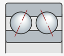
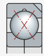
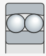
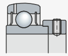
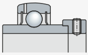
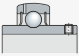
Electronic product catalogue medias

The standard range is described in detail in the online version **medias**. In addition, this tool for the design of bearing arrangements provides means of assistance such as a lubricant database, a calculation tool for determining rating life and other features.

Link to electronic product catalogue
The following link will take you to the Schaeffler electronic product catalogue: *https://medias.schaeffler.com*.

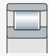
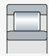
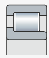
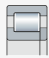
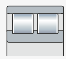
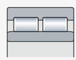
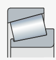
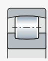
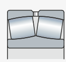
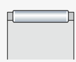
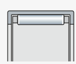
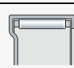
Compilation of standardised bearings

The following table shows a compilation of standardised bearings with their names and standard numbers (excerpt from DIN 623-1).

Name/design	Image	Standard number
Radial deep groove ball bearing, single row, without filling slot		DIN 625-1
Radial deep groove ball bearing, double row, with or without filling slot		DIN 625-3
Radial deep groove ball bearing, with flanged outer ring		DIN 625-4
Radial magneto bearing		DIN 615
Radial angular contact ball bearing, single row, without filling slot, non-separable		DIN 628-1
Radial angular contact ball bearing, single row, contact angle 15° and 25°		DIN 628-6
Radial angular contact ball bearing, double row, with or without filling slot		DIN 628-3
Angular contact ball bearing, four point contact bearing with split inner ring		DIN 628-4
Radial self-aligning ball bearing, double row		DIN 630
Radial insert ball bearing with inner ring extended on one side, curved outer ring outside surface and eccentric locking collar		DIN 626-1
Radial insert ball bearing with inner ring extended on both sides, curved outer ring outside surface and eccentric locking collar		DIN 626-1
Radial insert ball bearing with inner ring extended on both sides, curved outer ring outside surface and grub screw		DIN 626-1

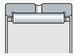
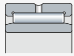
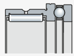
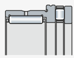
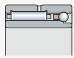
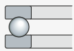
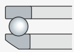
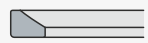
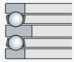
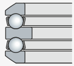
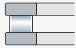
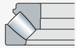
Continuation of table, see Page 648.

Continuation of table, Compilation of standardised bearings,
from Page 647.

Name/design	Image	Standard number
Radial cylindrical roller bearing, single row, two rigid ribs on inner ring, ribless outer ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, ribless inner ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, one rigid rib on inner ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, one rigid rib and one loose rib washer on inner ring		DIN 5412-1
L-section ring for cylindrical roller bearings		DIN 5412-1
Radial cylindrical roller bearing, double row, three rigid ribs on inner ring, ribless outer ring		DIN 5412-4
Radial cylindrical roller bearing, double row, three rigid ribs on outer ring, ribless inner ring		DIN 5412-4
Tapered roller bearing, single row		DIN 720
Radial spherical roller bearing, single row, barrel roller bearing		DIN 635-1
Radial spherical roller bearing, double row		DIN 635-2
Radial needle roller and cage assembly, single row		DIN 5405-1
Drawn cup needle roller bearing with open ends, single row		DIN 618
Drawn cup needle roller bearing with closed end, single row		DIN 618

Continuation of table, see Page 649.

Continuation of table, Compilation of standardised bearings, from Page 648.

Name/design	Image	Standard number
Radial needle roller bearing, single row, without inner ring		DIN 617
Radial needle roller bearing, single row, two rigid ribs on outer ring, ribless inner ring		DIN 617
Combined radial needle roller bearing/ axial deep groove ball bearing		DIN 5429-1
Combined radial needle roller bearing/ axial cylindrical roller bearing		DIN 5429-1
Combined radial needle roller bearing/ angular contact ball bearing		DIN 5429-2
Axial deep groove ball bearing, single direction, with flat housing locating washer		DIN 711
Axial deep groove ball bearing, single direction, with curved housing locating washer		DIN 711
Support washer for axial ball bearings		DIN 711
Axial deep groove ball bearing, double direction, with flat housing locating washer		DIN 715
Axial deep groove ball bearing, double direction, with curved housing locating washer		DIN 715
Axial needle roller and cage assembly, single row		DIN 5405-2
Axial bearing washer		DIN 5405-3
Axial cylindrical roller bearing, single direction		DIN 722
Axial spherical roller bearing, asymmetrical rollers		DIN 728

Basic structure of rotary rolling bearings

Rotary rolling bearings generally consist of the components in accordance with Figure 89:

- rolling elements (balls, ball rollers, rollers); see table Rolling element type
- inner and outer ring with rolling element raceways
- cage, see Page 618
- seals or sealing shields on one side or both sides of the bearing
- lubricant in the case of grease lubrication

Figure 89 shows standard parts of ball and roller bearings.

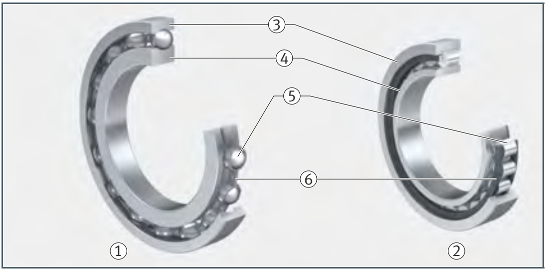
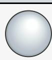
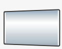


Figure 89
Structure of standard ball and roller bearings

① Ball bearing
② Roller bearing
③ Outer ring
④ Inner ring
⑤ Rolling elements (ball, cylindrical roller)
⑥ Cage (metal, polyamide)

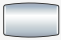
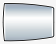
Rolling elements

Rolling elements are the connecting elements and contact elements between the stationary and the moving part of the rolling bearing. As the basis of the rolling element type, they essentially determine the characteristics of the bearing. Types of rolling elements, and the type of rolling bearing as derived from them, are shown in the following table.

Rolling element type		Bearing type
	Ball	Ball bearing
	Cylindrical roller	Cylindrical roller bearing
	Needle roller	Needle roller bearing, drawn cup needle roller bearing with open ends, drawn cup needle roller bearing with closed end, needle roller and cage assembly
	Tapered roller	Tapered roller bearing

Continuation of table, see Page 651.

Continuation of table, Rolling element type, from Page 650.

Rolling element type		Bearing type
	Symmetrical barrel roller	Barrel roller bearing, spherical roller bearing
	Long, slightly curved barrel roller	Toroidal roller bearing
	Asymmetrical barrel roller	Axial spherical roller bearing

Radial ball bearings

Radial ball bearings have balls as rolling elements, an operating contact angle between 0° and 45° and can support axial loads as well as radial loads.

Deep groove ball bearings

Deep groove ball bearings are versatile, self-retaining bearings with solid outer rings, inner rings and ball and cage assemblies, see Figure 90. These products, which are of simple design, robust in operation and easy to maintain, are available in single and double row designs and in open and sealed variants. Due to their very low frictional torque, deep groove ball bearings are suitable for high and very high speeds.

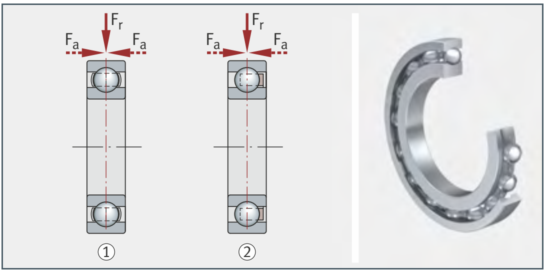


Figure 90
Deep groove ball bearings, single row

F_r = radial load
F_a = axial load

① Bearing with sheet metal cage
② Bearing with plastic cage

Combined load (axial and radial)

Due to the raceway geometry and the balls, the bearings can support axial loads in both directions as well as radial loads.

Single row deep groove ball bearings

Single row deep groove ball bearings are the most frequently used type of rolling bearing, see Figure 90. They are produced in numerous sizes and designs and are particularly economical. Open bearings are suitable for high to very high speeds, designs with gap seals are suitable for high speeds.

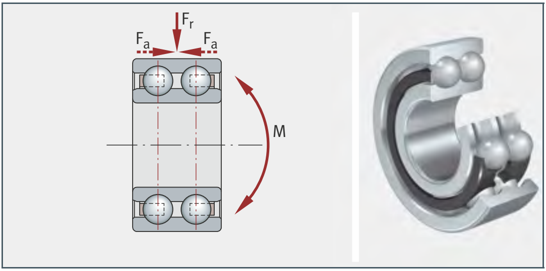
Double row deep groove ball bearings

Double row deep groove ball bearings correspond in their structure to a pair of single row deep groove ball bearings but have two raceway grooves each with one row of balls, see Figure 91. Due to the larger number of rolling elements, they can be subjected to higher loads than single row deep groove ball bearings.

Double row bearings are suitable for high to very high speeds and are used where the load carrying capacity of single row deep groove ball bearings is not sufficient.

Figure 91
Deep groove ball bearing, double row

M = tilting moment load

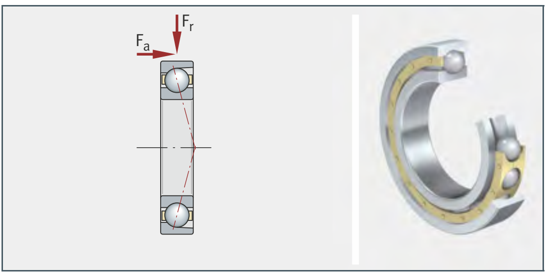


Magneto bearings

Magneto bearings correspond substantially to deep groove ball bearings without a slot (the slot is used to fill the bearing with the rolling elements). They have a shoulder on the outer ring and can therefore support axial forces in one direction only, see Figure 92. For axial guidance of the shaft, two bearings in a mirror image arrangement are always necessary. Magneto bearings are normally mounted with a small axial clearance; in this way, changes in length of the shaft and housing can be compensated.

Since the bearings are separable, it is possible to mount the outer ring separately from the inner ring with the ball and cage assembly.

Figure 92
Magneto bearing, single row



Angular contact ball bearings

The optimum load direction and the best flow of force can only be ensured using deep groove ball bearings if the active force acts in a vertical direction through the radial plane. In practice, it is frequently the case that the loads acting in a bearing arrangement are not purely radial but combined, where the loads comprise both radial and axial forces.
In contrast to deep groove ball bearings, the raceways on the inner and outer rings of angular contact ball bearings are offset from each other along the bearing axis, see Figure 93.

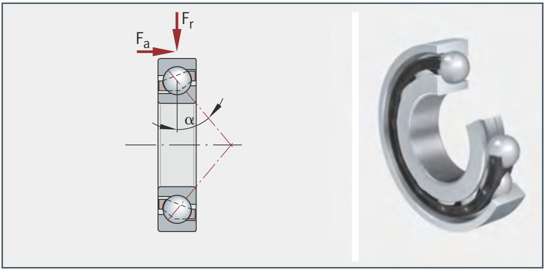
As a result, the forces are transmitted from one raceway to the other at a specific contact angle (oblique to the radial plane). They are therefore suitable for radial and axial loads acting simultaneously.

Contact angle

The contact angle is normally 15°, 25°, 30° or 40°. The axial load carrying capacity of the bearing increases with the contact angle. As a result, angular contact ball bearings are more suitable than deep groove ball bearings for supporting higher axial forces.

Figure 93
Angular contact ball bearing, single row

α = contact angle



Spindle bearings

Spindle bearings are self-retaining, single row angular contact ball bearings of a high precision design with contact angles of 15°, 20° or 25°. Bearings with a contact angle of 15° are particularly suitable for high radial loads, while those with a contact angle of 25° are more suitable for combined loads.

At high speeds, large centrifugal forces act on the balls which are superimposed on the operating loads and thus have a significant influence on the speed capacity of the bearings. In order to reduce the centrifugal forces and increase the speeds, bearings can be used that have smaller balls while retaining the normal external dimensions.

Due to special contact conditions between the balls and raceways, the friction and operating temperature of the bearing remain low.
For even higher speeds, hybrid bearings (bearings with ceramic balls) can be used.

Double row angular contact ball bearings

Double row angular contact ball bearings correspond in their structure to a pair of single row angular contact ball bearings in an O arrangement, see Figure 94. They can support high radial and axial loads and are therefore particularly suitable for the rigid axial guidance of shafts.

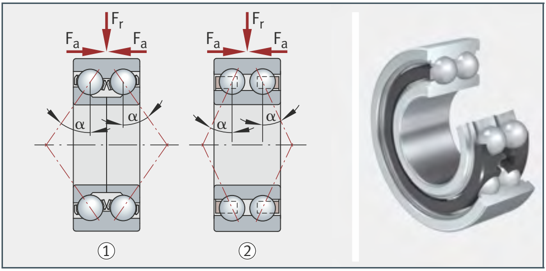
Smaller double row angular contact ball bearings do not have a filling slot and therefore have the same axial load carrying capacity on both sides. In the case of larger double row bearings, the position of the filling slot determines the load direction. The axial force should always be supported by the ball row without a filling slot.

Where high, alternating axial forces must be supported, double row angular contact ball bearings with a split inner ring can be used. These bearings have a larger contact angle and do not have a filling slot.

Figure 94
Angular contact ball bearings, double row

α = contact angle

① Bearing with split inner ring, large contact angle
② Bearing with unsplit inner ring, small contact angle



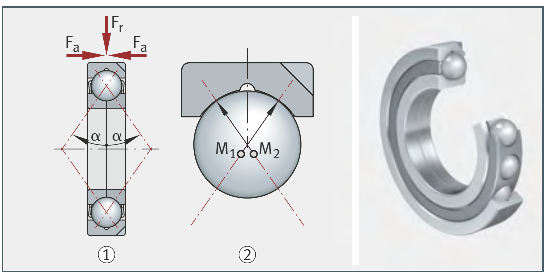
Four point contact bearings

Four point contact bearings are similar in their structure to double row angular contact ball bearings, see Figure 95. Since the centres of curvature of the arc-shaped raceways on the inner ring and outer ring are offset relative to each other, however, the balls are in contact with the bearing rings at four points under radial load. For this reason, four point contact bearings are only used under predominantly axial load. One of the two bearing rings, normally the inner ring, is split in order to allow filling with the balls.

Figure 95
Four point contact bearing

α = contact angle
M_1, M_2 = centres of curvature of outer ring raceway

① Four point contact bearing with retaining slot and split inner ring
② Raceway geometry



For axial loads
The bearings can support alternating, pure axial or predominantly axial loads. Due to the large contact angle (usually 35°), four point contact bearings are suitable for high axial forces in alternating directions.

If predominantly radial load is present, four point contact bearings should not be used due to the higher friction in the four point contact.

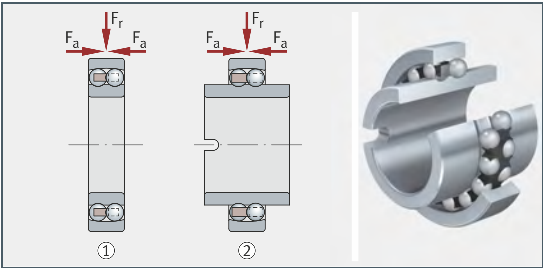
Self-aligning ball bearings

In the self-aligning ball bearing, two rows of rolling elements are held in two raceway grooves on the inner ring, see Figure 96. The raceway on the outer ring has a curved form. The cage combines the two rows of balls and the inner ring to form a unit that can align itself by swivelling relative to the outer ring.

Figure 96
Self-aligning ball bearings

Curved raceway of outer ring

① Bearing with cylindrical bore
② Bearing with extended inner ring and locating slot



Tolerant of misalignment
Due to the curved raceway on the outer ring, self-aligning ball bearings are tolerant of misalignment between the shaft and housing and deflections of the shaft. They can thus compensate static and dynamic angular defects within certain angular limits in a rotating shaft system.

They are used particularly in sectors such as agricultural machinery, conveying equipment, simple woodworking machinery and ventilators.

Thin section bearings

Thin section bearings have very small cross-sections relative to their diameters, see Figure 97. This allows designs with smaller design envelope and lower mass, while achieving high rigidity and running accuracy.

In contrast to the rolling bearing series standardised in accordance with DIN ISO, in which the cross-section increases with the bearing diameter, all sizes of bearing in one series have the same cross-section.

Deep groove ball bearings, four point contact bearings and angular contact ball bearings

Thin section bearings are available as deep groove ball bearings, four point contact bearings and angular contact ball bearings. The preferred diameter ranges are between 25 mm and 1000 mm.

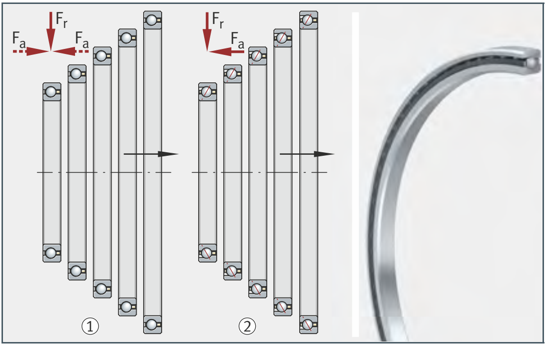


Figure 97
Thin section bearings

Curved raceway of outer ring

1 Deep groove ball bearing type
2 Angular contact ball bearing type

Radial insert ball bearings

Radial insert ball bearings are based on deep groove ball bearings of series 60, 62 and 63. The numerals identify the rolling element set and therefore the load carrying capacity. The inner ring is extended on one or both sides.

In conjunction with the corresponding housing, radial insert ball bearings allow the use of basic adjacent constructions. They are particularly easy to fit and are suitable for drawn shafts of grade h6 Ⓔ to h9 Ⓔ. For non-locating bearings, shafts of tolerance classes h5 Ⓔ to h7 Ⓔ are recommended.

Location and designs

Axial location can be carried out using eccentric locking collars, grub screws or integrated adapter sleeves, see Figure 98. Certain series can be located by means of a fit on the shaft.

The extended inner ring on both sides provides a seal running surface and prevents significant tilting of the inner ring.

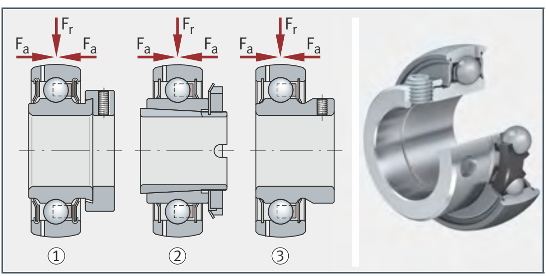


Figure 98
Location methods for radial insert ball bearings

1 Location by means of eccentric locking collar
2 Location by means of integral adapter sleeve
3 Location by means of grub screws in inner ring

Seals matched to application, numerous series with relubrication facility
A further, characteristic feature of radial insert ball bearings is the type of sealing used. The bearing is protected against contamination and the loss of lubricant by means of contact seals of a multi-piece design on both sides of the bearing. Additional protection of the seals can be provided by means of sheet steel flinger shields mounted in front of the lip seals.

The bearings can normally be relubricated via holes in the outer ring.

Compensation of static angular misalignments
Radial insert ball bearings are available in numerous designs with a curved or cylindrical outside surface of the outer ring. Bearings with a curved outside surface can, through adjusting motion of the outer ring in a curved bore, support angular misalignments (±2,5° with relubrication, ±5° without relubrication). Aligning rings with a curved bearing locating bore allow mounting of the bearings in housings with a cylindrical bore. The angular adjustment facility is thus maintained.

Radial insert ball bearings cannot be used for the compensation of dynamic angular misalignments.

Housing units

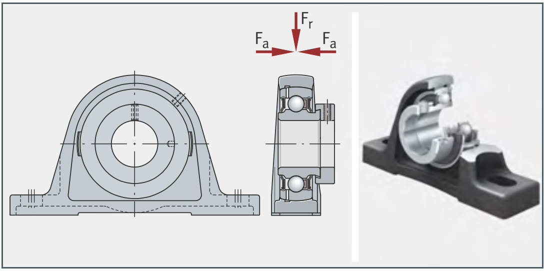
For radial insert ball bearings, Schaeffler can supply suitable plummer block and flanged housings that are made from flake graphite cast iron or sheet steel, see Figure 99. The housings can – like the radial insert ball bearings themselves – also be provided in a corrosion-resistant design. Cast iron housings are always one-piece units and can support high loads. Sheet steel housings are always two-piece units and are used where the priority is not the load carrying capacity of the housing but the low mass of the unit.

The housing units comprise radial insert ball bearings with a curved outer ring and a housing with a curved bore to form ready-to-fit units. The user is thus spared the need for costly production of the mounting environment required for these bearings. The areas of application correspond to those of the radial insert ball bearings. Bearings with a curved outside surface of the outer ring, when mounted in housings with a curved bore, can compensate static misalignments of the shaft, see section Compensation of static angular misalignments, Page 657.

The units are not suitable for supporting swivel or wobble type motion.

Figure 99
Plummer block housing unit

Flake graphite cast iron housing with integrated radial insert ball bearing



Radial roller bearings

Radial roller bearings are rolling bearings with cylindrical, needle, tapered or barrel rollers as rolling elements.

Cylindrical roller bearings

Cylindrical roller bearings are used as non-locating, semi-locating and locating bearings. They are available with a full complement of rollers, with a cage or with spacers.

Axial load carrying capacity

The axial load carrying capacity of a cylindrical roller bearing is dependent on:

- the size of the sliding surfaces between the ribs and the end faces of the rolling elements
- the sliding velocity at the ribs
- the lubrication of the contact surfaces
- tilting of the bearing

Ribs subjected to load must be supported across their entire height.

Cylindrical roller bearings must not be continuously subjected to pure axial loads. In order to prevent impermissibly high edge stresses on the rolling elements and raceways due to insufficient contact between the rolling elements and the raceways, a minimum radial load must always be present. The ratio F_a/F_r should not exceed 0,4. Cylindrical roller bearings with an optimised contact geometry allow a ratio F_a/F_r up to a value of 0,6.

Further information on the minimum radial load is given in the Schaeffler rolling bearing catalogues.

Cylindrical roller bearings with cage

Cylindrical roller bearings with cage are available in numerous designs, sizes and dimension series. In all standard designs, however, the cylindrical rollers are guided between rigid ribs by at least one bearing ring. Together with the cage and rollers, this forms a ready-to-fit unit. The other bearing ring can be removed. As a result, the inner ring and outer ring can be mounted separately. Tight fits can thus be achieved on both rings.

Tight fits increase the rigidity of the bearing arrangement and give precise radial guidance of the shaft.

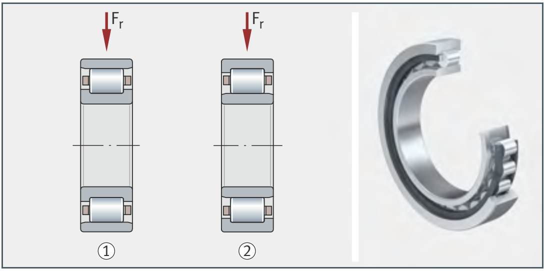
Non-locating bearings

Cylindrical roller bearings N and NU as well as NN and NNU are non-locating bearings and can support radial forces only, see Figure 100. In the case of series NU and NNU, the outer ring has ribs while the inner ring has no ribs. Bearings of the designs N and NN have ribs on the inner ring and an outer ring without ribs. The ring without ribs serves to compensate variations in the length of the shaft, for example as a result of temperature differentials.

Non-locating bearings NU can be combined with an L-section ring HJ to form a semi-locating bearing unit.

Figure 100
Non-locating bearings

① Cylindrical roller bearing N
② Cylindrical roller bearing NU



Semi-locating bearings
Cylindrical roller bearings NJ are semi-locating bearings, see Figure 101. They can support not only high radial forces but also axial forces in one direction and can therefore guide shafts axially in one direction. In the opposite direction, they act as non-locating bearings.

The outer ring with its two ribs provides axial guidance of the rollers. The L-shaped inner ring supports the rolling elements on one side against axial forces.

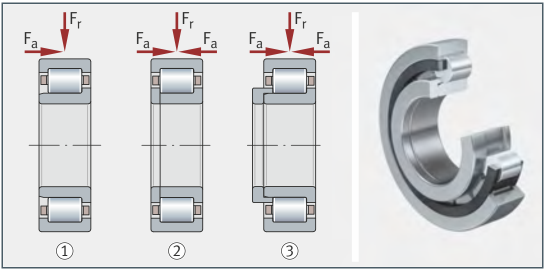
Semi-locating bearings NJ can be combined with an L-section ring HJ to form a locating bearing unit.

Locating bearings
Cylindrical roller bearings NUP are locating bearings, see Figure 101. They can support not only high radial forces but also axial forces in both directions and can therefore guide shafts axially in both directions.

The outer ring has two ribs, while the L-shaped inner ring has one rigid rib and one loose rib washer.

Figure 101
Semi-locating and locating bearings

① Cylindrical roller bearing NJ (semi-locating bearing)
② Cylindrical roller bearing NUP with rib washer (locating bearing)
③ Cylindrical roller bearing NJ with L-section ring HJ (locating bearing)



Disc cage or spacers

Figure 102 shows a semi-locating bearing of dimension series 23 with a space-saving disc cage. The flat brass disc cage and the plastic spacers prevent the cylindrical rollers from coming into contact with each other during rolling.

In the case of the disc cage, the rolling elements are guided between the ribs on the outer ring. Due to its low mass, the cage is subjected to only minimal load under acceleration.

The spacers are guided axially between the two outer ring ribs, see Figure 102. They are designed such that the rolling element set is self-retaining, so the bearing and inner ring can be mounted separately from each other. This gives easier mounting of the cylindrical roller bearings.

In comparison with conventional cage type bearings with roller retention, these cage designs allow a larger number of rolling elements to be accommodated in the bearing. Due to the lower frictional torques in comparison with full complement cylindrical roller bearings, less heat is generated in the bearing and higher speeds can thus be achieved.

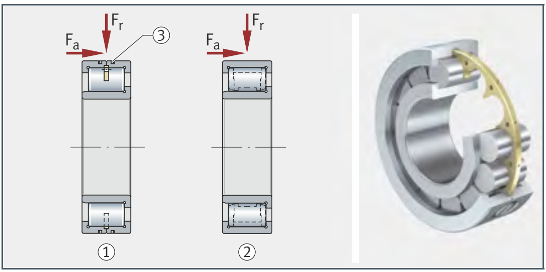


Figure 102
Cylindrical roller bearings with disc cage or spacers

1 Cylindrical roller bearing with disc cage
2 Cylindrical roller bearing with spacers
3 Retaining ring

Full complement cylindrical roller bearings

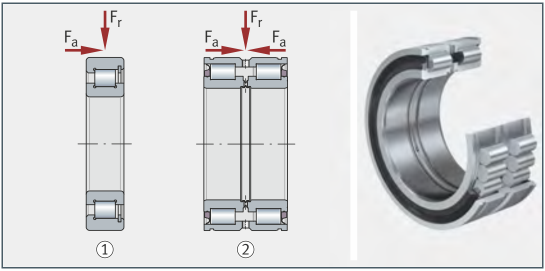
Full complement roller bearings are available as non-locating, semi-locating and locating bearings. They are of a single row or double row design and do not have a cage, see Figure 103.

Due to the lack of a cage, the bearing can accommodate more rolling elements. Since these bearings have the largest possible number of rolling elements, they have extremely high radial load carrying capacity, high rigidity and are suitable for particularly compact designs. Due to the kinematic conditions, however, they do not achieve the high speeds that are possible when using cylindrical roller bearings with cage.

Multi-row bearings have particularly high tilting rigidity but do not permit any skewing between the inner ring and outer ring.

Figure 103
Full complement cylindrical roller bearings

① Single row
② Double row, with annular slots in outer ring, sealed (rope sheave bearings)



Rope sheave bearings
Special bearing types are available for special applications. For example, double row full complement cylindrical roller bearings with annular slots in the outer ring are used in bearing arrangements for the support of rope sheaves, see Figure 103. These locating bearings have high rigidity and can support moderate axial forces in both directions as well as high radial forces.

Rope sheave bearings comprise solid outer and inner rings with ribs and rib-guided cylindrical rollers. Axial location of the bearings is achieved by means of retaining rings in the annular slots in the outer ring.

Sealing rings on both sides protect the bearing against contamination, moisture and the loss of grease.

Super precision cylindrical roller bearings

The single and double row bearings are used when the very highest precision is required under very high radial load. Typical areas of application include machine tools and printing machinery. In these cases, they facilitate bearing arrangements with very high precision, high radial rigidity and very high load carrying capacity. In machine tool building, they provide radial support for the main spindle, see Figure 124, Page 675. Super precision cylindrical roller bearings N, NN and NNU are in the accuracy classes SP and UP.

Since variations in length during rotary motion can be compensated between the rollers and the ribless raceway without constraining forces, the cylindrical roller bearings are highly suitable as non-locating bearings. Axial forces are supported by axial bearings, such as double direction axial angular contact ball bearings, see Figure 124, Page 675.

Tapered roller bearings

Tapered roller bearings are single row or multi-row units comprising a ribless outer ring, an inner ring with two ribs of different heights and a cage, see Figure 104. The cage contains truncated conical rollers.

Cages, ribs, roller profile, projected lines of contact

Pressed and stamped sheet steel cages are used as the standard cage. Cages made from glass fibre reinforced polyamide are also available. The lower rib retains – in conjunction with the cage – the rollers on the inner ring raceway. The high rib supports the axial force component arising from the tapered form of the rollers.

The sliding surfaces on the large rib and on the large roller end face are designed such that a lubricant film capable of supporting load is formed at the contact points between the roller and rib. The logarithmic profile induces the optimum distribution of stress at the contact between the rolling element and raceway and prevents stress peaks. The projected lines of contact of the tapered rollers intersect the projected raceways of the inner and outer ring at a point on the bearing axis, see Figure 104. This prevents any kinematic forced slippage at the rolling contact.

Open bearings are not self-retaining, so the outer ring can be removed in the case of these bearings. The outer ring and the inner ring with the roller and cage assembly can thus be mounted separately from each other.

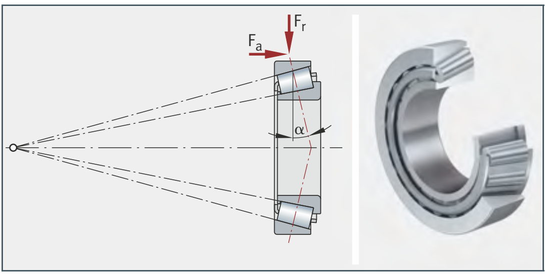


Figure 104
Single row tapered roller bearing, projected lines of contact of tapered rollers

α = contact angle

Load carrying capacity, adjustment, setting of clearance, preload

Single row tapered roller bearings can support radial loads, axial loads in one direction and combined loads (simultaneous radial and axial loads), while tapered roller bearings in an O or X arrangement can support high radial loads, axial loads in both directions and combined loads.

Due to the oblique position of the raceways, a single tapered roller bearing must not be subjected to pure radial load; an axial load or axial abutment must always be applied simultaneously. This is normally achieved by means of a second bearing mounted in a mirror image O or X arrangement.

In order to set the bearing clearance or preload, the bearings in bearing arrangements with two single row tapered roller bearings are adjusted against each other until the required value is achieved.

Nominal contact angle and axial load carrying capacity
The axial load carrying capacity is dependent on the nominal contact angle, i.e. the larger the angle, the higher the axial load to which the tapered roller bearing can be subjected. The size of the contact angle – and thus the magnitude of the axial load carrying capacity – is indicated by the bearing-specific value e in the product tables included in the rolling bearing catalogues.

Barrel roller bearings

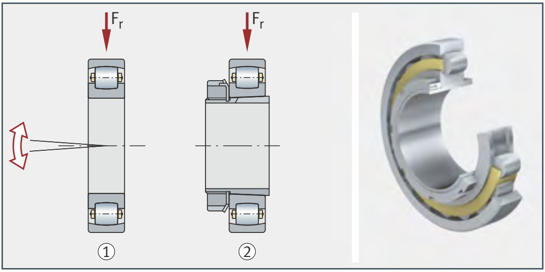
Barrel roller bearings are included in the group of self-aligning bearings, see Figure 105.

Barrel roller bearings have a concave outer ring raceway in the same way as self-aligning ball bearings and spherical roller bearings. As a result, the roller and cage assembly can align itself on the raceway by swivelling (in either a static or dynamic manner), such that the bearing can respond to misalignment and deflections of the shaft without problems.

The inner ring has a cylindrical or tapered bore and the bearings are not separable. They have only a low axial load carrying capacity.

Figure 105
Barrel roller bearings

① Bearing with cylindrical bore
② Bearing with tapered bore, adapter sleeve, tab washer and locknut

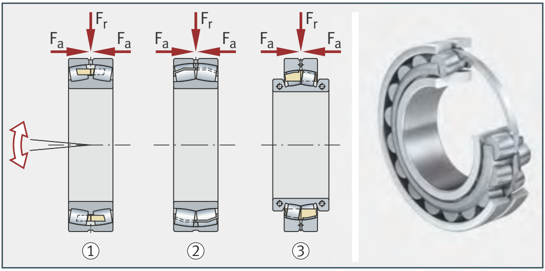


Spherical roller bearings

Spherical roller bearings have two rows of barrel rollers whose axes are inclined towards the rotational axis of the bearing, see Figure 106. The outer ring has a curved raceway in the same way as barrel roller bearings. The profile of the raceways is closely matched to the profile of the barrel rollers.

Figure 106
Spherical roller bearings

① Bearing with three rigid ribs on inner ring
② Spherical roller bearing E (increased capacity design without rigid ribs)
③ Axially split spherical roller bearing



For the supply of lubricant, the outer ring normally has a circumferential groove with radial holes between the rows of rollers. This gives improved lubrication of the bearing.

Load carrying capacity, dynamic compensation of angular misalignments
Spherical roller bearings have a very high radial load carrying capacity and a higher axial load carrying capacity than barrel roller bearings. Due to the concave design of the outer ring, the barrel rollers align themselves by swivelling on the outer ring raceway in the case of misalignments and shaft deflection.

In order to ensure problem-free operation, the bearings must be subjected to a minimum load. Values for this load are given in the Schaeffler rolling bearing catalogues.

Spherical roller bearings are used where high, shock type radial loads must be dynamically supported and misalignments or more pronounced deflections of the shaft are anticipated.

Toroidal roller bearings

Toroidal roller bearings have elongated barrel rollers and raceways that are matched to the lines of contact of the rolling elements, see Figure 107. The centre point of the outer ring raceway radius lies under the central axis of the bearing. The design of the bearing rings allows skewing between the rings within certain limits. At the same time, changes in the length of the shaft relative to the housing can be compensated.

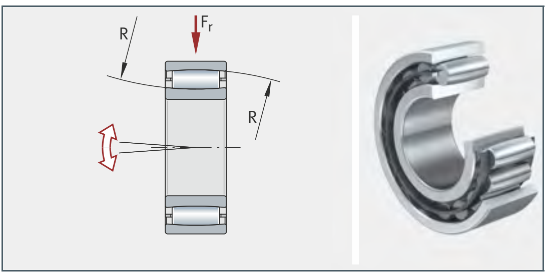
This design combines the dynamic self-alignment capacity of the spherical roller bearing with the axial displacement facility of cylindrical roller bearings. It is thus equally suitable for the dynamic and static compensation of misalignments.

Toroidal bearings are exclusively non-locating bearings and can support radial forces only.

The permissible skewing and axial displacement facility are restricted.

Figure 107
Toroidal roller bearing

R = raceway radius



Radial needle roller bearings

Radial needle roller bearings have a range of common characteristics, which are described in the following sections.

Needle roller bearings have needle rollers as rolling elements. In terms of rolling bearing technology, cylindrical rolling elements are classified as needle rollers if the rolling element diameter is $\leqq$ 6 mm and the ratio of the rolling element diameter to the rolling element length is $<$ 1:3.

Due to the small diameter of the rolling elements, all needle roller bearings have the common feature of a low radial section height. Due to the line contact, they are particularly suitable for bearing arrangements with high radial load carrying capacity and rigidity in a restricted radial design envelope.

Radial needle roller bearings can only be used as non-locating bearings. The defined axial displacement value in the case of bearings with inner rings allows axial motion between the shaft and housing. Where necessary, wider inner rings are available for larger displacement values.

In order to ensure slippage-free operation, a minimum radial load is necessary. This applies in particular to bearings running at high speeds since, if the radial load is not present, damaging sliding motion may occur between the rolling elements and raceways. Values for the minimum radial load are given in the Schaeffler rolling bearing catalogues.

Needle roller and cage assemblies

Apart from full complement needle roller sets, the needle roller and cage assembly is the simplest type of needle roller bearing, see Figure 108, Page 665. Needle roller and cage assemblies are of single or double row design and comprise a cage and needle rollers. Since they do not have an outer ring or inner ring, they run directly on the shaft and in the housing. The raceways must therefore be hardened and ground.

Since their radial section height corresponds directly to the diameter of the needle rollers, needle roller and cage assemblies allow bearing arrangements requiring only a very small radial design envelope. If the raceways are produced to high geometrical accuracy, high radial run-out accuracy can be achieved. The radial internal clearance is influenced by the shaft and housing tolerances as well as the grade of the needle rollers.

Needle roller and cage assemblies must be located axially by means of snap rings or an appropriate design of the adjacent construction with abutment shoulders.

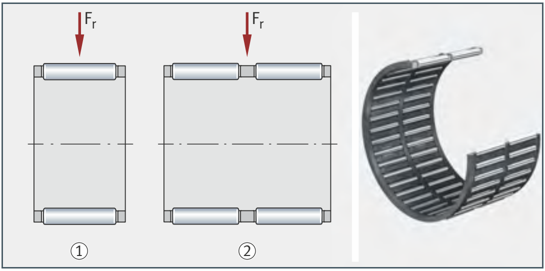


Figure 108
Needle roller and cage assemblies

1 Single row
2 Double row

Drawn cup needle roller bearings with open ends and with closed end

Drawn cup needle roller bearings with open ends and with closed end are needle roller bearings with a very small radial section height. They comprise thin-walled, drawn cup outer rings and needle roller and cage assemblies which together form a complete unit, see Figure 109.

The thin-walled outer rings adopt the dimensional and geometrical accuracy of the housing bore.

Drawn cup needle roller bearings are available either with both ends open or with one end closed. Drawn cup needle roller bearings with closed end are thus suitable for closing off the end of shafts in bearing positions. Where a rotating shaft is present, they give protection against injury, prevent the escape of lubricant and protect the rolling element system against contamination and moisture.

If the shaft cannot be produced as a raceway, the bearings can be combined with inner rings.

If axial locating elements such as shoulders and snap rings are not used, the housing bore can be produced easily and particularly economically. This also gives simpler mounting and dismounting of the bearings.

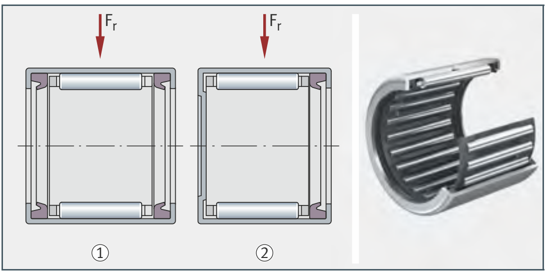


Figure 109
Drawn cup needle roller bearings with open ends and with closed end

① Drawn cup needle roller bearing with open ends, lip seal
② Drawn cup needle roller bearing with closed end, lip seal

Needle roller bearings

In comparison with drawn cup needle roller bearings with open ends and with closed end, the bearing rings of needle roller bearings are thicker, more rigid and are produced by machining methods. They place lower requirements on the dimensional accuracy, geometrical accuracy and hardness of the adjacent construction.

Needle roller bearings are subdivided into:

- needle roller bearings with ribs, with or without inner ring
- needle roller bearings without ribs, with or without inner ring
- aligning needle roller bearings
- combined needle roller bearings

Needle roller bearings with ribs
In these needle roller bearings, the outer ring and the needle roller and cage assembly form self-retaining, single or double row complete units, see Figure 110. The cage assembly is guided axially by ribs on the outer ring.

The bearings are available with and without a removable inner ring and in both sealed and open versions.

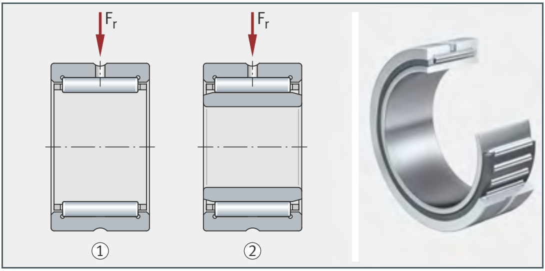


Figure 110
Needle roller bearings with ribs, single row

1 Without inner ring
2 With inner ring

Needle roller bearings without ribs
These complete units comprise outer rings without ribs, needle roller and cage assemblies and removable inner rings, see Figure 111.
Since the bearings are not self-retaining, the outer ring, needle roller and cage assembly and inner ring can be mounted independently of each other.

The bearings are available with and without an inner ring and in single and double row designs. The cage assembly is guided axially by thrust washers.

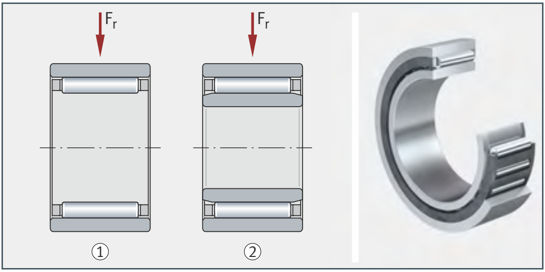


Figure 111
Needle roller bearings without ribs

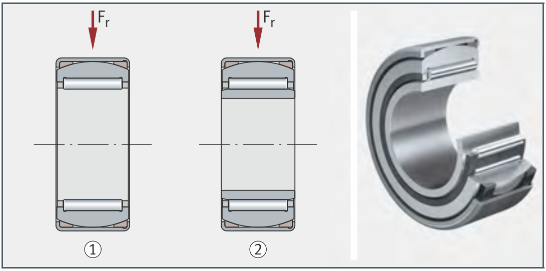
1 Without inner ring
2 With inner ring

Aligning needle roller bearings
Due to their internal construction, needle roller bearings of the types described previously only allow slight misalignment of the bearing axis. Aligning needle roller bearings have a raceway ring with a curved outside surface and a curved support ring and can compensate static misalignments of the bearing axis of up to 3°.

The bearings cannot support swivel or wobble type motion.

Figure 112
Aligning needle roller bearings

① Without inner ring
② With inner ring



Combined needle roller bearings

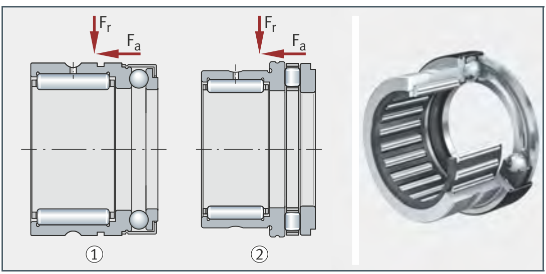
Radial needle roller bearings can support radial forces only. Combined needle roller bearings (radial needle roller bearings combined with a rolling element system capable of supporting axial loads) can additionally support axial forces.

Needle roller/axial deep groove ball bearings, needle roller/axial cylindrical roller bearings

Needle roller/axial ball bearings and needle roller/axial cylindrical roller bearings can support axial forces in one direction as well as high radial forces, see Figure 113. In order to support axial forces, the axial bearing component must be preloaded to 1% of the basic static axial load rating. Misalignments between the shaft and housing are not possible.

Figure 113
Needle roller/axial ball bearing, needle roller/axial cylindrical roller bearing

① Needle roller/axial deep groove ball bearing with end cap, without inner ring
② Needle roller/axial cylindrical roller bearing, without inner ring



Needle roller/angular contact ball bearings

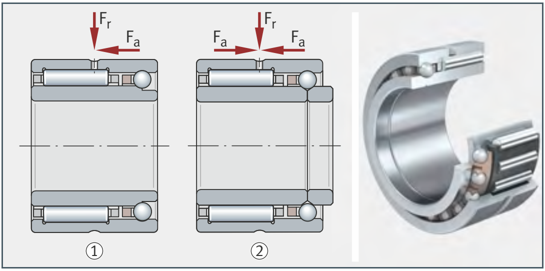
Needle roller/angular contact ball bearings can support axial forces in one or both directions, see Figure 114. Needle roller/angular contact ball bearings capable of supporting axial loads in one direction can be used as semi-locating bearings, while bearings capable of supporting axial loads in both directions can be used as semi-locating or locating bearings.

If the design capable of supporting axial loads in one direction is to support axial forces from alternating directions, two bearings must be adjusted against each other.

Where axial forces and misalignment between the shaft and housing are to be supported, the information in the section Needle roller/axial deep groove ball bearings, needle roller/axial cylindrical roller bearings must be observed.

Figure 114
Needle roller/angular contact ball bearings

① Capable of supporting axial loads in one direction, unsplit inner ring
② Capable of supporting axial loads in both directions, split inner ring



Bearings for screw drives

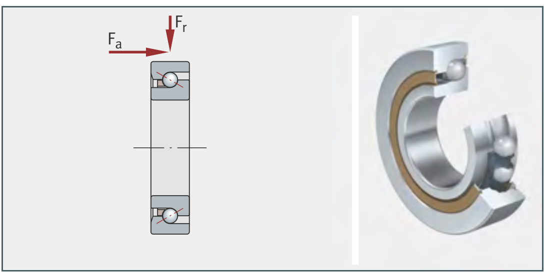
The essential requirements placed on bearings for screw drives are high axial run-out accuracy, axial and tilting rigidity as well as high load carrying capacity. The bearings that have proven effective in this area of application are single row and multi-row axial angular contact ball bearings and double direction needle roller/axial cylindrical roller bearings.

Single row axial angular contact ball bearings

The bearings can be universally combined in various bearing sets. Since the bearings are supplied already matched, a defined rolling bearing preload is achieved after mounting of the bearings that ensures clearance-free operation with high axial run-out accuracy and high axial rigidity. This bearing design can be used for a wide range of load requirements and is available in most cases in a greased and sealed design.

Single row axial angular contact ball bearings are particularly suitable for bearing arrangements for ball screw drives in machine tools, see Figure 115.

Figure 115
Axial angular contact ball bearing, single row



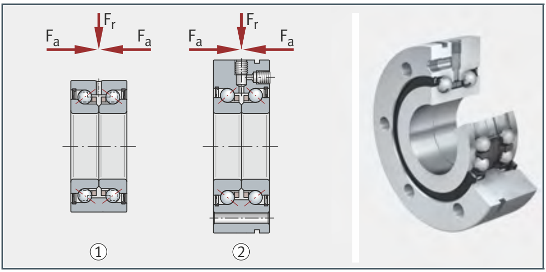
Double row axial angular contact ball bearings

Double row axial angular contact ball bearings are ready-to-fit, self-retaining, greased and sealed precision bearings in an O arrangement with a contact angle of 60°, see Figure 116. The bearing rings are matched to each other such that a defined preload is achieved when the rings are clamped in place using a precision locknut. The rolling element system is protected against contamination by means of contact seals on both sides of the bearing. For high speeds, minimal gap seals are available. Double row bearings are also available in matched pairs, so four-row bearing assemblies can be created. This gives an additional increase in the load carrying capacity and rigidity of the bearing arrangement.

The bearings are available with and without fixing holes in the outer ring, see Figure 116. Bearings with holes are screw mounted directly on the adjacent construction. This solution is particularly economical since there is then no need for the locating bore that would otherwise be required or for the bearing cover with the associated matching work.

Figure 116
Axial angular contact ball bearings, double row

① Double row, for mounting in locating bore
② Double row, for screw mounting to adjacent construction



Triple row axial angular contact ball bearings

Triple row designs have, in addition to two rows of balls with a 60° contact angle in an O arrangement, a third row of balls, see Figure 117, Page 671. This additional row allows higher loads in one direction. Due to the stepped outer ring, the bearings can be easily flange mounted on the adjacent construction. In order to reduce the radial design envelope required, the flange is flattened on two sides.

In order to make full use of the load carrying capacity, the bearings must be subjected continuously to load in the main load direction. They are therefore used mainly in screw drives with a locating/locating bearing arrangement and tensioned spindles or in vertically arranged screw drive bearing arrangements.

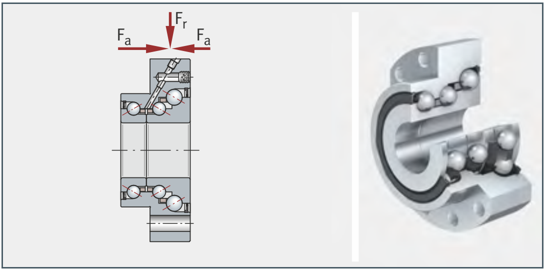


Figure 117
Axial angular contact ball bearing, triple row

For screw mounting to adjacent construction

Needle roller/axial cylindrical roller bearings

These bearings are double direction, precision cylindrical roller bearings with a radial bearing component and are not self-retaining. They comprise an outer ring with radial and axial raceways, a shaft locating washer, an inner ring, a radial needle roller and cage assembly and two axial cylindrical roller and cage assemblies, see Figure 118. The bearings are available in versions for screw mounting on the adjacent construction and for location in a housing bore. Since the outer ring can be screw mounted, there is no need for the cover that would otherwise be required and the associated matching work.

Needle roller/axial cylindrical roller bearings can support not only high radial forces but also axial forces in both directions and tilting moments. The outer ring, inner ring and axial cages are matched to each other such that the bearings are axially clearance-free after preloading by means of a precision locknut.

In comparison with axial angular contact ball bearings, they have higher load carrying capacity, rigidity and accuracy. For higher loads, bearings are available with a larger cross-section – and thus higher basic load ratings – as a heavy series.

If the axial abutment of the shaft locating washer is not sufficient or a seal raceway is required, bearings with a stepped shaft locating washer extended on one side are suitable, see Figure 118.

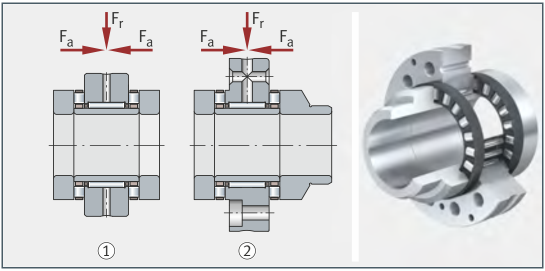


Figure 118
Needle roller/axial cylindrical roller bearings

1 For mounting in housing bores
2 For flange mounting, with stepped, extended shaft locating washer

Yoke type, stud type and ball bearing track rollers

Rotary rolling bearings are normally mounted in a housing bore. In this case, the outer ring supports the loads originating from the shaft and transmits these into the surrounding housing. In the case of yoke type, stud type and ball bearing track rollers, the outer ring runs freely on a flat or curved mating track (such as a rail, guideway or cam plate).

The characteristic feature of this rolling bearing type is the particularly thick-walled outer rings. These rings replace the housing and support deflections and stresses.

In practice, yoke type, stud type and ball bearing track rollers with a crowned outside surface are predominantly used, since tilting relative to the mating track often occurs and edge stresses must be avoided. The load carrying capacity of the bearing is increased by optimised profiles on the outer ring.

Yoke type track rollers

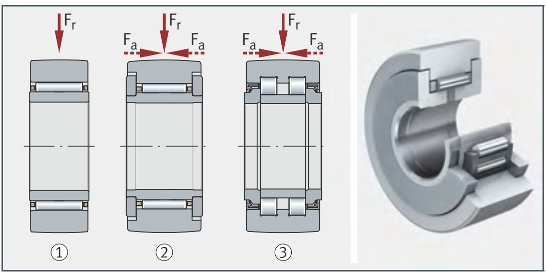
Yoke type track rollers are ready-to-fit needle or cylindrical roller bearings with a particularly thick-walled outer ring, see Figure 119. Yoke type track rollers are available with and without axial guidance of the outer ring and in both sealed and open versions. The outer rings have a crowned or cylindrical outside surface. They are mounted on shafts or studs and are supplied with or without an inner ring. They can support high radial loads. Yoke type track rollers with axial guidance can also tolerate axial loads resulting from slight misalignment defects, skewed running or brief contact running impacts.

Bearings without an inner ring require a raceway on the shaft or stud corresponding to the quality of a rolling bearing raceway.

The bearings are used in applications including cam gears, bedways, conveying equipment and linear guidance systems.

Figure 119
Yoke type track rollers

① Without axial guidance, with cage, open design
② With axial guidance, with cage, gap seals
③ With axial guidance, full complement roller set, lip seals



Stud type track rollers

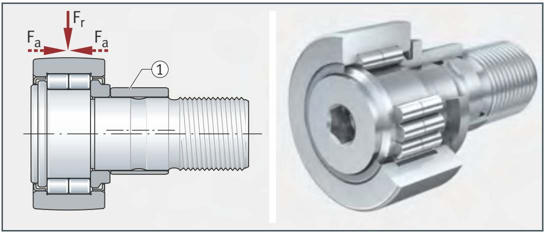
Stud type track rollers correspond in their design to yoke type track rollers with axial guidance but, in place of the inner ring, they have a heavy-section roller stud, see Figure 120. The stud has a fixing thread and, in many cases, a hexagonal socket on both sides for mounting of the stud type track rollers and adjustment in designs with an eccentric collar. The eccentric collar allows adjustment of the outer ring outside surface to match the mating track on the adjacent construction.

Stud type track rollers are available with various seals (such as labyrinth, gap or contact seals). The outer ring has a crowned or cylindrical outside surface.

Figure 120
Stud type track roller, double row, full complement roller set, with eccentric collar

Labyrinth seal

① Eccentric collar



Ball bearing track rollers

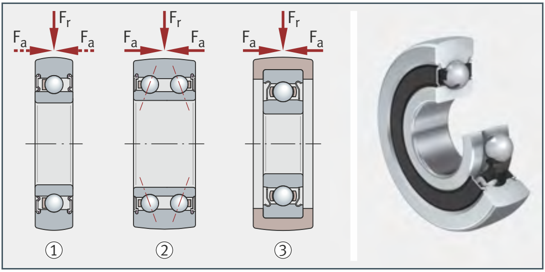
Ball bearing track rollers correspond in their design to sealed deep groove or angular contact ball bearings but have thick-walled outer rings with a crowned or cylindrical outside surface. They can support axial forces in both directions as well as high radial forces. Designs with a crowned outside surface are used where misalignments occur relative to the mating track.

Ball bearing track rollers are available with and without a stud, with a plastic tyre and with a profiled outer ring, see Figure 121.

Bearings without a stud are mounted on shafts or studs. Ball bearing track rollers with a plastic tyre can be used if particularly quiet running is required. Bearings with a profiled concave outer raceway are suitable for the design of robust shaft guidance systems, see Figure 147, Page 700.

Figure 121
Ball bearing track rollers without stud

① Single row, sealed
② Double row, sealed
③ With plastic tyre, sealed



Axial ball bearings

Axial ball bearings are pure axial bearings, which means that they may only be subjected to axial load.

Axial deep groove ball bearings

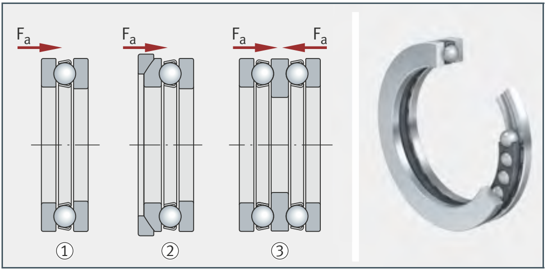
Axial deep groove ball bearings are single direction or double direction units that can support high axial forces and are not self-retaining.

Single direction bearings comprise a shaft locating washer and a housing locating washer between which a ball and cage assembly is arranged, see Figure 122, Page 674. They can support axial forces in one direction and support the shaft on one side.

Double direction bearings comprise a shaft locating washer, two housing locating washers and two ball and cage assemblies, see Figure 122. This design can support axial forces in both directions and can therefore guide the shaft on both sides.

Figure 122
Axial deep groove ball bearings

① Single direction
② Single direction, curved housing locating washer and support washer
③ Double direction



Angular adjustment facility
In addition to the series with flat housing locating washers, axial bearings with curved locating surfaces are also available for the compensation of static angular defects. These designs are normally used in conjunction with support washers and tolerate static misalignments of the shaft relative to the housing. They are not suitable, however, for wobble type motions of the shaft, since the friction on the curved locating surfaces is too high.

Axial angular contact ball bearings

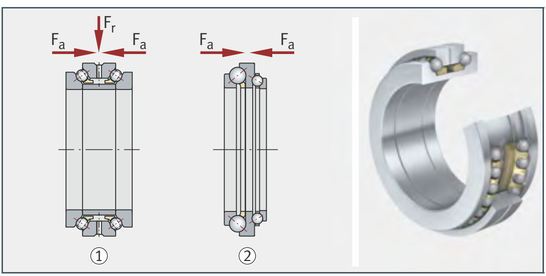
For axially rigid spindle bearing arrangements in machine tools, single row and double row axial angular contact ball bearings with increased accuracy are used, see Figure 123. The bearings can support axial forces in both directions.

Axial angular contact ball bearings transmit the forces from one raceway to the other by means of a defined contact angle. The ribs are sufficiently high that the contact ellipse does not contact or extend beyond the edge of the raceway even under the centrifugal force effect of high speeds and in axially preloaded bearings at high loads.

Figure 123
Axial angular contact ball bearings

Contact angle normally 60°

① Double direction, with spacer ring
② Double direction



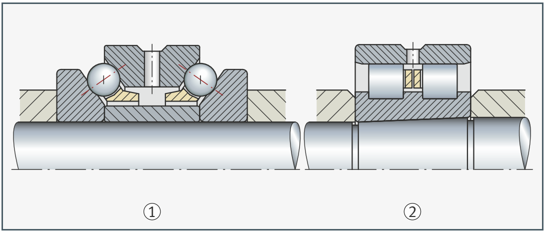
Double row axial angular contact ball bearings

Double row axial angular contact ball bearings have a ball-guided solid brass cage for each row of balls. A spacer ring is inserted between the shaft locating washers with a fit such that the bearing has the necessary preload once it is mounted and axially located.

In bearing arrangements for main spindles in machine tools, the radial forces are generally supported by a cylindrical roller bearing arranged adjacent to the axial angular contact ball bearing, see Figure 124.

Figure 124
Cylindrical roller bearing for support of radial forces

① Double direction axial angular contact ball bearing 2344
② Super precision cylindrical roller bearing NN30, double row



Axial roller bearings

Axial bearings of this type are single direction or double direction axial bearings based on cylindrical rollers or needle rollers.

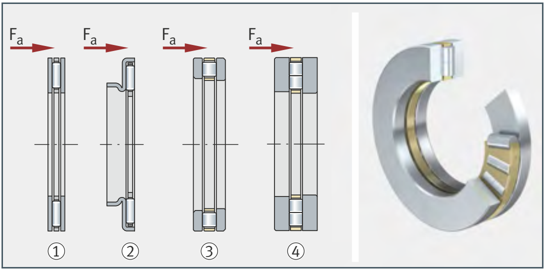
Axial cylindrical roller bearings, axial needle roller bearings

These axial bearings comprise flat, ribless washers between which axial needle roller and cage assemblies or axial cylindrical roller and cage assemblies are arranged, see Figure 125. Their axial section height corresponds to the diameter of the rolling elements plus the washer thickness values. They have high axial load carrying capacity, are extremely rigid and can support axial forces in one direction. The bearings can support axial forces only. Radial forces must be supported by an additional, suitable bearing.

For particularly low axial section heights, the cage assemblies can be integrated directly in the adjacent construction. The running surfaces for the rolling elements must therefore be produced as rolling bearing raceways.

Figure 125
Axial needle and axial cylindrical roller bearings

① Axial needle roller bearing
② Axial needle roller bearing with centring spigot
③ Axial cylindrical roller bearing, single row
④ Axial cylindrical roller bearing, double row



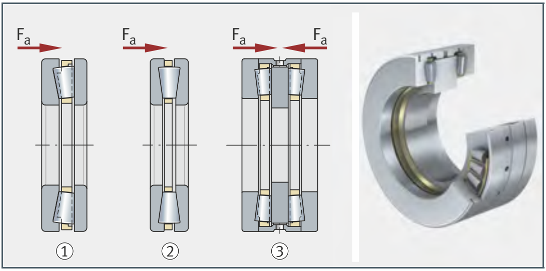
Axial cylindrical roller bearings and axial needle roller bearings are used where the load carrying capacity of axial deep groove ball bearings is not sufficient. Axial needle roller bearings are suitable for particularly low axial section heights, while axial cylindrical roller bearings can support higher loads. Misalignments between the shaft and housing are not permissible, but the bearings can align themselves radially.

Axial tapered roller bearings

The most frequently used type of axial tapered roller bearings has a shaft locating washer with a tapered raceway and a housing locating washer with a flat raceway, see Figure 126.

Figure 126
Axial tapered roller bearings

① Single direction, with flat housing locating washer
② Single direction, with two tapered raceways
③ Double direction



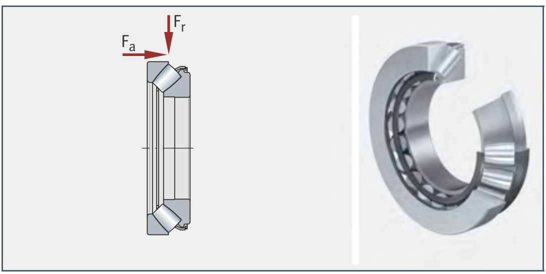
The rollers are guided by the rib of the shaft locating washer and are normally retained by a solid brass cage.

The bearings are used, for example, in rolling mill construction.

Axial spherical roller bearings

Axial spherical roller bearings comprise a housing locating washer, a shaft locating washer and asymmetrical barrel rollers, see Figure 127.

Figure 127
Axial spherical roller bearing



The barrel rollers abut the high rib of the inner ring and can align themselves on the curved raceway of the outer ring. As a result, the bearing is similar to other self-aligning bearings in being unaffected by misalignments and shaft deflections. The roller and cage assembly and the shaft locating washer are held together by solid brass cages or sheet steel cages.

Since the loads are transmitted from one raceway to the other at an angle inclined to the bearing axis, the bearings are also suitable for supporting additional radial loads ($F_{r\,max} = 0{,}55 \cdot F_a$) while an axial load is present.

In order to ensure problem-free operation, a minimum axial load is necessary. This applies particularly in the case of bearings running at high speeds with high accelerations and rapid load reversals. Due to the inertia forces of the rolling elements and cages, and the increasing frictional power, impermissible wear may occur between the rolling elements and raceways. In many cases (for example, in vertical bearing arrangements), the inherent mass of the bearing arrangement is sufficient, however, to apply the necessary minimum axial load.

Crossed roller bearings, dimension series 18

Crossed roller bearings of this type are open bearings for high precision applications, see Figure 128. The spacing between the cylindrical rollers is maintained by plastic spacers. The outer ring is split and is held together by retaining rings. The highly rigid bearings have high running accuracy and are supplied with normal clearance, low clearance or preload. The outer rings are fixed in the adjacent construction by means of clamping rings. Sealing of the bearing position can be freely designed as necessary.

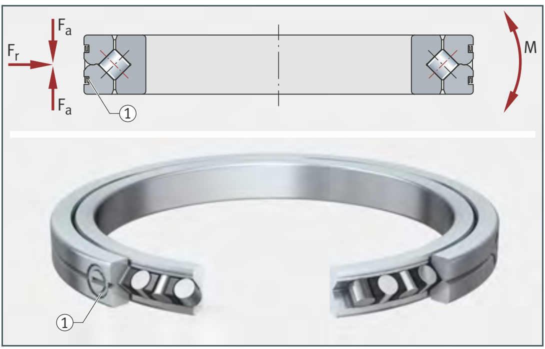
The dimensions of the crossed roller bearings conform to ISO dimension series 18 in accordance with DIN 616.

Due to the X arrangement of the cylindrical rollers, these bearings can support axial forces in both directions, radial forces and any combination of loads as well as tilting moments by means of a single bearing position. As a result, designs with two bearing positions can in many cases be reduced to a single bearing position.

Figure 128
Crossed roller bearing

F_r = radial load
F_a = axial load
M = tilting moment load

① Retaining ring



Swivel bearings (slewing rings)

Slewing rings are large size bearings with high load carrying capacity for oscillating and slow rotary motions, see Figure 129. The bearing rings are available without teeth, with internal teeth or with external teeth and are generally screw mounted directly to the immediate parts of the adjacent construction.

Swivel bearings are predominantly mounted in a horizontal position and are used to support axial forces and large tilting moments. In the applications, radial loads only occur to a subordinate extent.

Slewing rings are normally subjected to load during infrequent rotary motion, slow swivel motion, slow rotation, or while stationary, and are preferably dimensioned on the basis of their static load carrying capacity.

Four point contact bearings and crossed roller bearings

Swivel bearings can be designed in the form of ball or roller bearings. The cage comprises segments or spacers. These maintain the spacing between the rolling elements. In swivel bearings with a diameter of several metres, the rings are often split into segments for transport and mounting reasons. The raceways are subjected to induction or flame hardening. Figure 129 shows sealed four point contact bearings (designs without teeth and with internal teeth).

Crossed roller bearings can be used in joints for industrial robots and in bearing positions that make demands beyond the load carrying capacity, rigidity and accuracy of ball bearings. The rolling elements on the raceways comprise rollers arranged with a roller axis that is alternately offset by 90°. Crossed roller bearings have segment cages or a full complement of rollers. The inner or outer ring can be provided with teeth to facilitate drive solutions.

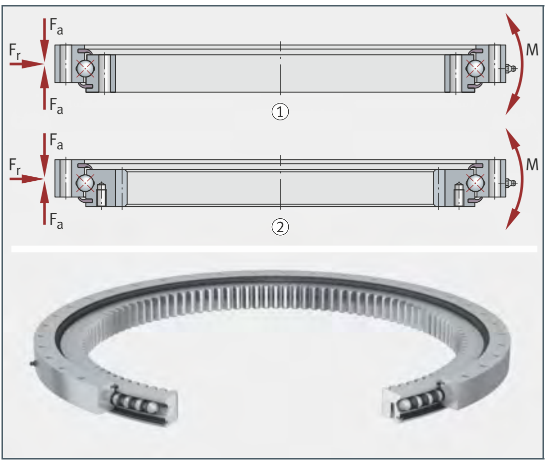


Figure 129
Four point contact bearings

M = tilting moment load

1 Without teeth, sealed
2 With internal teeth, sealed

Rotary table bearings (bearings for combined loads)

Rotary table bearings can support axial loads in both directions and radial loads as well as tilting moments without clearance. Due to the fixing holes in the bearing rings, mounting of the units is very simple. After mounting, the bearings are radially and axially preloaded. They are particularly suitable for bearing arrangements requiring high running accuracy, such as rotary tables, reversible clamps, face plates and milling heads.

Axial angular contact ball bearings

Axial angular contact ball bearings are ready-to-fit and greased bearing units with particularly low friction, with high accuracy for very high speeds, high axial and radial loads and high demands on tilting rigidity. They comprise a one-piece outer ring, a two-piece inner ring and two ball and cage assemblies, see Figure 130. The contact angle is 60°. The outer ring and inner ring have fixing holes for screw mounting of the bearing on the adjacent construction. The unit is secured by means of retaining screws for transport and safe handling.

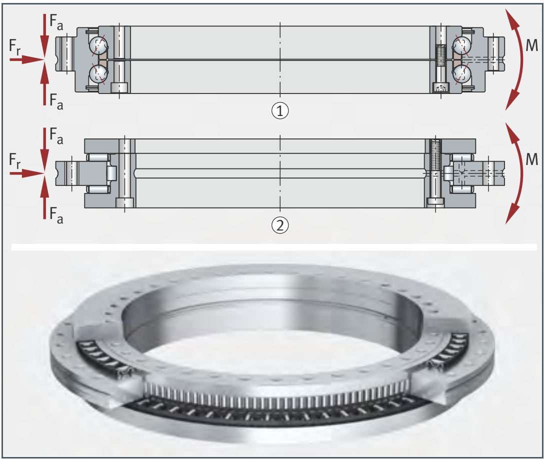
Axial/radial bearings

Axial/radial bearings are double direction axial bearings with a radial guidance bearing that are suitable for screw mounting, see Figure 130. These ready-to-fit, greased units are very rigid, have high load carrying capacity and run with particularly high accuracy. Designs with an integrated angular measuring system can measure the angular position of the rotary axis to an accuracy of a few angular seconds.

Figure 130
High precision bearings for combined loads

M = tilting moment load

① Axial angular contact ball bearing
② Axial/radial bearing



Guiding elements in a rotary direction – plain bearings

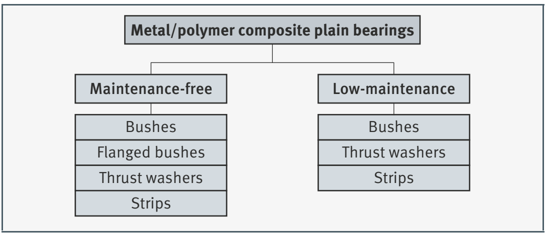
In contrast to rolling bearings, where low friction is achieved through the rolling of rolling elements between the raceways, plain bearings exhibit pure sliding motion between the shaft and bearing cup. Friction is minimised by the sliding contact surface, surface modification or suitable lubrication. While there is a wide range of different plain bearings, only dry running plain bearings are covered here in brief.

In addition to the metal/polymer composite plain bearings described below, the medium performance segment can be served using plain bearings with ELGOTEX and the upper performance segment using plain bearings with ELGOGLIDE.

Metal/polymer composite plain bearings

Plain bearings are bearings for very small radial and axial design envelopes. They have a steel or bronze backing and are available as bushes, flanged bushes, thrust washers and strips, see Figure 131 and Figure 132. Designs with a bronze backing are highly resistant to corrosion, have very good thermal conductivity and are antimagnetic.

Figure 131
Plain bearing types and designs – overview

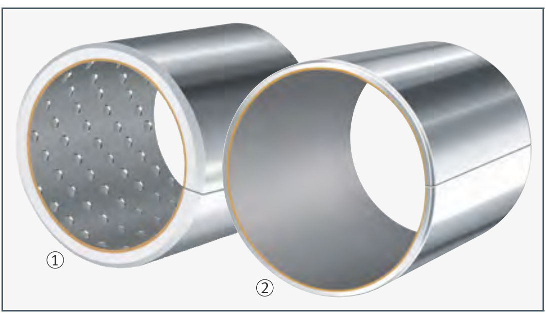


Materials E40, E40-B, E50

The material used in metal/polymer composite plain bearings is the maintenance-free E40 (with steel backing), E40-B (with bronze backing) or the low-maintenance E50. The dry lubricant is based on polytetrafluoroethylene PTFE with embedded chemically non-reactive additives. The materials fulfil the regulations on lead-free plain bearings and thus the Directive 2000/53/EC (End-Of-Life Vehicles Directive) and the Directive 2011/65/EU (RoHS-II).

Figure 132
Metal/polymer composite plain bearings (bush)

① E50, low-maintenance
② E40, maintenance-free

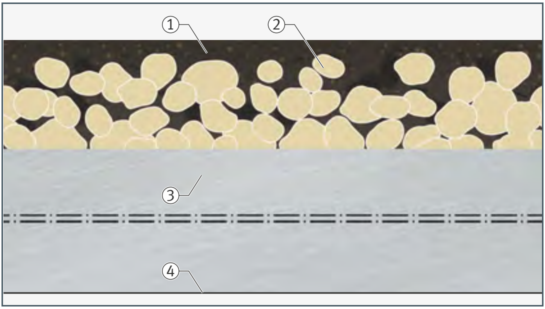


Maintenance-free plain bearing material

The maintenance-free plain bearing material E40, see Figure 133, is intended for dry running due to the use of PTFE as a dry lubricant. These bearings are thus particularly suitable where the bearing position must be maintenance-free, there is a risk of lubricant starvation or where lubricant is unacceptable or undesirable. The material E40 can be used not only for rotary and oscillating motion but also for short stroke linear motions. Typical areas of application can be found, for example, in fluid technology, in sports gear, in medical or electrical equipment as well as in automotive engineering.

Figure 133
Structure of maintenance-free plain bearing material E40 – three-layered, with steel backing

① Running-in layer
② Sliding layer
③ Steel backing
④ Tin layer as surface protection

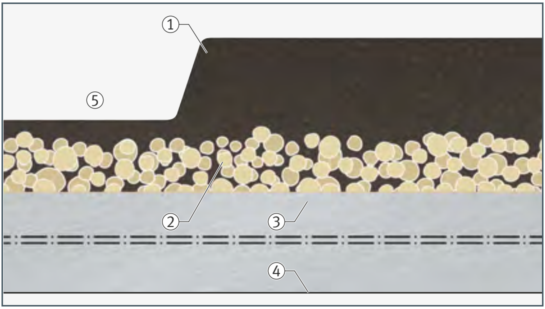


Low-maintenance plain bearing material

The low-maintenance plain bearing material E50, see Figure 134, is a low-wear material with good damping characteristics and long relubrication intervals. The sliding layer is made from polyoxymethylene POM. E50 can be used for rotary and oscillating motion and is recommended for long stroke linear motions. It is only slightly sensitive to edge loads and is insensitive to shocks. Application examples can be found in particular in the area of production machinery, construction and agricultural equipment as well as commercial vehicles.

Figure 134
Structure of low-maintenance plain bearing material E50 – three-layered, with steel backing

① Sliding layer
② Intermediate layer
③ Steel backing
④ Tin layer as surface protection
⑤ Lubrication pocket



Plain bearing range from Schaeffler

The complete range is described in the Schaeffler catalogue HG 1, Plain Bearings, in Technical Product Information TPI 211 and in the online version **medias**: *https://medias.schaeffler.com*.

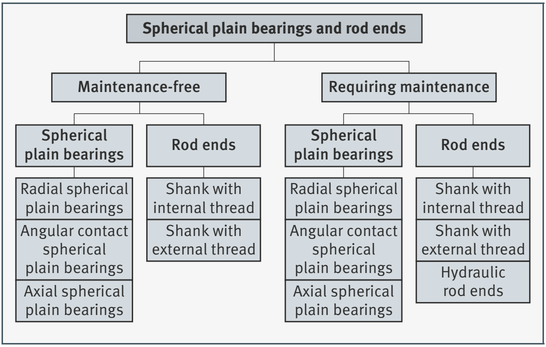
Guiding elements in a rotary direction – spherical plain bearings

Spherical plain bearings

Spherical plain bearings are ready-to-fit, standardised machine elements. Due to the concave outer ring bore and the curved inner ring geometry, they allow spatial adjustment motions. The bearings can support static loads and are suitable for tilting and swivel motion. They can compensate for shaft misalignment, are not subject to edge stresses under misalignment and permit substantial manufacturing tolerances in the adjacent construction.

An overview of the important types and designs is shown in Figure 135:

Figure 135
Types and designs of spherical plain bearings – overview



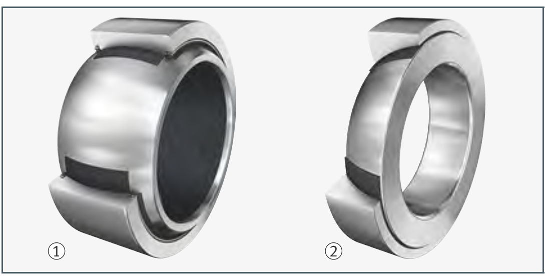
Maintenance-free spherical plain bearings

Maintenance-free spherical plain bearings are available as radial, axial and angular contact spherical plain bearings, see Figure 136. The sliding layer between the inner ring and outer ring is ELGOGLIDE, PTFE composite or PTFE film.

Radial spherical plain bearings are preferably used to support radial forces. Certain series are also suitable for alternating loads up to a contact pressure of $p = 300\ N/mm^2$. The bearings are used where particular requirements for operating life apply in conjunction with maintenance-free operation or where, for reasons of lubrication, bearings with metallic sliding contact surfaces are not suitable, for example under unilateral load. Radial spherical plain bearings are available in open and sealed designs.

Figure 136
Maintenance-free spherical plain bearings, sliding layer ELGOGLIDE

① Radial spherical plain bearing, lip seal on both sides
② Angular contact spherical plain bearing, open design



Angular contact spherical plain bearings conform to DIN ISO 12240-2. They have inner rings with a curved outer slideway and outer rings with a concave inner slideway to which an ELGOGLIDE sliding layer is attached by adhesive. The bearings can support radial and axial forces and are suitable for alternating dynamic loads. Preloaded units can be achieved using paired arrangements. Angular contact spherical plain bearings are used to support high loads in conjunction with small motions. In this case, they are an alternative to tapered roller bearings.

Axial spherical plain bearings conform to DIN ISO 12240-3. In these units, the shaft locating washer is supported in the ball socket-shaped sliding zone of the housing locating washer. The sliding material on the housing locating washer is ELGOGLIDE. The bearings are preferably used to support axial forces. They are suitable as support or base bearings and can also be combined with radial spherical plain bearings of dimension series E to DIN ISO 12240-1.

Spherical plain bearings requiring maintenance

Spherical plain bearings requiring maintenance can be designed as radial, axial and angular contact spherical plain bearings and must be lubricated using oil or grease via the outer or inner ring or the housing locating washer as appropriate. The bearings comprise inner rings and outer rings with a steel/steel or steel/bronze sliding contact surface. The inner rings have a cylindrical bore and a curved outer slideway. The outer rings have a cylindrical outside surface and a concave inner slideway.

Radial spherical plain bearings can support radial forces, transmit motion and loads with low moment levels and thus keep bending stresses away from the construction elements. They are particularly suitable for alternating loads with impact and shock type stresses and support axial loads in both directions. The bearings are available in open and sealed designs.

Angular contact spherical plain bearings conform to DIN ISO 12240-2. The sliding contact surface is steel/steel. For a further description, see section Maintenance-free spherical plain bearings, Page 682, and Schaeffler catalogue HG 1, Plain Bearings.

Axial spherical plain bearings conform to DIN ISO 12240-3. The sliding contact surface is steel/steel. For a further description, see section Maintenance-free spherical plain bearings, Page 682, and Schaeffler catalogue HG 1, Plain Bearings.

Rod ends

Rod ends are spherical plain bearing units. They comprise a housing and integral shank, into which a spherical plain bearing is integrated, and have an external or internal thread. They are used as connecting levers and connecting rods and as connecting elements between cylinders and their adjacent parts in hydraulic and pneumatic cylinders.

Spherical plain bearing range from Schaeffler

The complete range is described in the Schaeffler catalogue HG 1, Plain Bearings, and in the online version **medias**: *https://medias.schaeffler.com*.

Guiding elements in a translational direction – linear rolling element guidance systems

Linear rolling element guidance systems, also known as linear guidance systems, are translational guidance systems. They are based on the principle of the rolling of rolling elements (balls, rollers, needle rollers) between moving guidance elements.

Such guidance systems ensure the translation with particularly low friction of one or more movable subassemblies while maintaining a direction of motion on a linear track (profiled guideway, guideway, cylindrical shaft).

Linear guidance systems are responsible for the guidance and transmission of force between machine parts moving in a translational direction and exert a substantial influence on the performance capability and accuracy of a machine.

An overview of common linear rolling element guidance systems is shown in Figure 137:

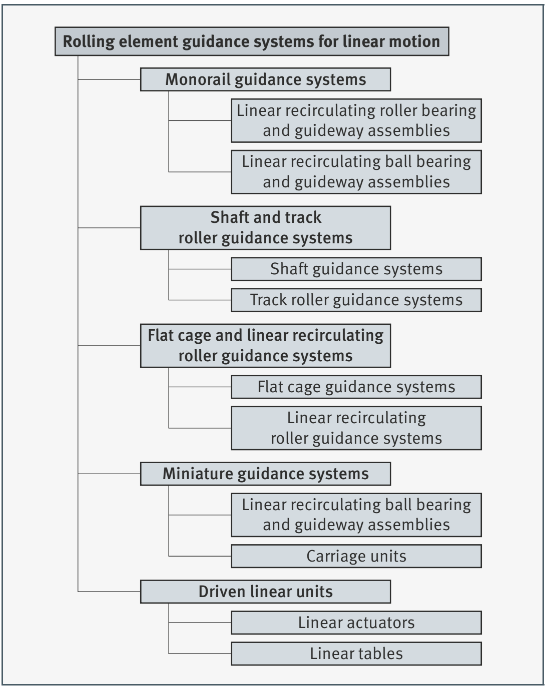


Figure 137
Rolling element guidance systems for linear motion – overview

Differentiation of linear rolling element guidance systems

Linear rolling element guidance systems are available in numerous designs, for example:

- flat cage guidance system
- linear roller bearing
- monorail guidance system (linear recirculating ball bearing unit, linear recirculating roller or ball guidance system)
- shaft guidance system
- track roller guidance system
- miniature guidance system
- driven linear unit

Guidance systems with balls or rollers contain balls or rollers running between the moving component and stationary component, while track roller guidance systems contain profiled track rollers supported by rolling bearings.

Linear rolling element guidance systems can be differentiated in terms of:

- the type of rolling element motion (with/without recirculation of the rolling elements)
- the type of rolling contact on the raceways (point contact or line contact)

Linear rolling element guidance systems without rolling element recirculation

In the case of linear rolling element guidance systems without rolling element recirculation, the rolling elements are guided between the moving table and the stationary guideway in a normally rigid cage. The stroke length of the moving component is therefore restricted by the difference in length between the table and the rolling element cage (based on the motion of the cage relative to the two raceways).

Flat cage guidance systems

Flat cage guidance systems are linear bearings without rolling element recirculation. The rolling elements move at half the velocity of the table and thus cover only half the distance.

Areas of application

Due to their design, flat cage guidance systems are particularly suitable for oscillating motion and where linear locating or non-locating bearings with extremely high load carrying capacity, restricted stroke length and very smooth running are required. Bearing arrangements with these guidance elements have high rigidity, high accuracy, low friction and a smaller design envelope relative to comparable linear guidance systems.

Guideway/cage combinations

Flat cage guidance systems comprise a pair of guideways between which angled needle roller flat cages, angled cylindrical roller flat cages or needle roller flat cages are arranged. The guideways are available in various profiles, examples of which are shown in M/V and J/S combinations in Figure 138.

Guideways with an adjusting gib are suitable for setting of preload, and guideways with an integral toothed rack for positive control of the angled flat cage are suitable where there is a risk of cage creep.

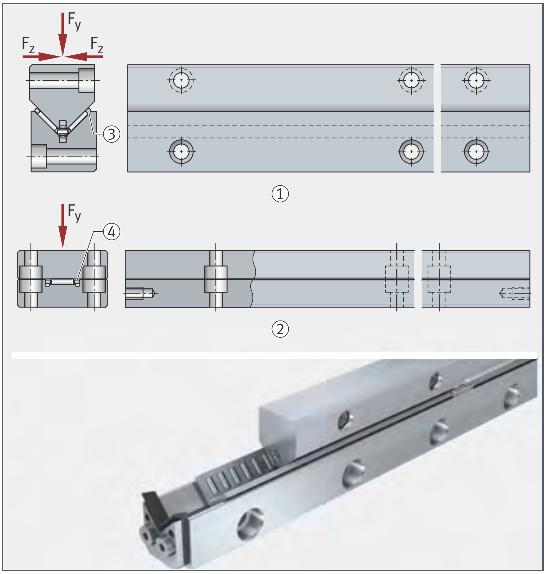
End pieces at the ends of the guideways hold the cage in its nominal position at the ends of the stroke length to prevent the cage from creeping out of the load zone.

The cages have a basic component made from light metal, steel, brass or plastic. A large number of rolling elements are guided by precise cage pockets.

Figure 138
Flat cage guidance systems

F_y, F_z = load directions

① M/V guideways
② J/S guideways
③ Angled needle roller flat cage
④ Needle roller flat cage



Linear rolling element guidance systems with rolling element recirculation

In the case of guidance systems with rolling element recirculation, the rolling elements in the carriage are recirculated by means of channels and special return elements, see Figure 139. This does not restrict the stroke length of these guidance systems, which is generally limited by the length of the guideways.

Areas of application

Such guidance systems are designed on the basis of balls or rollers and intended for applications with unlimited stroke length. This functional principle applies to linear roller bearings and monorail guidance systems.

Linear roller bearings with guideways

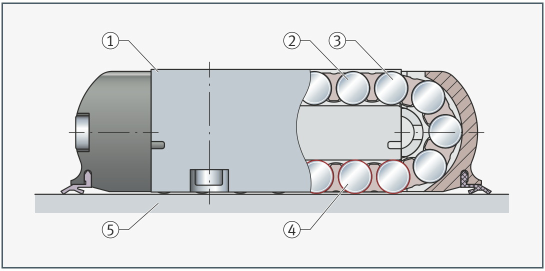
These linear guidance systems with very high load carrying capacity for linear locating/non-locating bearing arrangements comprise linear roller bearings with cylindrical rollers and guideways that have up to four raceways for the linear roller bearings, see Figure 139. Guideways with four raceways can support forces in the main load direction, together with forces in the opposing direction if a counterstay is fitted, as well as lateral forces in two directions.

In a closed arrangement, they can support forces from all directions and moments about the axis. They run with high precision, have low friction and allow compact designs.

In a preloaded design, guidance systems with linear roller bearings achieve extremely high rigidity values. Adjusting gibs are suitable for preloading. These transmit the defined values uniformly to the whole length of the linear roller bearing.

Figure 139
Linear roller bearing with rolling element recirculation

① Carriage
② Rolling element
③ Rolling element in return zone
④ Rolling element in load zone
⑤ Guideway



Monorail guidance systems

Monorail guidance systems are among the most important designs of linear rolling element guidance system. As high performance components, they are almost indispensable in general machine building.

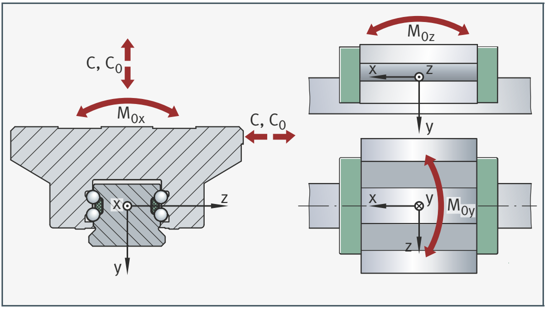
Dimensioning – load carrying capacity and life

The size of a monorail guidance system is determined by the demands made on its load carrying capacity, life and operational security.

Load carrying capacity

The load carrying capacity is described in terms of the basic dynamic load rating C, the basic static load rating C_0 and the static moment ratings M_{0x}, M_{0y} and M_{0z}, see Figure 140.

Figure 140
Load carrying capacity and load directions



Calculation of basic load ratings according to ISO

The calculation of the basic dynamic and static load ratings given in the product tables of Schaeffler catalogues is based on DIN ISO 14728-1 and 2. The information on the basic dynamic load rating C in the product tables corresponds to the basic dynamic load rating C_{100}.

Suppliers from the Far East frequently calculate basic load ratings using a basic rating life based on a displacement distance of only 50 km in contrast to 100 km in accordance with DIN.

The basic load ratings C_{50}, C_{100} for linear recirculating ball bearing and guideway assemblies can be converted using the following equations:

Equation 74

$$C_{50} = 1{,}26 \cdot C_{100}$$

Equation 75

$$C_{100} = 0{,}79 \cdot C_{50}$$

For linear recirculating roller bearing and guideway assemblies, conversion is as follows:

Equation 76

$$C_{50} = 1{,}23 \cdot C_{100}$$

Equation 77

$$C_{100} = 0{,}81 \cdot C_{50}$$

Legend

C_{100} N
Basic dynamic load rating C for displacement distance of 100 km – definition according to DIN ISO 14728-1

C_{50} N
Basic dynamic load rating C for displacement distance of 50 km – definition according to DIN ISO 14728-1.

Dynamic load carrying capacity and life

The dynamic load carrying capacity is described in terms of the basic dynamic load rating and the basic rating life. The basic dynamic load rating is the load in N at which the guidance system achieves a displacement distance of 100 km (C_{100}) with a survival probability of 90%.

Basic rating life

The basic rating life L and L_h is achieved or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue occurs (for an explanation of the symbols used, see Page 691).

Equation 78

$$L = \left(\frac{C_{100}}{P}\right)^{p} \cdot 100$$

Equation 79

$$L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C_{100}}{P}\right)^{p}$$

Equation 80

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C_{100}}{P}\right)^{p}$$

According to DIN ISO 14728-1, the equivalent dynamic load P should not exceed the value 0,5 · C.

Equivalent load and velocity

The equations for calculating the basic rating life assume that the load P and the velocity v_m are constant. Non-constant operating conditions can be taken into consideration by means of equivalent operating values. These have the same effect as the loads occurring in practice.

Equivalent dynamic load

Where the load varies in steps and the velocity varies in steps, the equivalent dynamic load is calculated as follows:

Equation 81

$$P = \sqrt[p]{\frac{q_1 \cdot v_1 \cdot F_1^p + q_2 \cdot v_2 \cdot F_2^p + \ldots + q_z \cdot v_z \cdot F_z^p}{q_1 \cdot v_1 + q_2 \cdot v_2 + \ldots + q_z \cdot v_z}}$$

Mean velocity

Where the velocity varies in steps, the mean velocity is calculated as follows:

Equation 82

$$v_m = \frac{q_1 \cdot v_1 + q_2 \cdot v_2 + \ldots + q_z \cdot v_z}{100}$$

Combined load

If the direction of the load acting on an element does not coincide with one of the main load directions, an approximate value for the equivalent load is calculated as follows:

Equation 83

$$P = |F_y| + |F_z|$$

If an element is simultaneously subjected to a force F and a moment M, an approximate value for the equivalent dynamic load is calculated as follows:

Equation 84

$$P = |F| + |M| \cdot \frac{C_0}{M_0}$$

Symbols, units and definitions

The following values are used in calculation of the equivalent load and velocity as well as the basic rating life:

Legend

C_{100}	N	M_0	Nm
Basic dynamic load rating C for displacement distance of 100 km – definition according to DIN ISO 14728-1		Static moment rating	
C_0	N	n_{osc}	min^{-1}
Basic static load rating in the direction of the force acting on the element		Number of return strokes per minute	
F	N	P	N
Force acting on the element		Equivalent dynamic bearing load	
F_y	N	p	–
Vertical component		Life exponent: Monorail guidance systems based on balls: p = 3 Monorail guidance systems based on rollers: p = 10/3	
F_z	N		
Horizontal component			
H	m	q_z	%
Single stroke length for oscillating motion		Duration as a proportion of the total operating time	
L, L_h	km, h	v_z	m/min
Basic rating life for a distance of 100 km or in operating hours		Variable velocity	
M	Nm	v_m	m/min
Acting moment		Mean velocity.	

Operating life

The operating life is defined as the life actually achieved by monorail guidance systems. It may differ significantly from the calculated life.

The following influences can lead to premature failure through wear or fatigue:

- excess load due to misalignment as a result of temperature differences and manufacturing tolerances (elasticity of the adjacent construction)
- contamination of the guidance systems
- inadequate lubrication
- oscillating motion with very small stroke length (false brinelling)
- vibration while stationary (false brinelling)
- overloading of the guidance system (even for short periods)
- plastic deformation

Static load carrying capacity

The static load carrying capacity of the monorail guidance system is limited by:

- the permissible load on the monorail guidance system
- the load carrying capacity of the raceway
- the permissible load on the screw connection between the upper and lower components
- the permissible load on the adjacent construction

For design purposes, the static load safety factor S_0 required for the application must be observed, see tables starting Page 693.

Basic static load ratings and moment ratings

The basic static load ratings and moment ratings are those loads under which the raceways and rolling elements undergo permanent deformation equivalent to one tenth of a thousandth of the rolling element diameter.

Static load safety factor

The static load safety factor S_0 is the security against permanent deformation at the rolling contact:

Equation 85

$$S_0 = \frac{C_0}{P_0}$$

Equation 86

$$S_0 = \frac{M_0}{M}$$

Legend

S_0 –
Static load safety factor

C_0 N
Basic static load rating in the load direction

P_0 N
Equivalent static bearing load in the load direction

M_0 Nm
Static moment rating in the load direction (M_{0x}, M_{0y}, M_{0z}) according to the product tables in the Schaeffler catalogues

M Nm
Equivalent static moment rating in the load direction.

The equivalent static bearing load is determined in approximate terms from the maximum loads:

Equation 87

$$P_0 = F_{max}$$

Equation 88

$$M_0 = M_{max}$$

Application-oriented static load safety factor
In machine tool applications, the static load safety factor S_0 is in accordance with the following tables. The data given in the tables are subject to the precondition that the specifications for the strength of the connection as given in Schaeffler catalogues are fulfilled.

Preconditions	S_0
Critical case ■ High dynamic loading with one axis stationary ■ Severe contamination ■ Actual load parameters are not defined.	8 to 12
Normal case ■ Not all load parameters are completely known. or: ■ Loads are estimated from the performance data of the machine.	5 to 8
■ All load parameters are known.	4 to 5
■ All load parameters are known (and definitely correspond to reality).	3 to 4

In machine tool applications, safety factors of $S_0 > 10$ are normally required for reasons of rigidity. For the precise design of the guidance system, Schaeffler offers ➔ Bearinx-online or design by the “Schaeffler Technology Center” in conjunction with Application Engineering facilities. In the precise design process, the displacement of the tool point can also be analysed.

In general applications, the static load safety factor S_0 is in accordance with the following table:

Preconditions	S_0
■ Predominantly oscillating load with stationary guidance system	20
■ All load parameters are completely known, running is smooth and free from vibrations.	3 to 4

In general applications with a suspended arrangement (where a suspended arrangement is present, a drop guard is recommended), S_0 is in accordance with the following table:

Preconditions	S_0
■ Not all load parameters are known and a coherent weight is supported by fewer than 4 carriages.	20
■ Not all load parameters are known and a coherent weight is supported by at least 4 carriages. or: ■ All load parameters are known and a coherent weight is supported by fewer than 4 carriages.	8 to 12
■ All load parameters are known and a coherent weight is supported by at least 4 carriages.	5 to 8

Fracture strength of guidance systems

If the fixing screw threads are of a sufficient size, monorail guidance systems can be subjected to loads up to the static load carrying capacity C_0 and M_0 according to the product tables.

The load must be transmitted via locating surfaces. The basic load ratings can only be achieved when utilising the full thread lengths.

Lubrication – oil or grease lubrication

Monorail guidance systems must be lubricated. Technical, economic and ecological factors will determine whether oil or grease should be used and which lubrication method should be applied.

A significant factor in selecting the type of lubrication is the environmental conditions (such as contamination) acting on the guidance system.

Suitable lubricants

Monorail guidance systems are supplied with preservative or with initial greasing. The guidance systems run exclusively under mixed friction conditions. As a result, doped lubricants (type P to DIN 51502) should be used in preference.

Design of a monorail guidance system

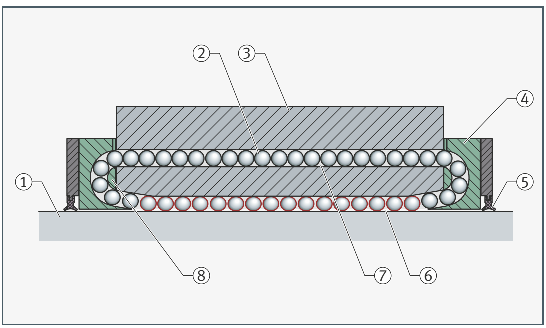
Monorail guidance systems comprise one or more carriages running on a profiled guideway. Depending on the type of rolling element, monorail guidance systems are differentiated into linear recirculating ball bearing and guideway assemblies or linear recirculating roller bearing and guideway assemblies.

The technical characteristics and area of application of the monorail guidance system are determined by the number, arrangement and contact geometry of the rolling element rows.

The carriage comprises the functional components of the saddle plate, end pieces, rolling elements, end seals and sealing strips, see Figure 141. The rolling elements in the carriage are guided by a rolling element recirculation system comprising a forward section and reverse section.

Figure 141
Monorail guidance system (linear recirculating ball guidance system)

① Guideway with raceway profile
② Rolling elements (balls)
③ Saddle plate (steel part)
④ End piece with outer return device
⑤ End seal
⑥ Load zone (raceway)
⑦ Recirculation channel
⑧ Inner return device



The saddle plates and guideways are made from hardened rolling bearing steel and have ground raceways. The rolling elements are in point or line contact with the guideway and carriage (depending on the rolling element type).

The end pieces of the carriage contain return devices that direct the rolling elements from the forward section into the reverse section. They also support the end seals.

The rolling element system is protected against contamination by sealing elements. The guidance systems are lubricated via lubrication connectors in the end pieces and/or lubrication pockets within the carriages.

Nominal contact angle

The load carrying capacity and rigidity of the monorail guidance system are influenced by the arrangement of the rolling element raceways. The raceways and contact points are therefore arranged at a specific angle, the nominal contact angle. The nominal contact angle specifies the direction of force flow relative to the horizontal plane of the guidance system and is stated for loads in the main load direction. In two-row and four-row linear recirculating ball bearing and guideway assemblies, it is normally 45°.

Arrangement of rolling element rows – X or O arrangement

Linear recirculating ball bearing and guideway assemblies can be designed with 2, 4 or 6 rows of rolling elements. Linear recirculating roller bearing and guideway assemblies predominantly have 4 rows of rolling elements.

Four-row systems are constructed in an X or O arrangement, see Figure 142, and table Monorail guidance system, Page 696. Guidance systems in an O arrangement have a higher moment rigidity about the guidance system axis than systems in an X arrangement.

In the case of linear recirculating roller bearing and guideway assemblies, the cylindrical rollers are in an X or O arrangement on the raceways.

Two-row linear recirculating ball bearing and guideway assemblies have an O arrangement, four-row systems an X or O arrangement and six-row systems an X and O arrangement.

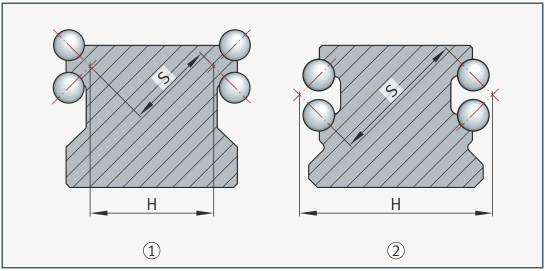
The higher the rigidity of the guidance system, the greater the effect of very small tilting, as a result of inaccuracies in the adjacent construction for example, in exerting very high internal constraining forces on the rolling elements.

Units in an X arrangement permit greater skewing about the guidance system axis. The rigidity in a compressive, tensile and lateral direction is not influenced by the X or O arrangement.

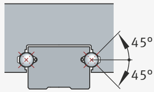
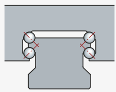
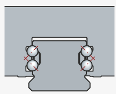
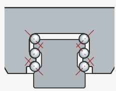
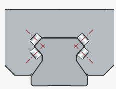
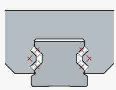
Figure 142
X or O arrangement

H = support distance
S = lever arm

① X arrangement
② O arrangement



Typical arrangements of rolling elements and contact geometries in monorail guidance systems are shown in the following table. This presents a selection of commonly available monorail guidance systems.

Monorail guidance system	Rolling contact	Number of rolling element rows	Arrangement
	Ball, 4 point	2	O
	Ball, 2 point	4	X
	Ball, 2 point	4	O
	Ball, 2 point	6	X and O
	Roller, line	4	X
	Roller, line	4	O

Running accuracy/ accuracy classes

Monorail guidance systems are divided into various accuracy classes. The specific class is defined by the different tolerances for the maximum deviations in height and lateral dimensions.

The accuracy requirements placed on the linear elements used increase with the requirements for the accuracy of an application. The accuracy class is determined by the application conditions of the guidance system.

Preload Monorail guidance systems are predominantly preloaded. As a result, significantly greater rigidity is achieved than in clearance-free systems (increasing preload leads to an increase in static rigidity). The preload is induced by specific sorting of the rolling elements with oversize and the spring rate at the rolling contact is set.

When selecting the preload class, it must be taken into consideration that a high preload induces additional loads on the rolling element set and leads to a reduction in the basic rating life. It must therefore be critically assessed whether it is always advisable to select very high preload classes.

Rigidity of monorail guidance systems Rigidity is an important characteristic of monorail guidance systems. The rigidity is dependent on the type and size of the guidance system.

Rigidity is defined as the ratio of load to deflection. A distinction is drawn between the compressive, tensile and lateral rigidity of a guidance system.

Factors influencing the rigidity The rigidity is influenced by:

- the rolling element type (ball or roller)
- the arrangement of the rolling elements (number of rows, nominal contact angle)
- the osculation
- the carriage design (normal, long, low, narrow, high)
- the size of the guidance system (size 5 to 100)
- the load direction (compressive, tensile, lateral load)
- the preload class

Friction The frictional force F_R is influenced by the load, preload, travel velocity, design of the rolling element recirculation channels, lubricant (quantity and viscosity), temperature, misalignment and sliding motion components of the seals. It is the product of the normal force F_N and the friction coefficient μ. The friction coefficient is dependent on the system used (ball or roller guidance system) and differs in magnitude.

Linear recirculating roller bearing and guideway assemblies The units have a full complement roller set, see Figure 143, Page 698. Since they have the maximum possible number of rolling elements, bearings with a full complement of rollers have extremely high load carrying capacity and particularly high rigidity.

Linear recirculating roller bearing and guideway assemblies are used wherever linear guidance systems must support extremely heavy loads, where particularly high rigidity is required and where very precise travel is also necessary. In preloaded form, they are particularly suitable for machine tools.

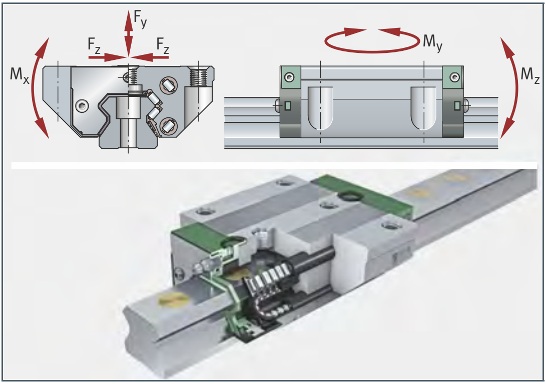


Figure 143
Full complement linear recirculating roller bearing and guideway assembly

F_y, F_z = load-bearing component in y and z direction
M_x, M_y, M_z = moment about x, y and z axis

Two-row linear recirculating ball bearing and guideway assemblies

These linear recirculating ball bearing and guideway assemblies have two rows of balls and a full complement ball rolling element system, see Figure 144. The rolling elements are in four point contact with the raceways.

Since the load carrying capacity and rigidity are lower than that of the other linear recirculating ball bearing and guideway assemblies, they are used where there are lower requirements for load carrying capacity and rigidity of the guidance system. Two-row units can be used to achieve economical linear guidance systems in the lower and medium range of load carrying capacity.

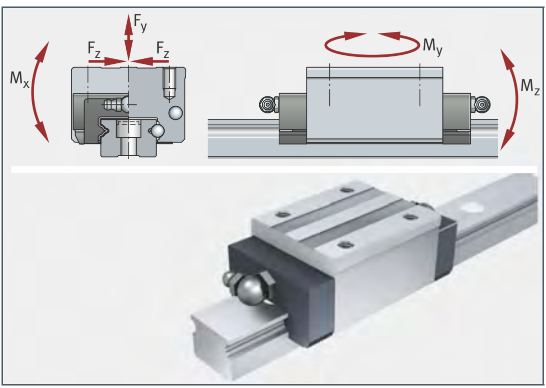


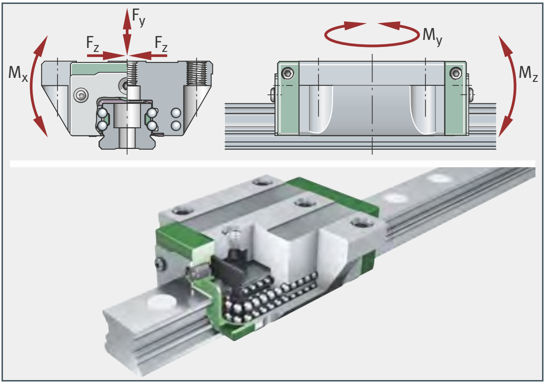
Figure 144
Two-row linear recirculating ball bearing and guideway assembly

Four-row linear recirculating ball bearing and guideway assemblies

Four-row linear recirculating ball bearing and guideway assemblies with a full complement ball set represent the most extensive and complex group within the range of monorail guidance systems, see Figure 145. Since they have the maximum possible number of rolling elements, bearings with a full complement of balls have extremely high load carrying capacity and particularly high rigidity.

Linear recirculating ball bearing and guideway assemblies are used where linear guidance systems with high load carrying capacity and rigidity must move heavy loads with high running and positional accuracy as well as low friction. The guidance systems are preloaded and – depending on the application – can be used at accelerations up to 150 m/s^2 and velocities up to 360 m/min.

Figure 145
Four-row linear recirculating ball bearing and guideway assembly

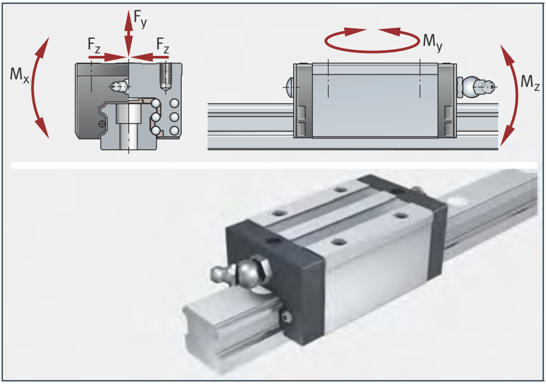


Six-row linear recirculating ball bearing and guideway assemblies

The rolling elements are in two point contact with the raceways, see Figure 146, Page 700. Four outer rows of balls support compressive loads while the inner rows of balls support tensile loads and all the rows support lateral loads. The guidance systems are preloaded in order to increase their rigidity.

Due to their six rows of balls, these recirculating guidance systems are the ball monorail guidance systems with the highest load carrying capacity and rigidity.

Figure 146
Six-row linear recirculating ball bearing and guideway assembly



Track roller guidance systems

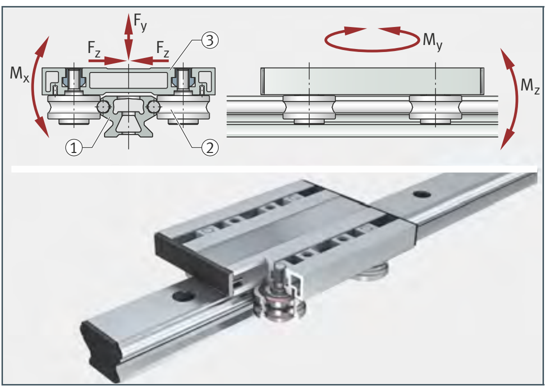
Track roller guidance systems in accordance with Figure 147 are highly suitable, due to their lightweight construction, for applications in handling systems where low-noise running, high velocities and long displacement distances are required together with low, uniform displacement resistance. The guidance systems can support forces from all directions, except in the direction of motion, and moments about all axes (in the case of track roller guidance systems with non-locating bearing carriages, the load directions are restricted).

The system elements, namely carriages, composite guideways and track rollers, can be combined to achieve economical designs that are precisely matched to the application.

Figure 147
Track roller guidance system

F_y, F_z = load-bearing component in y and z direction
M_x, M_y, M_z = moment about x, y and z axis

① Guideway with hollow section profile
② Profiled track roller
③ Hollow section carriage



The carriages are available as lightweight hollow section carriages, open carriages for high performance guidance systems of a simple design, closed carriages for guidance systems in contaminated environments, non-locating bearing carriages for locating and non-locating bearing applications with two guideways in parallel and as bogie carriages for curved tracks or guidance systems in the form of closed oval or circular tracks.

Composite guideways are available as solid and hollow section guideways, with a support rail of high bending rigidity, as a half guideway, a curved guideway element or a flat type. Guideways are also available with slots for toothed racks or toothed belts.

Profiled track rollers are used for the guidance of carriages and support of forces. These double row angular contact ball bearings have an outer ring with a gothic arch profile raceway, are sealed on both sides and are greased for life. They can support axial loads from both sides and high radial forces due to the thick-walled outer ring.

Under static loading, both the permissible radial load of the bearing and the permissible radial load of the mating track must be taken into consideration.

In order to ensure that the outer ring is driven, that no slippage occurs and that the track roller does not lift from the mating track, the track rollers must be subjected to a minimum load in dynamic operation. Values are indicated in the Schaeffler product catalogues.

Dimensioning – load carrying capacity and life

The loads present in track roller guidance systems differ from those in supported, rotary rolling bearings. When calculating their load carrying capacity, additional parameters must therefore be taken into consideration.

Permissible radial loads

The thick-walled outer rings of the track rollers can support high radial loads. If these track rollers are used against a shaft as a raceway, the outer rings undergo elastic deformation.

Compared to rolling bearings supported in a housing bore, track rollers have the following characteristics:

- Modified load distribution in the bearing.
  This is taken into consideration by means of the basic load ratings C_{rw} and C_{0rw} that play a definitive role in rating life calculation.
- Bending stress in the outer ring.
  This is taken into consideration by means of the permissible radial loads $F_{r\,per}$ and $F_{0r\,per}$. The bending stresses must not exceed the permissible strength values of the material (due to the risk of fracture).

Permissible radial load under dynamic loading

For rotating bearings under dynamic load, the effective basic dynamic load rating C_{rw} is used. C_{rw} is used to calculate the basic rating life. The permissible dynamic radial load $F_{r\,per}$ must not be exceeded. If the basic static load rating C_{0rw} is lower than the basic dynamic load rating C_{rw}, then C_{0rw} applies.

Permissible radial load under static loading

For bearings under static load, when stationary or with only infrequent motion, the effective basic static load rating C_{0rw} is used. The value C_{0rw} is used to calculate the static load safety factor S_0. The permissible static radial load $F_{0r\,per}$ must not be exceeded. In addition to the permissible radial load of the bearing, the permissible radial load of the mating track must also be taken into consideration. The basic load ratings stated are valid only in conjunction with a shaft as a mating track that is hardened (to at least 670 HV) and ground (Ra = 0,3 μm).

Fatigue limit load

The fatigue limit load C_{urw} is defined as the load below which – under laboratory conditions – no fatigue occurs in the material.

Calculation of the rating life

The general methods for calculating the rating life are:

- the basic rating life in accordance with DIN ISO 281
- the adjusted rating life in accordance with DIN ISO 281
- the expanded calculation of the adjusted reference rating life in accordance with DIN ISO 281-4

These methods are described in the Schaeffler catalogues and the Technical Pocket Guide STT, see Page 703.

Rating life of track rollers

In comparison with the Schaeffler catalogue data, the following values must be replaced:

- $C_r = C_{rw}$
- $C_{0r} = C_{0rw}$
- $C_{ur} = C_{urw}$

The carriages LFCL, LFL..-SF, LFLL, LFKL and the bogie carriage LFDL contain four track rollers LFR.

The equivalent principle applies here. The corresponding parameters are taken into consideration in the basic load ratings C_y, C_{0y}, C_z, C_{0z} and the permissible moment ratings M_{0x}, M_{0y} and M_{0z}.

Legend

C_y N
Basic dynamic load rating in y direction

C_{0y} N
Basic static load rating in y direction

C_z N
Basic dynamic load rating in z direction

C_{0z} N
Basic static load rating in z direction

M_{0x} Nm
Static moment rating about x axis

M_{0y} Nm
Static moment rating about y axis

M_{0z} Nm
Static moment rating about z axis.

In the case of track rollers with a profiled outer ring, calculation is carried out exclusively by means of the basic rating life in accordance with DIN ISO 281.

Equations for the basic rating life

Equation 89

$$L_s = 0{,}0314 \cdot D_a \left(\frac{C_{rw}}{P_r}\right)^p$$

Equation 90

$$L_h = 26{,}18 \cdot \frac{D_a}{H \cdot n_{osc}} \left(\frac{C_{rw}}{P_r}\right)^p \qquad L_h = 52{,}36 \cdot \frac{D_a}{v_m} \left(\frac{C_{rw}}{P_r}\right)^p$$

Rating life for carriages with four track rollers

Equation 91

$$L_s = \left(\frac{C_y, C_z}{P}\right)^p$$

Equation 92

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C_y, C_z}{P}\right)^p \qquad L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C_y, C_z}{P}\right)^p$$

Legend

L_s — 10^5 m
Basic rating life in 10^5 metres

L_h — h
Basic rating life in operating hours

C_{rw}, C_y, C_z — N
Effective basic dynamic load rating

P_r — N
Equivalent dynamic bearing load (radial load)

P — N
Equivalent dynamic bearing load in corresponding load direction

D_a — mm
Rolling contact diameter of track roller, see product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems

H — m
Single stroke length for oscillating motion

n_{osc} — min^{-1}
Number of return strokes per minute

v_m — m/min
Mean travel velocity

p — –
Ball: p = 3;
Needle roller (non-locating bearing track roller or carriage): p = 10/3.

Operating life

The operating life is defined as the life actually achieved by a track roller. It may differ significantly from the calculated life.

This may be due to wear or fatigue as a result of:

- deviations in the operating data
- insufficient or excessive operating clearance (roller, guideway)
- contamination, inadequate lubrication, operating temperature too high or too low
- overloading of the guidance system
- vibration stress – false brinelling; oscillating bearing motion with very small stroke length, which can lead to false brinelling.
- very high shock loads
- prior damage during installation

Due to the variety of possible mounting and operating conditions, the operating life cannot be precisely calculated in advance. The most reliable way of arriving at a close estimate is by comparison with similar applications.

Static load safety factor

The parameter for the static load is the static load safety factor S_0. This indicates the security against impermissible permanent deformations in the bearing and is determined by means of the following equation:

Equation 93

$$S_0 = \frac{C_{0rw}}{F_{0r}}$$

Static load safety factor for carriages with four track rollers

Equation 94

$$S_0 = \frac{C_{0r}}{F_0} \qquad S_0 = \frac{M_0}{M}$$

Legend

S_0 N
Static load safety factor

C_{0rw} N
Effective radial basic static load rating of track roller, in accordance with product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems

F_{0r} N
Static force acting in a radial direction

C_{0r} N
Basic static load rating in accordance with product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems

F_0 N
Static force acting in y and z direction

M_0 Nm
Permissible static moment rating in x, y, z direction

M Nm
Moment acting in load direction (M_x, M_y, M_z).

At a static load safety factor of $S_0 < 4$, track rollers are regarded as highly loaded. For applications with normal operating conditions, a value $S_0 > 4$ is required.

When using individual track rollers, for example in conjunction with guideways, the permissible load of the guideway should be taken as decisive where necessary.

Static load safety factors $S_0 < 1$ cause plastic deformation of the rolling elements and the raceway, which can impair smooth running. This is only permissible for bearings with small rotary motions or in secondary applications.

Minimum load

In order to ensure that the outer ring is driven, that no slippage occurs and that the track roller does not lift from the mating track, the track rollers must be subjected to a minimum load in dynamic operation.

In general, the ratio for the minimum load is $C_{0rw}/F_r < 60$.

Lower hardness of raceway

If shafts with a lower surface hardness are used (such as X46, X90), a hardness factor (reduction factor) must be applied, see following equations and Figure 148, Page 706.

Equation 95

$$C_H = f_H \cdot C$$

Equation 96

$$C_{0H} = f_{H0} \cdot C_0$$

Legend

C N
Basic dynamic load rating

C_0 N
Basic static load rating

C_H N
Effective basic dynamic load rating

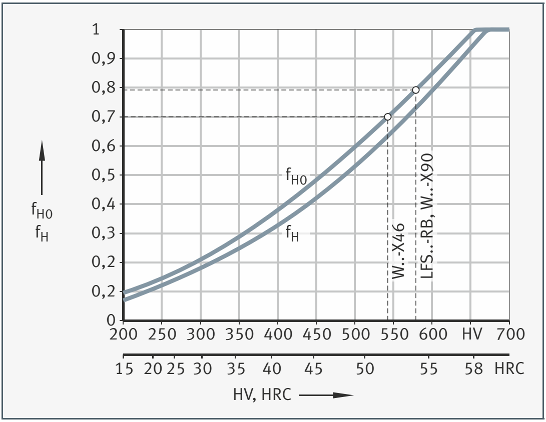
C_{0H} N
Effective basic static load rating

f_H –
Dynamic hardness factor (reduction factor), see Figure 148, Page 706

f_{H0} –
Static hardness factor (reduction factor), see Figure 148, Page 706.

Figure 148
Static and dynamic hardness factors for lower hardness of raceways

f_{H0} = static hardness factor
f_H = dynamic hardness factor
HV, HRC = surface hardness



Lubrication of raceways

The guideway raceways must be lubricated (even before first use). Lubrication can be carried out by means of lubrication and wiper units.

These units are already integrated in the compact carriage LFKL..-SF. For carriages LFL..-SF and LFCL, the lubrication and wiper unit AB is available as an accessory.

The guideway raceway is lubricated by an oil-soaked felt insert. Oil can be fed to the felt inserts via lubrication nipples in the end faces. At delivery, the felt inserts are already soaked with oil (H1 approval for the food industry). For relubrication, an oil of viscosity $\nu = 460\ \text{mm}^2/\text{s}$ is recommended.

Lubrication intervals

The lubrication intervals for guideway raceways are dependent on the environmental influences. The cleaner the environment, the smaller the quantity of lubricant required. The time and quantity can only be determined precisely under operating conditions since it is not possible to determine all the influences by calculation. An observation period of adequate length must be allowed.

Fretting corrosion is a consequence of inadequate lubrication and is visible as a reddish discolouration of the mating track or outer ring. Inadequate lubrication can lead to permanent system damage and therefore to failure of the linear unit. It must be ensured that the lubrication intervals are reduced accordingly in order to prevent fretting corrosion.

In general, a thin film of oil should always be present on the guideway.

Lubrication of track rollers

Track rollers LFR have an initial greasing of a high quality lithium soap grease. Track rollers of smaller diameters are lubricated for life.

Design of bearing arrangements

The accuracy of the guidance system as achieved by the manufacturer can only be properly utilised if the adjacent construction fulfils certain requirements.

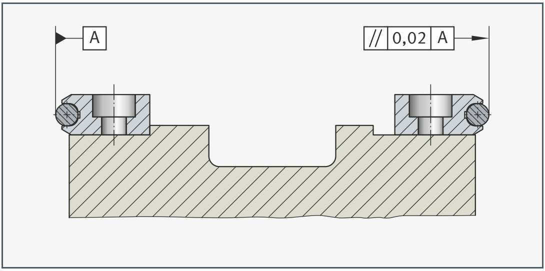
Requirements for the adjacent construction

The running accuracy of the linear guidance system is essentially dependent on the straightness, accuracy and rigidity of the mounting surfaces.

The higher the requirements for accuracy and smooth running of a track roller guidance system, the more attention must be paid to the geometrical and positional accuracy of the adjacent construction. The adjacent surfaces should be flat and have parallel faces.

If two guideways are present, parallelism in accordance with Figure 149 is recommended.

Figure 149
Parallelism of guideways



Displacement force

The displacement force is dependent on the preload, the lubrication and the particular application.

Location of carriages and guideways

If lateral loads are present, it is recommended that the guideways and carriages should be located against locating surfaces. In the case of guideways comprising multiple sections joined together, it is recommended that the guideways should be aligned by means of the shaft. If necessary, the guideways should be located on the adjacent construction by means of dowels.

If two guideways are arranged in parallel, the first guideway should be clamped against a stop, see Figure 149. The second guideway should then be aligned accordingly. Any gaps between the guideway and the adjacent construction should be filled with resin.

Shaft guidance systems, linear ball bearings

Shaft and round guidance systems with linear ball bearings are among the oldest guidance systems based on rolling elements. Such guidance systems comprise a hardened and ground shaft and one or more low-friction linear bearings, see Figure 150.

In shaft guidance systems, linear ball bearings are used for the support and transmission of forces. These bearings can support high radial loads while having a relatively low mass and allow the construction of linear guidance systems with unlimited travel.

The shaft is generally mounted on a support rail. The shafts can be either solid or hollow shafts, while the support rails are solid. Bearings and units are available in various series (light, heavy duty, compact, etc.).

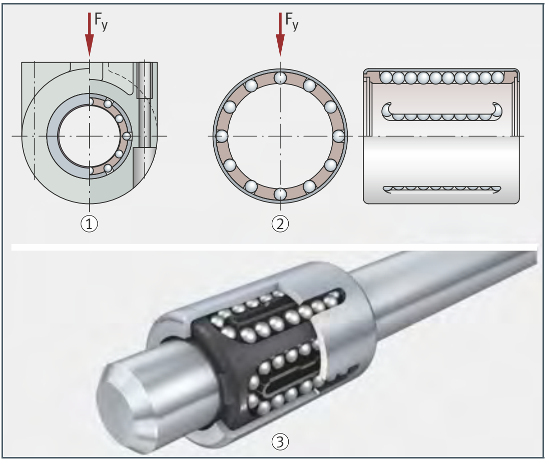


Figure 150
Shaft guidance system, closed housing

F_y = load-bearing component in y direction

① Linear ball bearing in closed housing
② Closed linear ball bearing
③ Shaft guidance system with linear ball bearing and solid shaft

Linear ball bearings

The structure of linear ball bearings is shown in Figure 151. While rotary ball bearings perform rotary motion, linear ball bearings run back and forth on the shaft as linear motion elements. The unlimited stroke length and return of the balls from the loaded zone to the unloaded zone is facilitated by the cage.

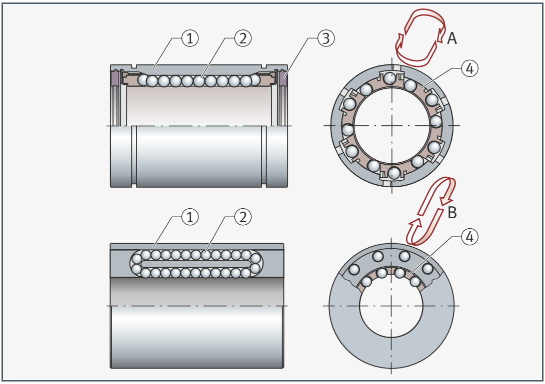


Figure 151
Linear ball bearings with tangential and radial return

A = tangential return
B = radial return

1 Steel sleeve or load plates
2 Balls made from rolling bearing steel
3 Sealing rings
4 Plastic or steel cage

Linear ball bearings without compensation of shaft deflection and misalignments

Linear ball bearings of series 1 in accordance with ISO 10285 are also designated, due to the very small radial section height, as sleeve type linear motion ball bearings, see Figure 152 1, Page 710. They comprise a formed and hardened outer sleeve with an integrated plastic cage. The balls undergo return travel along the openings in the outer sleeve.

Linear ball bearings of series 3 in accordance with ISO 10285 have larger radial section heights than series 1. The best known is the machined series, the so-called linear ball bearing with a solid housing, see Figure 152 2, Page 710.

Linear ball bushings comprise solid, ground and hardened outer rings with a plastic cage in which the rolling elements circulate.

Sleeve type linear ball bearings and linear ball bushings of these designs cannot compensate for deflections and misalignments of the shaft or misalignments of the bearings. This must be taken into consideration in the design of bearing arrangements.

Linear ball bearings with compensation of shaft deflection and misalignments

Linear ball bearings of ISO series 3 (linear ball bushings with single row adjustable load plate) have several load plate segments distributed around the circumference arranged such that they can swivel in an axial direction, see Figure 152 ③. This facilitates self-alignment of the bearing by up to ±30 angular minutes. Each load plate has a ball raceway that is deflected in each case by a return channel made from plastic in the stationary part of the housing.

The segments are supported centrally on a retaining ring. Their common contact point is also the centre point of the rocking motion.

Linear ball bearings of this design are intended for moderate loads.

The linear ball bearing design in ISO series 3 with the highest load carrying capacity allows self-alignment by up to ±40 angular minutes by means of several segments arranged around the circumference that themselves constitute independent linear bearings, see Figure 153, Page 711. Each segment has its own housing sections, return areas and recirculation channels. In contrast to the design described above for moderate loads, the segments in this case have two rows of balls, see Figure 152 ④.

In contrast to conventional linear ball bearings, the self-alignment function has the advantage that, even if misalignments are present, constraining forces are prevented from acting on the bearing and thus reducing the rating life. Due to the self-alignment function, the balls run without difficulty into the load zone. At the same time, load is more uniformly distributed over the entire ball row. This leads to smoother running, allows higher accelerations and prevents overloading of the individual balls.

Figure 152
Cross-sections of the type series in accordance with ISO 10285

① Series 1, sleeve type linear ball bearing
② Series 3, machined linear ball bushing
③ Series 3, linear ball bushing for moderate load and self-alignment
④ Series 3, linear ball bushing for very high load carrying capacity and self-alignment

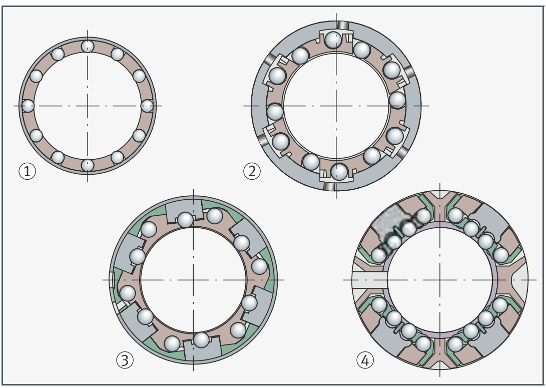
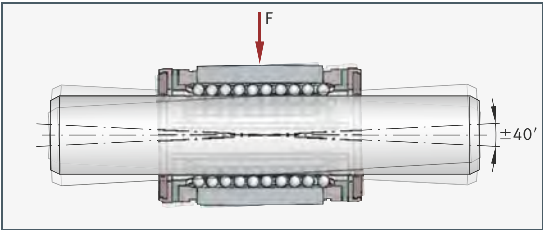


Figure 153
Angular compensation (self-alignment) in linear ball bushing, ISO series 3

Angular compensation
±40 angular minutes
F = load



Open and closed design of linear ball bearings
Linear ball bearings of all type series are available in open and closed designs. While the closed variant completely encloses the circumference of the shaft, the open design covers only a part thereof. In the case of the open design, a recess (segment cutout) allows underpinning or support of the shaft in this area. With the aid of this support, it is possible to prevent – particularly at higher operating loads – sagging of the shaft. Open bearings are designed for applications incorporating a supported shaft.

In addition to the designs for supported shafts, complete housing units are also available for open and closed bearings. In this case, the bearing is integrated in a strong, rigid housing. The housings are available in closed, open, slotted and tandem versions. Due to their low total mass, the units are particularly suitable for reduced mass designs with high loads and where higher accelerations and travel velocities are required.

Dimensioning – load carrying capacity and life

The size of a linear ball bearing is determined by the demands made in terms of load carrying capacity, rating life and operational security.

The load carrying capacity is described in terms of:

- the basic dynamic load rating C
- the basic static load rating C_0

The calculation of the basic dynamic and static load ratings given in the product tables of Schaeffler catalogues is based on DIN ISO 14728-1 and 2.

Basic rating life
The basic rating life L is reached or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue occurs.

Equation 97

$$L = \left(\frac{C}{P}\right)^3$$

Equation 98

$$L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C}{P}\right)^3$$

Equation 99

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C}{P}\right)^3$$

Legend

L — 10^5 m
Basic rating life L in 10^5 m

L_h — h
Basic rating life in operating hours

C — N
Basic dynamic load rating

P — N
Equivalent dynamic bearing load

H — m
Single stroke length

n_{osc} — min^{-1}
Number of return strokes per minute

v_m — m/min
Mean travel velocity.

Operating life

The operating life is defined as the life actually achieved by a shaft guidance system. It may differ significantly from the calculated life.

The following influences can lead to premature failure through wear or fatigue:

- misalignment between the shafts or guidance elements
- contamination
- inadequate lubrication
- oscillating motion with very small stroke length (false brinelling)
- vibration during stoppage (false brinelling)

Due to the wide variety of mounting and operating conditions, it is not possible to precisely predetermine the operating life of a shaft guidance system. The safest way to arrive at an appropriate estimate of the operating life is comparison with similar applications.

Static load safety factor

The static load safety factor S_0 indicates the security against impermissible permanent deformations in the bearing and is determined in accordance with the following equation:

Equation 100

$$S_0 = \frac{C_0}{P_0}$$

Legend

S_0	–	C_0	N
Static load safety factor		Basic static load rating.	
P_0	N		
Equivalent static load			

For linear ball bearings KH and KN..-B, the value must be $S_0 \geqq 4$.

In relation to guidance accuracy and smooth running, the value $S_0 \geqq 2$ is regarded as permissible. If $S_0 < 2$, please contact us.

Influence of the shaft raceway on basic load ratings

The basic load ratings are only valid if a ground (Ra = 0,3 μm) and hardened shaft (at least 670 HV) is provided as a raceway.

Lower hardness of raceway

If shafts with a surface hardness lower than 670 HV are used (such as shafts made from X46 or X90), a hardness factor (reduction factor) must be applied, see Figure 148, Page 706:

Equation 101

$$C_H = f_H \cdot C$$

Equation 102

$$C_{0H} = f_{H0} \cdot C_0$$

Legend

C	N	C_{0H}	N
Basic dynamic load rating		Effective basic static load rating	
C_0	N	f_H	–
Basic static load rating		Dynamic hardness factor (reduction factor), see Figure 148, Page 706	
C_H	N	f_{H0}	–
Effective basic dynamic load rating		Static hardness factor (reduction factor), see Figure 148, Page 706.	

Load direction and orientation of the ball rows

The effective load rating of a linear ball bearing is dependent on the position of the load direction in relation to the orientation of the ball rows:

- The lowest basic load rating C_{min} and $C_{0\ min}$ will occur in the apex orientation, see Figure 154.
- The highest basic load rating C_{max} and $C_{0\ max}$ will occur in the symmetrical orientation, see Figure 154.

If the bearings are mounted in correct alignment, the maximum basic load rating can be used. If mounting in correct alignment is not possible or the load direction is not defined, the minimum basic load ratings must be assumed.

Main load direction

For linear ball bearings and linear ball bearing and housing units, where the fitting position of the ball rows is defined, the basic load ratings C and C_0 in the main load direction are stated, see Figure 155. For other load directions, the effective basic load ratings can be determined using the load direction factors in the Schaeffler catalogue WF 1, Shaft Guidance Systems.

If the mounting position of the ball rows is not defined, the minimum basic load ratings are stated.

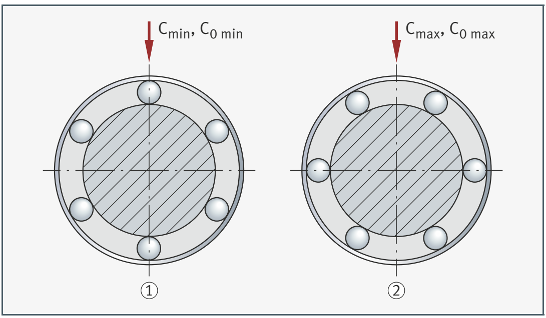


Figure 154
Load carrying capacity as a function of the orientation of the ball rows

C_{min}, $C_{0\ min}$ = lowest basic dynamic or static load rating
C_{max}, $C_{0\ max}$ = highest basic dynamic or static load rating

1 Apex orientation
2 Symmetrical orientation

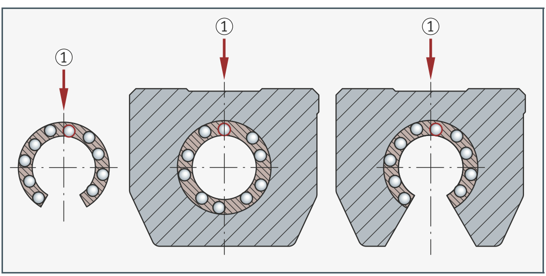


Figure 155
Main load direction for bearings and units

1 Main load direction

Lubrication

Open linear ball bearings are supplied with a wet or dry preservative and can be lubricated using either grease or oil. The oil-based preservative is compatible and miscible with lubricants with a mineral oil base, which means that it is not generally necessary to wash out the bearings before mounting. Bearings with a dry preservative must be greased or oiled immediately after they are removed from the packaging (due to the risk of corrosion).

Grease lubrication

Grease lubrication should be used in preference to oil lubrication, since the grease adheres to the inside of the bearing and thus prevents the ingress of contamination. This sealing effect protects the rolling elements against corrosion.

In addition, the design work involved in providing grease lubrication is less than that for providing oil, since design of the sealing arrangement is less demanding.

Composition of suitable greases

The greases for linear ball bearings have the following composition:

- lithium or lithium complex soap
- base oil: mineral oil or poly-alpha-olefin (PAO)
- special anti-wear additives for loads $C/P < 8$, indicated by “P” in the DIN designation KP2K–30
- consistency to NLGI grade 2 in accordance with DIN 51818

Initial greasing and operating life

Based on experience, the operating life is achieved with the initial greasing when bearings are operated in normal ambient conditions ($C/P > 10$), at room temperature and with $v \leqq 0,6 \cdot v_{max}$. If it is not possible to achieve these conditions, relubrication must be carried out.

Sealed linear ball bearings are already adequately greased when delivered and are therefore maintenance-free in many applications.

Influence of the adjacent construction on the running accuracy of the guidance system

The good running characteristics of shaft guidance systems are dependent not only on the bearings. They are also influenced to a large extent by the geometrical and positional tolerances of the adjacent construction as well as the mounting of the guidance systems. The higher the accuracy to which the adjacent construction is produced and assembled, the better the running characteristics.

Applications

Examples of the design of bearing arrangements
The design and layout of bearing arrangements for widely varying areas of engineering requires fundamental and extensive knowledge and experience of the application of the corresponding bearings. This section describes applications as examples to show designers and students how bearing arrangement tasks can be realised.

Bearing selection
The issues to be clarified in preparing a design solution for a bearing arrangement are derived from the type of machine and its working conditions. The designers must therefore clarify first the tasks for the bearings as a result of the operational function of the machine, plant, etc. and the operating conditions. They must therefore know the direction and magnitude of the forces to be supported. This will then give reference points to indicate the type of bearing.

Dimensioning
The correct bearing size is generally defined by the calculation of the fatigue life L_{10h}. Initially, the focus here is on bearing arrangements that have already proved successful in practice.
When dimensioning linear guidance systems, a sufficiently high static load safety factor must be taken into account, which is dependent on the application.

Internal clearance and running accuracy
The design of the bearing in relation to radial or axial internal clearance and the running accuracy is dependent not only on the operating conditions but also on how closely the bearing arrangement should guide the rotating part.

Accuracy and preload
With linear guidance systems, the accuracy class must be selected with regard to the running accuracy and the tolerances of the external dimensions. By choosing the appropriate preload, the rigidity required for the application can be achieved.

Design of the adjacent construction
Once the bearing type, bearing size and bearing design have been defined, the next task is the design of the bearing position. This includes the design of the adjacent parts and the selection of the fits.

Lubrication, sealing
The operating conditions determine the type of lubrication and sealing. Since a high percentage of bearing failures can be attributed to defects in lubrication and sealing, these components must be selected with particular care. The use of lubrication monitoring systems can significantly extend the operating life of the bearing.

Mounting and dismounting
When considering all the aspects described above, attention must also be paid to the correct mounting and dismounting of the bearing.

Special requirements There are cases in which certain requirements acquire particular importance and therefore move to being the starting point for the considerations. In this way, a decisive influence from the very beginning may be exerted by high speeds or temperatures, unusual load conditions or accuracy requirements, etc.
In order to fulfil these special requirements, Schaeffler offers an extensive range of variants and accessories, enabling the selection of special coatings, materials, sealing concepts, lubricants and additional elements, for example.

Electronic selection system As a means of support to designers, Schaeffler offers a bearing selection system in the form of the software program **medias**, see section **medias** – knowledge database, electronic selection and information system, Page 744, and: *https://medias.schaeffler.com*.

Bearinx calculation software The Schaeffler calculation software ➔ Bearinx facilitates the detailed analysis and design of rolling bearing arrangements, see section Bearinx calculation software from Schaeffler, Page 739.

Monitoring The appropriate condition monitoring system regularly checks parameters such as movements, loads and lubricant quality during operation and detects incipient changes or damage to the bearings at an early stage, thus allowing maintenance to be carried out at the optimum time.

Examples describing typical bearing arrangement issues The following examples are taken from various areas of application. The selection was made in such a way that typical issues relating to bearing arrangements can be discussed. The examples are not provided for the purpose of giving designers specific advice for their own areas of work beyond these issues; they are intended rather to convey knowledge of bearing technology in numerous areas, in order that designers can draw on these to develop suggestions for their own work.

Equations for calculation The equations for calculating the rating life of rotational rolling bearings are given in the section Dimensioning – load carrying capacity and life, starting Page 571.

The rating life equations for translational rolling bearings are given in the section Guiding elements in a translational direction – linear rolling element guidance systems, starting Page 684.

Attention The applications provide example solutions only. Their content is exclusively for the purposes of information and cannot replace technical advice on the application of bearings in individual cases. In practice, the specifications of the actual application and the information provided by bearing manufacturers must always be observed.

The publisher assumes no liability for compliance between the content and any legal regulations.

Bearing arrangement for the rotor shaft in a threephase motor

Electric motors transform electrical energy into mechanical energy. They convert the force that is exerted by a magnetic field on the wire in a coil carrying current into rotary motion.

The quality of the motor is assessed to a significant extent on the basis of its quiet running in operation. For electrical machinery, the noise limit values are defined in VDE 0530 and the maximum permissible mechanical vibrations are specified in DIN ISO 2373.

Noise and vibration are influenced principally by the rolling bearings fitted. In addition to the geometrical accuracy of the rolling bearing raceways once mounted, the radial internal clearance of the bearings has a decisive effect on the running noise. The best results are achieved with bearing arrangements that are almost clearance-free when warm from operation. Due to the tolerances of the joined parts, however, this is only possible with considerable effort. Such bearing arrangements are therefore frequently fitted with an axial spring element to achieve clearance-free adjustment. This is situated between the housing cover and the outer ring of the non-locating bearing.

Application data

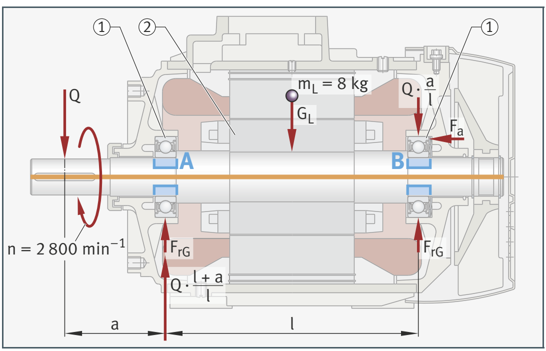
The technical data of the motor are as follows:

- drive power of the belt drive 3 kW
- mass of the rotor $m_L = 8$ kg
- nominal speed $n = 2\,800$ min^{-1}
- size 100L
- surface cooling in accordance with 42673, page 1, type B3, protection class IP44, insulation class F
- requisite rating life 20 000 h

Figure 156
Load diagram and bearing arrangement

A, B = bearing positions

① Deep groove ball bearing 6206-2Z
② Rotor



Bearing selection

The bearing arrangement should be simple, economical, maintenance-free and quiet. These requirements are best fulfilled by deep groove ball bearings. In DIN 42673, size 100L is defined as having a shaft end diameter of 28 mm. A bore diameter of 30 mm is thus specified.

For the bearing arrangement, a bearing of series 62 is suitable for both bearing positions (bearing position A and B). These bearings guide the rotor shaft on the output side and ventilation side. The spring element on the ventilation side (bearing position B) gives clearance-free adjustment of the bearing arrangement and also provides the axial counter-guidance of the rotor shaft. The clearance-free adjustment of the bearings prevents the internal clearance from having a deleterious effect on the noise behaviour.

Dimensioning

Further calculation of the bearing arrangement is carried out in a slightly different way from normal practice. The magnitude of the load on the shaft end is not known to the manufacturer of the motors, hence the permissible radial load is indicated in the manufacturer's catalogues. In order to determine the radial load carrying capacity, the deep groove ball bearing on the output side is considered. The calculation is based on a requisite rating life L_{10h} of 20 000 hours. The rotor mass, the unilateral magnetic attraction and the imbalance must also be taken into consideration.

Since the latter two criteria are not known, the rotor mass is multiplied by a safety factor of 1,5. As a result, a permissible radial load of 1 kN is calculated for the centre of the shaft end.

Since the operating load is less than the permissible load in most cases, this gives an achievable rating life of more than 20 000 hours: $L_{10h} = (16\,666/n) \cdot (C/P)^3$. The operating life is thus normally determined by the grease operating life and not by the material fatigue.

Machining tolerances

The inner rings are subjected to circumferential load. They therefore have a tight fit on the shaft seats machined to k5. The outer rings are fitted in end cap bores machined to H6. The outer ring on the non-locating bearing side (bearing position B) can therefore be axially displaced in the bore and compensate thermal expansion under spring preload.

Internal clearance

Since the inner ring has a tight fit on the shaft, the radial clearance is reduced. Due to heating of the rotor, the inner ring is at a higher temperature during operation than the outer ring. This also gives a reduction in the radial internal clearance. In order to prevent distortion of the bearings, the radial internal clearance C3 is selected.

Lubrication The grease filling as supplied is sufficient for the entire operating life of the bearings. Since increased temperatures must be taken into consideration as a result of the insulation class F, a high temperature grease is used.

Sealing In small to medium-sized electric motors, bearings with sealing shields (non-contact seals) on both sides have proved effective. The sealing shields prevent the escape of grease and also protect the rolling element system from foreign bodies in the motor body. In order to prevent the ingress of dust and moisture, the shaft opening on the drive side is designed as an extended gap and covered by a protective cap. This fulfils the requirements of protection class IP44.

Mounting and dismounting The thermal method using a suitable induction heating device, for example from the HEATER series, is recommended for the mounting of bearings. Smaller bearings can be mounted by mechanical means using a mounting sleeve set such as the IMPACT-33.
In dismounting, the shaft is driven out and the bearing is removed at the opposite bearing position using a mechanical or hydraulically supported extractor.

Monitoring The use of a vibration-based condition monitoring system (e.g. OPTIME) allows incipient damage to the bearings to be detected at an early stage. The use of one sensor per bearing position is recommended, positioned in the respective load zone of the bearing arrangement.

Bearing arrangement for the main spindle in a CNC lathe

The heart of a machine tool is the main or work spindle and its bearing arrangement. The quality of the main spindle bearing arrangement is measured in terms of the cutting volume and machining precision.

The only bearings used as main spindle bearings are rolling bearings with increased accuracy; these are mainly angular contact ball bearings or spindle bearings (radial angular contact ball bearings with contact angles of 15° and 25°), double direction axial angular contact ball bearings, radial and axial cylindrical roller bearings and occasionally tapered roller bearings.

Application data

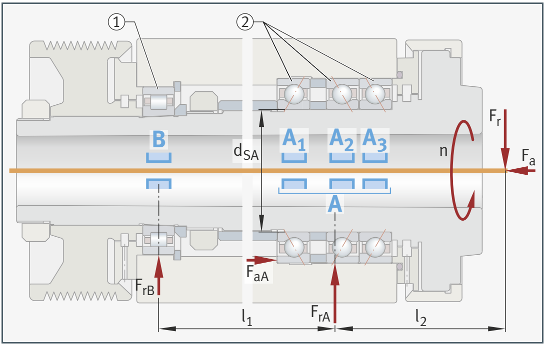
The technical data of the application are as follows:

- drive power P = 25 kW
- maximum spindle speed n = 5 000 min^{-1}

Figure 157
Load diagram and bearing arrangement

A, B = bearing positions

① Cylindrical roller bearing N1016-K-M1-SP
② Spindle bearing set in tandem O arrangement B7018E-T-P4S-TB-TL



Bearing selection

Depending on the requisite performance data of the machine tool, the spindle bearing arrangement is designed with ball or roller bearings in accordance with the criteria of rigidity, friction behaviour, accuracy, speed capacity, lubrication and sealing.

The bearings must give precise radial and axial guidance of the spindle and must exhibit high rigidity. This is achieved by using the largest possible shaft diameter and a corresponding bearing arrangement, see Figure 157. The bearings are additionally preloaded to increase the rigidity and have increased accuracy.

Dimensioning

The bearing size is determined principally by the requisite rigidity of the spindle. While the fatigue life is included as a factor in dimensioning, it plays only a subordinate role in practice.

The decisive factors for the operating duration of the bearings are the Hertzian pressure p_0 and the grease operating life t_{fG}. For deep groove ball bearings with rolling elements made from the rolling bearing steel 100Cr6, the fatigue strength for example at p_0 is $\leqq$ 2 000 N/mm^2.

Main spindle bearings generally fail not as a result of material fatigue but of wear. The objective of the design is therefore to give rolling bearings that are fatigue-resistant at very high cleanliness and with a hydrodynamic lubricant film capable of supporting load at the contact points of the rolling contact parts (wear-free running).

Fatigue resistance can be ensured with a ratio $S_0^* = (C_0/P_0^*) \geqq 8$. P_0^* is calculated from the dynamic load forces in accordance with the equation for the equivalent static load: $P_0 = F_{0r}$ at $F_{0a}/F_{0r} \leqq 1{,}3$. Since the ratio is $F_a/F_r \leqq 1{,}3$, P_0^* is assigned the value for F_r. Calculation shows that the bearings are fatigue-resistant under the stated operating conditions ($S_0^* \geqq 8$).

Design of the adjacent construction, machining tolerances

The table gives the machining tolerances of the bearing seats for the spindle bearings and cylindrical roller bearings in this application.

Bearing	Seat	Diameter tolerance	Cylindricity tolerance (DIN EN ISO 1101) µm	Total axial run-out tolerance of abutment shoulder µm
Spindle bearing	Shaft	+5/−5 µm	1,5	2,5
	Housing	−4/+8 µm	3,5	5
Cylindrical roller bearing	Shaft, tapered	Taper 1:12	1,5	2,5
	Housing	−15/+3 µm	3,5	5

Bearing arrangement, preload

The work side is fitted with a locating bearing comprising a spindle bearing set in a tandem O arrangement with slight preload. This preload fulfils the normal requirements.

A single row cylindrical roller bearing is fitted as a non-locating bearing on the drive side. Due to its conical inner ring bore, the bearing is set almost clearance-free when it is pressed axially onto the spindle.

The bearing combination and arrangement ensure the high speeds and cutting performance values required.

Lubrication The spindle bearings and cylindrical roller bearings are greased for life with a high quality rolling bearing grease. The bearing interiors are filled with grease to approx. 35% in the case of the spindle bearings and approx. 20% in the case of the cylindrical roller bearings. Since the fatigue strength is ensured, the grease operating life t_{fG} is the decisive factor for the operating duration of the bearings.

Sealing The bearings are protected against contamination by a labyrinth with defined narrow radial gaps.

Mounting and dismounting The spindle bearing assembly is mounted in accordance with the thermal method using an induction heating device. Heating devices from the HEATER series or devices using medium frequency technology are suitable. A bearing-specific enveloping circle gauge, the appropriate hydraulic nut (HYDNUT) and a suitable pressure generator are needed to mount the cylindrical roller bearing.
Depending on the design, mechanical or hydraulically supported dismounting equipment can be used for dismounting.

Monitoring The use of carefully routed wired acceleration sensors in combination with an online condition monitoring system such as Schaeffler ProLink is recommended in order to monitor the main spindle bearing arrangement for damage. It is vital that sensor cables are routed securely to avoid potential interference or damage. The deployed sensors should also be resistant to the cooling lubricant used in order to ensure reliable and long-term functionality.

Bearing arrangement for radial support rollers in a rotary kiln

Rotary kilns are manufactured in versions with direct and indirect heating and are used for process plant applications. Materials transport is carried out – in a longitudinal direction through the kiln with rotation of the drum – from the inlet side to the outlet side. The drum body is therefore slightly inclined in the longitudinal direction. The feed materials (such as solids, dusts, slurries) may vary widely in consistency and granularity.

Two opposing radial support rollers form a support angle φ and are incorporated in one station. Depending on the length of the drum body, support is provided using two or more stations. At each station, a support ring is arranged around the drum body that rolls on the support rollers as the drum rotates.

For axial guidance of the drum and support of the axial loads, the normal arrangement comprises two axial support rollers on one support ring.

Application data

Rotary kiln:

- total weight (drum plus charge) $G = 5\,100$ kN
- number of stations $Z = 2$
- support angle of station $\varphi = 60°$
- inclination angle of drum $\beta = 2°$
- dimensions of support ring $D \times b = 6\,600$ mm $\times$ 600 mm

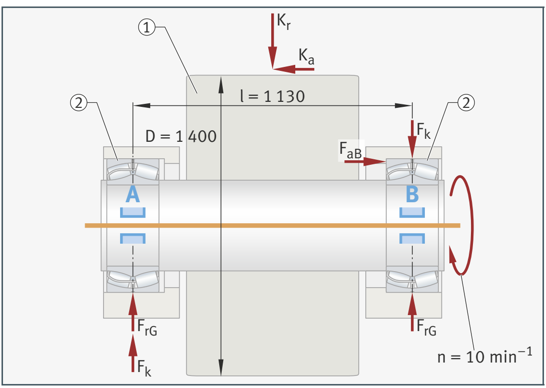
Radial support rollers:

- dimensions of radial support rollers $D \times b = 1\,400$ mm $\times$ 630 mm
- speed of radial support rollers $n = 10$ min^{-1}
- diameter of bearing seat for bearings in radial support rollers $d = 320$ mm
- requisite rating life 50 000 h to 110 000 h

Figure 158
Load diagram and bearing arrangement

A, B = bearing positions

① Radial support roller
② Spherical roller bearing 24164-E1



Bearing selection

For the bearing arrangement of the radial support rollers, spherical roller bearings 24164-E1 are selected, whose basic rating life (load carrying capacity) must be checked as a function of the bearing diameter. Spherical roller bearings are selected since, in this application, shaft deflections can occur and misalignments due to the adjacent construction must be anticipated. The bearings selected are highly suitable for compensation of these influences. In addition, they can support combined loads, since not only are high radial loads present (at low speed) but axial loads can also occur due to displacement of the kiln.

The spherical roller bearing series 241 has a high radial and axial load carrying capacity. Shocks and vibrations are effectively compensated by the double row line contact in conjunction with a suitable grease. The bearings are catalogue products and thus available rapidly worldwide.

Dimensioning

The strength specifications for the axis determine the shaft and journal diameters (and therefore the bearing sizes) for the bearings. However, the requisite rating life L_{10h} of the rolling bearings must be checked for the specified diameter d.

The bearing arrangement of the radial support rollers requires a basic rating life of 50 000 h to 110 000 h. After calculation of the radial, axial and tilting moment load F_r, F_a and F_k and the equivalent dynamic bearing load P, the rating life equation is used to check whether the spherical roller bearings defined by the strength specification for the shaft are sufficiently well dimensioned. The rating life is calculated: $L_{10h} = (16\,666/n) \cdot (C/P)^{10/3}$ and compared with the above value. With a value of 144 900 h, the bearings significantly exceed the requirement. This is due to the specified shaft diameter.

Design of the adjacent construction, machining tolerances

The bearing arrangement used is a floating bearing arrangement. The outer rings undergo point load, so the housing bores can have a loose fit (machining tolerance H7, clearance fit). Due to the spacing between the housing cover and the bearing outer ring, the outer rings of bearing A and B are not in contact with each other. The bearing rings can therefore be axially displaced by 2 mm to 3 mm and are abutted on the relevant inner shoulders of the housings. The inner rings undergo circumferential load. They therefore have a tight fit (interference fit) on the axis machined to n6 and are clamped in place using caps. The housings for the bearings are made from flake graphite cast iron.

Lubrication The bearings in the radial support rollers are subjected to heavy load at low speeds. They are therefore lubricated using greases of high base oil viscosity and containing EP additives. Relubrication can be performed manually at regular intervals, however, the use of an automatic relubrication system is advantageous as it provides a constant supply of fresh lubricant, extends the life of the bearing and minimises maintenance outlay. The precise lubrication intervals must be determined by lubrication interval calculation.

Sealing The bearing arrangement is sealed by means of felt rings, in front of which are labyrinths with a relubrication facility.

Mounting and dismounting Mounting is carried out in accordance with the thermal method using an induction heating device, for example from the HEATER series. Hydraulically assisted extraction devices or the hydraulic method are used for dismounting, depending on the application design. The hydraulic method requires oil holes in the shaft.

Monitoring The use of wired acceleration sensors combined with an online condition monitoring system is recommended for monitoring radial support rollers in a rotary kiln. Due to the low speeds involved, the measurement technology used should offer high signal resolution and sensitivity.
In addition, the sensors used should exhibit a high degree of resistance to the ambient conditions typically present in a rotary kiln, such as high temperatures, vibrations and contamination.

Bearing arrangement for connecting rods (crank pins) in piston compressors

Piston compressors operate on the displacement principle. Gases such as air or nitrogen can be used as the compression medium. The gas to be compressed is enclosed in a chamber where it is then compressed. Compression is carried out by means of pistons lubricated by oil or grease.

Piston pins supported by means of bearings connect the pistons to the connecting rods. The latter are supported on the crankshaft by means of the crank pin bearing arrangement. The connecting rods support the forces occurring and transmit these to the crankshaft in order to build up the torque. The crankshaft converts the oscillating linear motion of the piston into rotary motion with the aid of connecting rods and thus transmits the motor torque generated from the piston force to the drive.

Application data

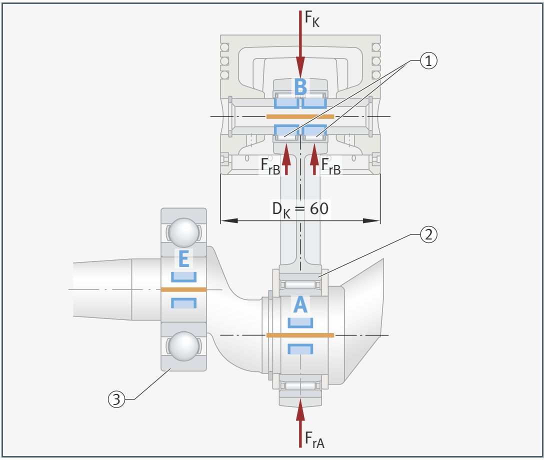
The technical data of the crank pin bearing arrangement are as follows:

- offset of the crank pins = 180°
- piston diameter D_K = 60 mm
- nominal pressure p = 15 bar
- operating speed (nominal speed) n = 660 min^{-1}
- requisite rating life of the bearings 5 000 h to 35 000 h

Figure 159
Load diagram and bearing arrangement

A, B = bearing positions

1 Drawn cup needle roller bearings with open ends
2 Needle roller bearing NAO 30×47×16
3 Deep groove ball bearing



Bearing selection

Connecting rod bearing arrangements are, due to their unusual force, motion and lubrication conditions, some of the most challenging bearing positions in automotive engineering and machine building. Due to the inertia forces, the mass of the connecting rod with the bearings and the piston mass must be as small as possible. The design envelope for the bearings is thus very small. For this reason, and due to the non-uniform rotary and swivel motion, crank pin and piston pin bearing arrangements in piston compressors are realised predominantly by means of needle roller bearings, in which the rolling elements run directly on the hardened crank pin or piston pin and in the hardened connecting rod eyes. In the piston compressor described here, the crankshaft and connecting rod are not necessarily hardened. A direct bearing arrangement is therefore not possible for the crank pin bearing arrangement. For this reason, ribless needle roller bearings NAO $30 \times 47 \times 16$ with an inner ring are selected.

The connecting rod is to be guided axially on the bearing (crank end guidance). The outer ring is somewhat narrower than the inner ring. The outer ring, needle roller and cage assembly and the connecting rod eye run between hardened axial contact washers.

Dimensioning

For the compressor, the size and rating life of the crank pin bearings A should be checked. First, the nominal pressure p is used to determine the maximum piston force F_K. The radial load F_{rA} (bearing A) corresponds to the maximum piston force F_K ($F_{rA} = F_K$). The piston force F_K can vary with time.

The load pattern varying with time can be summarised in its effect on the dynamic bearing load for bearing A in the equivalent dynamic load P. Since no axial load is present, the following applies to bearing A: $P = 0{,}55 \cdot F_{rA}$.

In the low-speed piston compressor described here, the inertia forces can be disregarded; this means that calculation of the bearing load must only take account of the gas forces due to the equivalent dynamic load P. The needle roller bearing is thus subjected to a load P = 2,33 kN.

The calculation results are used to check whether the needle roller bearing will achieve the requisite rating life. Checking is carried out by comparing the requisite rating life with the basic rating life: $L_{10h} = (16\,666/n) \cdot (C/P)^{10/3}$. The bearing size is suitable for the application, since the calculation gives 98 697 h.

Machining tolerances

The crank pin bearings are subjected to high levels of shocks and non-uniform loads. The inner and outer rings therefore have a tight fit. The bores in the connecting rod have a fit in accordance with N6, while the bearing seats on the crank pin have a fit in accordance with k5.

Internal clearance

A tight fit on the shaft and in the housing will reduce the radial internal clearance of the bearings. The needle roller bearings therefore have the internal clearance C3.

Lubrication The open bearings are lubricated by the motor lubrication system. In order that the spray oil enters the bearings and reaches the lateral connecting rod surfaces, the axial contact washers have bowstring-shaped sections.

Sealing The crank pin bearings are sealed from the outside by the compressor housing.

Mounting and dismounting The thermal method using a suitable induction heating device, for example from the HEATER series, is recommended for the mounting of bearings. Smaller bearings can be mounted by mechanical means using a mounting sleeve set such as the IMPACT-33.
Dismounting is carried out using a mechanical or a hydraulically supported extractor.

Monitoring A continuous condition monitoring system with vibration and temperature sensors (such as Schaeffler ProLink CMS) regularly monitors connecting rod movements, crank pin load and lubricant quality.

The recorded vibration and temperature data are used to detect discrepancies at an early stage and prevent possible damage. Automated warnings enable timely maintenance and reduce unplanned downtime to ensure compressor reliability.

Bearing arrangement for paper guide rolls in web offset printing machines

In web offset printing machines, the paper webs are fed – in contrast to the sheet offset process – through a system of rotating cylinders. The blanket cylinder transfers the image to the paper. This process is stabilised by an impression cylinder that is pressed against the paper and the blanket cylinder. Modern web offset printing machines run at more than 40 000 revolutions of the cylinders per hour.

Paper guide rolls guide the paper web on its path through the machine. The bearing arrangement must have low frictional torque, since there is a risk of slippage, especially at a small wrap angle. In this case, the paper web would slide over the roll surface to a greater or lesser degree and could no longer drive the paper guide roll by friction. This can in turn impair the quality of the print results or even lead to tearing of the paper web.

In order to prevent contamination of the paper by lubricant, the bearing arrangement must be well sealed and maintenance-free. A printing machine contains numerous paper guide rolls. The bearing arrangement must therefore be easy to mount and economical.

Application data

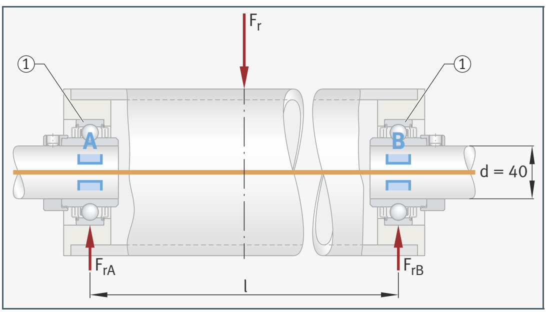
The technical data of the example are as follows:

- operating speed (nominal speed) $n = 2\,000\ \text{min}^{-1}$
- total radial load $F_r = 800$ N
- diameter of the shaft stubs $d = 40$ mm
- operating temperature $\vartheta = +120$ °C
- requisite rating life of the bearings 40 000 h

Figure 160
Load diagram and bearing arrangement

A, B = bearing positions

1 Radial insert ball bearing with locking collar E40-KLL



Bearing selection

The requirement for the smallest possible frictional torque of the bearing can be fulfilled by the selection of a ball bearing with a small grease quantity and the use of a free-running grease. Since the sealing arrangement of the bearing has a considerable influence on the bearing frictional torque, a bearing with non-contact seals is selected. With this information and the application data, preliminary selection of a ball bearing can be made. As a result, the radial insert ball bearing with locking collar E40-KLL and integrated gap/labyrinth seals was selected.

Dimensioning

The axial load can be disregarded, since the paper web generates no axial load and the inherent mass of the paper guide roll acts only in a radial direction. The radial load F_r comprises the inherent mass of the roll and the maximum web tension in downward guidance of the web. It acts equally on both bearings.

The equivalent dynamic load P is determined according to the same method as for deep groove ball bearings, since radial insert ball bearings are based on the same construction as deep groove ball bearings (series 60, 62 and 63). Since no axial load is present, P corresponds to the highest radial load F_r (F_{rA}, F_{rB}) for each bearing.

The values determined can now be used to calculate the basic rating life: $L_{10h} = (16\,666/n) \cdot (C/P)^3$. The result ($>$ 40 000 h) fulfils the requirement for basic rating life. The radial insert ball bearing is thus suitable for the bearing arrangement for the paper guide rolls.

Design of the adjacent construction, machining tolerances

The selected radial insert ball bearing places only slight requirements on the adjacent construction. Due to the circumferential load on the outer ring, this bearing ring must have a tight fit (transition fit). The bore in the cylinder is therefore produced in accordance with K7. For the shaft stubs, drawn shafts of the tolerance class h6 to h9 can be used.

On the locating bearing side, the inner ring is located on the shaft by means of the locking collar and secured by means of grub screws. The bearing mounted in this way now acts as a locating bearing. In order to support the thermal expansion of the cylinder, the opposing bearing must be mounted as a non-locating bearing. The locking collar is therefore loosened from the inner ring by the amount of the cylinder expansion and not clamped on the shaft but instead fixed in place only by means of the grub screws. In this way, the inner ring can be displaced on the shaft if there is thermal expansion of the cylinder.

Lubrication

The bearings are greased with a grease optimised for friction (free-running grease). The grease quantity is selected such that maintenance-free operation is ensured for the entire operating life of the bearings. The grease operating life is 40 000 h.

Sealing

The non-contact seal in the bearing causes no seal friction and thus supports the requirement for a low-friction bearing arrangement. It reliably prevents the ingress of contamination and foreign matter into the bearing and the escape of lubricant from the bearing.

Mounting and dismounting

The radial insert ball bearing gives very easy mounting and simple location. The radial insert ball bearing can be pushed onto the shaft manually. As a result, no special tools are required for mounting and dismounting.

Monitoring

The early detection of bearing damage can be achieved by using a vibration-based condition monitoring system. The monitoring system used and the acceleration systems should be able to cover a high frequency bandwidth (e.g. Schaeffler ProLink).

Linear bearing arrangement in the main axes of machine tools

Modern multi-axis machining centres offer a variety of machining options which allow workpieces to be manufactured to the highest precision. The individual axes must therefore exhibit a high degree of accuracy and rigidity, which also reflects the demands on the linear guidance systems installed within them.

There are also requirements governing the service life and robustness of the linear guidance system in terms of adaptation to the respective ambient conditions during machining. Such conditions include contamination with cooling lubricants, swarf or, in the case of dry machining, contact with abrasive material particles. The right lubrication is also of fundamental importance, as supply to the rolling contact must be ensured even under adverse conditions to avoid the risk of premature failure. The configuration of the main axes with their proximity to the machining point presents multiple challenges for the linear guidance systems.

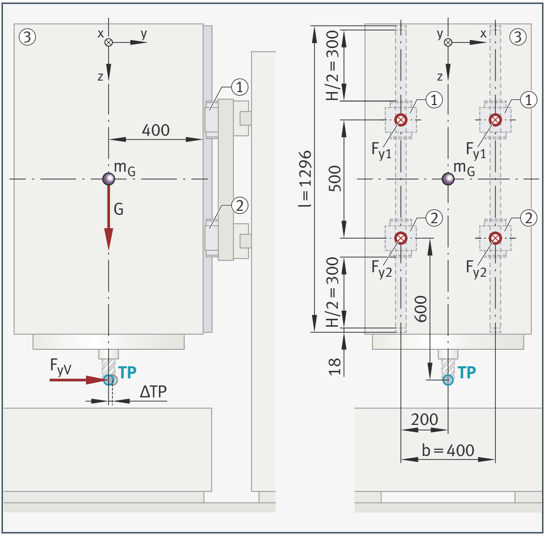
Application data

The following requirements apply to the z-axis of the machine tool:

- available installation space for 2 linear guidance systems: guideway spacing b = 400 mm, guideway length l = 1 296 mm
- required stroke length H = 600 mm
- carriage spacing (carriage centre) on each guideway 500 mm
- operating point position TP (in the centre position of the z-axis): centred between the guideways, 600 mm below the lower carriages
- max. machining force at operating point F_{yV} = 13 kN
- weight force of z-axis G = 15 kN
- centre of mass position centred between the carriages, distance of 400 mm from the screw mounting surface of the z-axis guideways
- resistance to material particles
- maximum permissible displacement of the operating point:
  - during finishing (machining force F_{yS} = 4 kN): $\Delta TP \leq 10\ \mu m$
  - at max. feed (machining force F_{yV} = 13 kN): $\Delta TP \leq 20\ \mu m$
- requisite rating life of the carriages > 10 000 km

Figure 161
Load diagram and bearing arrangement

① Upper carriage
② Lower carriage
③ Z-axis



Selection of the guidance system

In order to ensure a high load capacity and rigidity of the axis, linear recirculating roller bearing and guideway assemblies should be used in preference in the main axes of machine tools. Unlike balls, rollers have line contact and are therefore capable of transmitting higher forces. Linear recirculating roller bearing and guideway assemblies are preloaded to a higher degree than ball guidance systems which, in combination with line contact, creates an extremely rigid linear guidance system. Linear recirculating roller bearing and guideway assemblies also meet the requirement for a long operating life courtesy of their high basic load ratings, additionally offer extensive sealing variants and can also be connected to a central lubrication facility.

Dimensioning

The most heavily loaded carriage and the forces acting on it can be determined from the specified machining forces and the geometric arrangement of the carriages. The linear guidance systems used in machine tools often have a static load safety factor of approx. S_0 = 8 to 10, as this is the value at which the required rigidity and rating life is usually achieved. An approximate, initial determination of the size of the linear guidance system can therefore be carried out on this basis.

The loads on the individual carriages are calculated on the basis of the specified carriage arrangement and forces. In this application example, the carriages must support the following forces:

- upper carriages: F_{y1} = –13,8 kN in each case
- lower carriages: F_{y2} = 20,3 kN in each case

The static equivalent force acting on the most heavily loaded carriage is used in initial dimensioning of the guidance system. $P_0 = F_{max}$ applies, therefore, in this case P_0 = 20,3 kN. With a static load safety factor $S_0 = 8$, the guidance system should therefore have a basic static load rating of approx. 8 · 20,3 kN = 162,4 kN. As a result, the first guidance system to be selected is linear recirculating roller bearing and guideway assembly RUE45-F with a basic static load rating C_0 = 215 kN.

To determine the rating life, the acting forces are converted into an equivalent force P. For only one load case and one load direction, $P = F_{max}$ = 20,3 kN. Using the formula for calculating the basic rating life $L = (C/P)^{10/3} \cdot 100$, a rating life $L = (92\ \text{kN}/20{,}3\ \text{kN})^{10/3} \cdot 100$ = 15 400 km is obtained here for linear recirculating roller bearing and guideway assembly RUE45-F (C = 92 kN). However, as only the rating life at the maximum machining force was taken into account here, the actual rating life is significantly higher. The requisite rating life is thus observed.

The deflections δ of the carriages, which can be found in the catalogue on deflection curves, are taken into account when determining the displacement of operating point ΔTP. Standard preload class V3 is usually used with linear recirculating roller bearing and guideway assemblies, however, a higher preload can be selected if required.

In the application described, only the displacements caused by the machining forces are relevant, as the displacement caused by weight force is a constant and can therefore be compensated. For this reason, the carriage loads F were additionally calculated for the load cases with and without machining forces and the displacements compared:

Calculation parameter			**Load case**		
			1	**2**	**3**
Active forces (weight force and machining forces)			G	$G + F_{yS}$	$G + F_{yV}$
		kN	15	15 + 4	15 + 12
Bottom carriage (preload V3)	Load F	kN	6	10,4	20,3
	Deflection δ	μm	3	6	11
Top carriage (preload V3)	Load F	kN	-6	-8,4	-13,8
	Deflection δ	μm	-3	-5	-9
Calculated displacement of operating point ΔTP		μm	10	19	30
Result: Displacement of operating point compared with load case 1		μm	–	+9	+20

The requirements governing rigidity of the axis are therefore met. Only the deflection of the linear guidance system was calculated, the adjacent structure was assumed to be completely rigid.
The machine manufacturer can factor in deformation of the adjacent construction with the aid of an FEM calculation.

If a requirement criterion is not met, the analysis must be repeated with another linear recirculating roller bearing and guideway assembly. By the same token, overdimensioning should also be avoided. In such instances (e.g. S_0 › 10), a smaller size should be selected.

Accuracy When selecting the accuracy class for a monorail guidance system, not only the tolerances for height and for the distance dimension from the locating edge of the guideway to the locating edge of the carriage should be taken into account, but also the running accuracy of the carriage on the guideway. The higher accuracy classes G1 or G0 are generally used for the main axes of machine tools.

Demands on the adjacent construction Strict requirements are defined for the adjacent construction in order to be able to maintain the precision of the linear guidance system in the application. Force stresses between the carriages, which would have a negative effect on the displacement resistance and life of the axis are also avoided as a result.

Sealing The main axes come into contact with a wide variety of particles and media in the machining area of a machine tool, such as very fine swarf or cooling lubricant that splashes onto the guideways and carriages of the linear guidance systems. In order to select the appropriate sealing concept, it is necessary to know which method of shielding the customer intends to use for the linear guidance systems. If telescopic covers are planned for shielding the guidance systems, the standard wipers on the carriages will be sufficient. If no cover is envisaged for the guidance systems, additional wipers must be attached to the carriage.
In the application under consideration, additional wipers made from NBR are used.

Lubrication Schaeffler monorail guidance systems can be lubricated with grease, flowable grease or oil. The relubrication quantities and intervals must be calculated in accordance with the catalogue. The values apply under clean ambient conditions and must be adjusted in the event of heavy contamination.

Monitoring The lubrication and condition of the linear guidance system can also be monitored using the Schaeffler DuraSense system, which allows the lubrication status of each carriage to be determined individually and lubrication to be initiated where required. In addition, data analysis provides an early means of identifying whether the condition of the carriages is deteriorating, thus enabling the timely planning of maintenance activities and avoiding unplanned machine downtimes. These features render the Schaeffler DuraSense system indispensable in applications where lubrication plays a critical role.

Bearing arrangement for a wheelset in rail vehicles

Vehicles that run or are guided on one or more rails are known as rail vehicles (for example trains or trams). The rail and vehicle are closely matched to each other and are normally described as a wheel/rail system.

Application data

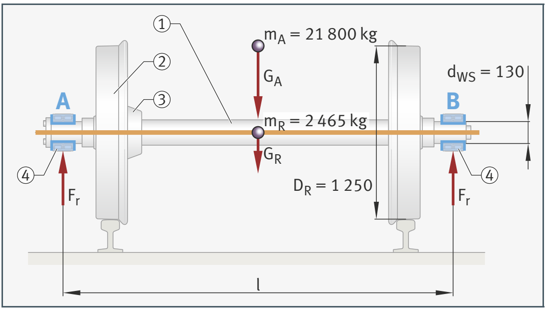
The technical data of the wheelset bearing arrangement are as follows:

- proportion of the vehicle mass acting on the wheelset bearing, $m_A = 21\,800$ kg
- mass of the wheelset $m_R = 2\,465$ kg
- wheel diameter (rolling diameter) $D_R = 1\,250$ mm
- diameter of the shaft journal $d_{WS} = 130$ mm
- number of bearings per axle $i_R = 2$
- maximum travel velocity v = 200 km/h
- ambient temperature $\vartheta = -50$ °C to +50 °C
- requisite rating life of the bearings in travel kilometres 5 000 000 km

Figure 162
Load diagram and bearing arrangement

A, B = bearing positions

① Wheelset shaft
② Wheel disc
③ Hub seat
④ Wheelset bearing unit F-801804.ZL-L055-M32AX



Bearing selection

Wheelset bearings are components with safety implications; the operational security of rail vehicles depends to a large extent on them.

Wheelset bearing units comprise a housing with integrated rolling bearings. Since the bearings are subjected to high loads and high operational security is required, the only bearings used are radial roller bearings (cylindrical and tapered roller bearings). The bearings transmit the forces from the vehicle or bogie frame to the wheelset and thus to the rail.

For the 6-axle locomotive with three bogies in this example, wheelset bearing units with cylindrical roller bearings, plastic cages, sheet metal caps and preset radial internal clearance are used. Cylindrical roller bearings are radial bearings but they can also support the axial forces present by means of the ribs. The greased and sealed units are ready-to-mount and simply require pressing onto the wheelset shaft.

Dimensioning

The guidelines from the international railways association UIC envisage shaft journals from d = 120 mm to d = 130 mm. The bearing size is to be checked for wheelset bearing units with cylindrical roller bearings d = 130 mm. The axle load is taken as the basis for determining the bearing size. This is the proportion of the vehicle mass that is transmitted per wheelset to the rails. The static load on the wheelset bearings is calculated from the axle load m_A reduced by the wheelset mass m_R.

In addition to the static load, the roller bearings must also support dynamic forces occurring during travel, for example when passing over points, crossings, etc. Additional, radial dynamic loads are taken into consideration by means of the safety factor f_z. This is now normally between 1,2 and 1,5.
The radial load F_r on a roller bearing is calculated by splitting the total load uniformly over the number of roller bearings i_R per axle.

In order to calculate the rating life L_{km}, the equivalent dynamic load P must be determined. The equivalent dynamic load P corresponds to the radial load F_r, since the factor f_a for the axial load is 1 (the axial forces have a not inconsiderable effect on the bearings).

The basic rating life in travel kilometres is determined from the equivalent dynamic load P, the basic dynamic load rating C and the wheel diameter D_R: $L_{km} = (C/P)^{10/3} \cdot D_R \cdot \pi$. The calculation gives a value of $5{,}44 \cdot 10^6$ km. Since the application requires a nominal operating life of 5 million kilometres, the wheelset bearing unit is adequately dimensioned.

Design of the adjacent construction, machining tolerances

The housing must, as a connecting part between the vehicle bogie frame and the wheelset, reliably transmit the forces present. The material for the housings is dependent essentially on the operating and application conditions. Vehicles for commuter transport are subjected to frequent acceleration and braking intervals. As a result, preference is given to the use of light metal housings. In other cases, the predominant choice is spheroidal graphite cast iron.

Since the inner rings of the roller bearings are subjected to circumferential load, they have a tight fit on the shaft journals. For the shaft diameter $>$ 105 mm, the tolerance p6 has proved successful. The outer rings are subjected to point load. The housing bore normally has the tolerance H7 or H6.

Lubrication The wheelset bearing units are supplied already greased. The lubricants used are lithium soap greases of consistency grade (NLGI grade) 2 or 3 with suitable additives. The grease filling is normally renewed at the time of general inspection of the vehicle. The good running characteristics of the plastic cages increase the grease operating life and thus the maintenance intervals.

Sealing In addition to contact seals, the use of non-contact seals has proved effective for higher velocities and speeds. An arrangement of several gaps in the form of a labyrinth is also possible. In many cases – such as this application – gap seals by means of sheet metal caps are implemented.

Mounting and dismounting The thermal method using a suitable induction heating device, for example from the HEATER series, is recommended for the mounting of bearings. Alternatively, the bearings can be mounted using a hydraulically assisted press-on unit (TOOL-RAILWAY tools).
The bearings are dismounted using a hydraulically assisted extraction unit (TOOL-RAILWAY tools).

Monitoring An implemented monitoring system records vibrations, temperature and wheel load distribution. The condition of the bearing arrangement, the wheelset temperature and the wheel loads are continuously monitored by the system to enable the early detection of wear, overheating and irregularities.

Automated warnings ensure timely maintenance, thus extending the life of the wheelset. Proactive condition monitoring minimises downtimes and ensures the reliable and efficient operation of rail vehicles

Digitalisation Axlebox bearings are assigned a data matrix code (DMC), which is applied by laser. This allows product-specific data on the respective bearing to be recorded and facilitates the exchange of information between component suppliers, vehicle manufacturers and operators.

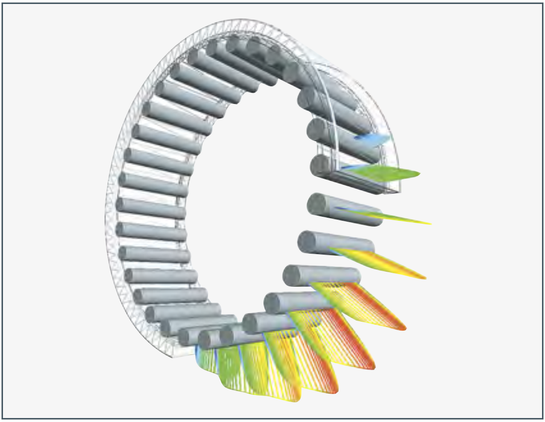
Data are exchanged in database form via the GS1 EPCIS interface standard. The data matrix code supports precise condition monitoring diagnosis in the rail vehicle, as the data recorded in the monitoring system can be assigned to the individual axlebox bearings.

Bearinx calculation software from Schaeffler

➔ Bearinx is one of the leading programs for the calculation of rolling bearings. It facilitates the detailed analysis of rolling bearing arrangements – from the individual bearing through complex shaft and linear guidance systems to complex transmissions. The complete calculation is carried out in a comprehensive calculation model. Even in the case of complex transmissions, the contact pressure on each individual rolling element is included in the calculation. ➔ Bearinx takes account of factors including:

- the non-linear elastic deflection behaviour of the bearings
- the elasticity of shafts and housings
- the influences of fit, temperature and speed on the operating clearance or preload of the bearings and on the contact angle
- the profiling of rollers and raceways as well as raceway osculations
- the displacement of contact angles as a function of load in ball bearings
- the actual contact pressure taking account of the misalignment and profiling of rolling elements
- the influence of lubrication conditions, contamination and actual contact pressure on the fatigue life

Figure 163
Precise detail: even the contact pressure on each individual rolling element is included in the calculation



The analysis of design variants is aided by the transparent documentation of results, the graphical representation of shaft reactions and the internal load distribution of bearings. Thanks to an online tutorial and a detailed help system, it is possible to utilise the full potential of ➔ Bearinx.

The uniform calculation of the fatigue life using computer-aided calculation methods corresponding to the state of the art is defined in ISO/TS 16281. This calculation method is of course also included in the online version.

Customer version – Bearinx-online Rotative

Commonly available calculation tools normally use heavily simplified calculation methods. In these cases, the misalignment of bearings as a result of shaft deflection and the different deflection behaviour of various bearing types is largely disregarded. In general, the internal load distribution of bearings – which is decisive for the fatigue life – is generally determined only by means of approximation methods.

It is possible using Bearinx-online Rotative to determine the actual load taking account of shaft deflection and the deflection behaviour of the rolling bearings. The internal load distribution of the bearings is of course also calculated precisely – to the level of the contact pressure taking account of the actual rolling element profile.

The algorithms in Bearinx-online Rotative are identical to those applied in Bearinx, which is used at Schaeffler. Bearinx-online Rotative facilitates the calculation of single axis shaft systems supported at several points. Input of data is supported by a user-oriented interface. 3D visualisation and 2D representation with dimensioning simplify data control.

The component assembly data and the rolling bearing geometries are made available by the integrated Schaeffler catalogue. The actual calculation is then performed by the powerful Schaeffler calculation servers. The input files created using Bearinx-online Rotative are compatible with Bearinx. As a result, further communication with Schaeffler advisory engineers is easier and duplicate work is avoided.

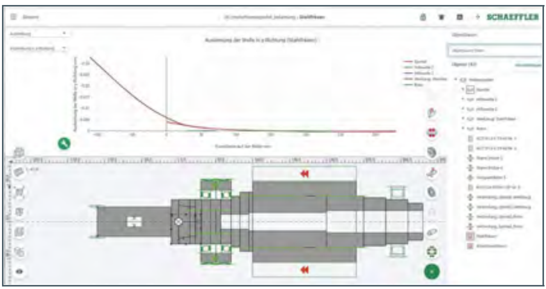


Figure 164
Bearinx-online Rotative, shaft reactions presented in graphical form

The software is not provided with the intention of transferring advice and calculation services from Schaeffler to customers. The reality is the opposite: with this arrangement, Schaeffler wishes to work even more closely with its customers. The objective is to make a suitable preliminary selection of rolling bearings in the early design phase, in order to reduce development times at the customer.

Bearinx-online Rotative– an overview

- calculation of bearing rigidity at the operating point taking account of all relevant influences
- graphical representation of shaft reactions (shaft deflection and shaft inclination)
- rigid and elastic adjustment of bearings in the relevant shaft system
- calculation of fatigue life in accordance with ISO/TS 16281
- simple modelling of shaft systems by means of integrated wizards

The terms of use of the software and the access to additional necessary services such as training and support are defined in a mutual contractual agreement.

Training on Bearinx-online Rotative is available on payment of a fee. In the case of technical colleges, the introduction of software usage is free of charge.

Information about the customer version and the possibility of applying for registration/usage can be found on the Schaeffler Internet portal at: *https://www.schaeffler.de/Calculation*.

Other modules

In addition to Bearinx-online, other specialised modules are also available from Schaeffler under the Easy series, which can be accessed free of charge:

- Bearinx-online Easy Linear
- Bearinx-online Easy LinearSystem
- Bearinx-online Easy Friction
- Bearinx-online Easy BallScrew
- Bearinx-online Easy RopeSheave
- Bearinx-online Easy EMachine
- Bearinx-online Easy Pump

Friction calculation with Easy Friction

The Easy Friction module can be used to determine the friction values of Schaeffler rolling bearings using a detailed method. The internal load distribution of the bearings and the contact pressures on the raceways and ribs together with the actual rolling element profile is of course taken into consideration. The calculation is based on a theory of friction calculation that employs physical algorithms and has been confirmed by means of extensive test values. The bearing rating life is calculated in accordance with ISO/TS 16281.

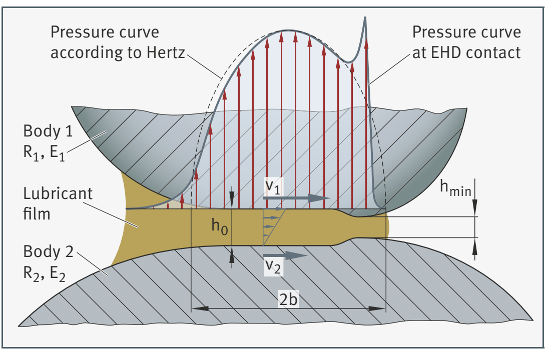
The algorithms in Bearinx-online Easy Friction take account in particular of the following parameters:

- losses in rolling and sliding contacts
- losses in the load-free zone
- splash losses
- seal friction

Since the module is seamlessly incorporated in the “parent software” Bearinx used at Schaeffler, other typical influencing factors are also taken into consideration:

- radial and axial load
- tilting of bearing rings
- lubricant (viscosity class)
- temperature
- precise internal bearing geometry
- bearing clearance
- profiling of the bearing components
- rib geometry

Figure 165
Elastohydrodynamic contact (EHD contact)



Rapid comparison of friction between various bearing arrangement concepts

Since bearings can be exchanged, different bearing arrangement concepts can be quickly and easily compared with each other. This makes it possible to achieve a bearing arrangement that is efficient and has optimised friction. All input data can be stored locally. As a result, modifications can be quickly made to an existing application case without the need for duplicated input of data. Furthermore, the stored file can be exchanged with the Schaeffler engineering service in order to achieve an optimum bearing design.

The most important results are displayed directly in a results window. In addition, the input data and the calculation results can be documented in a PDF file.

Bearing types suitable for calculation
Bearinx-online Easy Friction can be used for calculation of the following types:

- deep groove ball bearings
- angular contact ball bearings
- tapered roller bearings
- spherical roller bearings
- needle roller bearings
- cylindrical roller bearings

The calculation software is available only in an online format. Initial registration takes only a short period of time.

Information about the customer version and the possibility of applying for registration/usage can be found on the Schaeffler Internet portal at: *https://www.schaeffler.de/Calculation*.

medias – knowledge database, electronic selection and information system

In addition to the Schaeffler product catalogue, the **medias platform** provides information on all product and service solutions including a knowledge database and online training. Calculation and configuration tools assist with product selection, while numerous industrial solutions are shown that link products with possible applications.
medias is a comprehensive, reliable system to help you help yourself in answering practically all questions on bearing technology.

Digital product catalogue

Users of **medias** receive all product information from a single data source. The digital product catalogue contains e-commerce functions and has been expanded to include detailed technical information on products and service solutions in the form of a comprehensive knowledge database. CAD data relating to the products can be downloaded and imported into dedicated applications.

Engineering tools

medias integrates numerous tools that assist with the calculation and needs-based configuration of the desired products.
This saves time in selection of products and gives simpler handling.

These tools include:

- Bearing Selection Assistant
- Housing Selection Assistant
- Linear configurators
- medias interchange
- Bearinx calculation modules (see Page 739 to Page 743)
- Heating Manager
- Bearing Frequency Calculator
- Grease Selection Guide

Figure 166
medias
Bearing Selection Assistant

medias The three-stage user model configuration of **medias** offers a suitable account for all users. All advisory functions and all product- and sector-specific content from Schaeffler Industrial are available in the public domain of **medias**.

medias Plus With **medias** Plus, members receive access to exclusive additional content.

medias Business With **medias** Business, business customers can expect an expanded product catalogue and even more efficient ordering and communication processes.

The following link will take you to **medias**:
https://medias.schaeffler.com.

A

C

D

E

F

G

J

K

L

M

N

O

P

Q

R

U

V

W

X

Y

Schaeffler Technical Pocket Guide

Contents

- Units of measurement and scripts
- Mathematics
- Technical statistics
- Chemistry
- Physics
- Mechanics
- Mechanical vibrations and acoustics
- Hydraulics and pneumatics
- Mechatronics
- Strength of materials
- Engineering materials
- Product development
- Technical drawing guideline
- Tolerances and fits
- Design elements

In line with the philosophy of a structured reference work, the chapters are composed in a comprehensive yet concise form and are oriented towards direct, practical usability. With its compact and succinct manner of presentation, the Schaeffler Technical Pocket Guide thus successfully bridges the gap between a comprehensive textbook and purely tabular work.

Target groups

On the basis of its concept, the STT is aimed primarily at students attending technical universities, colleges and technical colleges as well as students at technical institutes specialising in mechanical engineering. Due to its didactic concept, the book is suitable both for teaching purposes and for independent study. In addition, the compendium is a proven and reliable companion for experienced designers in the fields of vehicle, mechanical, instrument and plant engineering.

Publisher

Schaeffler is a leading global manufacturer of rolling bearings and a renowned supplier to the automotive industry. This preeminent position is also evidenced by the large number of patent applications filed each year.

The STT app:
apps.schaeffler.com

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